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A SURVEY OF THE LITERATURE ON THE SIZE EFFECT ON MATERIAL STREN--ETC(U)
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A SURVEY OF THE LITERATURE ON THE SIZE EFFECT ON MATERIAL STRENGTH

APPLIED MATHEMATICS GROUP
ANALYSIS AND OPTIMIZATION BRANCH
STRUCTURAL MECHANICS DIVISION

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This technical report has been reviewed and is approved for publication.

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Nearly 500 years ago, Leonardo da Vinci observed that long wires are weaker than short wires of the same diameter. Over 100 years ago, Karmarsch found that the tensile strength of wires of a given length can be represented by an expression of the form $F = A + B/d$, where d is the diameter and A and B are positive constants. In the present century, studies of the size effect have proliferated. Weibull (1939) was the first to give a reasonably satisfactory explanation of the volume effect on material strength. Daniels (1945) studied the statistical theory of bundles of threads. The statistical → next page

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theory of extreme values ("weakest-link" theory) plays a very important role in studies of the size effect; competing theories include the energy theory and the technological theory. Summaries are given of relevant publications identified in the course of a literature survey on the size effect. Since this survey was motivated by concern about the reliability of large composite aircraft structures, which are now coming into use, special attention is given to the size effect on composite materials and structures. An attempt is made to summarize the present state of knowledge and to identify unsolved problems requiring further research.

PREFACE

This report summarizes the results of a literature survey performed under Project 7071, "Research in Applied Mathematics", which became a part of Project 2304 on 1 October 1976. Work was begun while the author was employed at the Aerospace Research Laboratories (AFSC) and continued at the Air Force Flight Dynamics Laboratory (AFSC), where the author has been employed since the disestablishment of ARL on 30 June 1975.

This literature survey was requested in April 1972 by Dr. Stephen W. Tsai, then Chief Scientist of the Air Force Materials Laboratory. It was motivated by concern about the reliability of large composite aircraft structures then beginning to come into use, and it is therefore applicable to the composites program of the Air Force Flight Dynamics Laboratory, which was begun as a joint AFML/AFFDL venture.

The author began this work with the somewhat naive idea that the literature on the size effect on material strength is merely a subset of the literature on order statistcs, on which he was then (and still is) engaged in preparing a chronological annotated bibliography. He soon discovered, however, that the "weakest-link" theory, based on the statistical theory of extreme values, is only one of several theories that have been advanced to explain the various size effects that have been observed in static tests and fatigue tests of materials and structures. Interdisciplinary research is needed to synthesize these theories into a unified theory, and experimental tests of both large and small specimens of composites and other structural materials are needed for verification.

Nearly seven hundred publications relevant to the size effect are delineated in Sections I-IV. A summary and recommendations are given in Section V, followed by a list of the references and a supplementary bibliography on extreme-value theory. A rich source of information about relevant publications since 1947 is Applied Mechanics Reviews (AMR). The volume and reference numbers of each publication identified from this source are given following the listing.

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SECTION I

PRE-WORLD WAR II PERIOD (1859-1939)

The phenomenon known as the size effect on material strength has been known for more than a century. Karmarsch (1859) represents the tensile strength of metal wires by an expression of the form F = A + B/d, where d is the diameter and A and B are constants. Trautwine (1872) notes the effect of the length of a specimen of constant cross section on its strength. On page 174 of his 1881 edition (the only one the present author has seen), Trautwine tabulates data on the endwise average ultimate crushing loads (in lbs./in.^{2}) for different kinds of wood. The data are the results of experiments by Eaton Hodgkinson on cylinders 1 inch in diameter and 2 inches high, with flat ends. Trautwine writes: "... It appears that SEASONED white and yellow pines, spruce, and ordinary oaks, which are the woods most employed in the United States for bridges, roofs, &c., crush with from 5000 to 7000 lbs per sq inch, WHEN IN SHORT BLOCKS (short in proportion to their diam,), the average being 6000 lbs. In practice, however, the crushing load rapidly becomes less AS THE PIECE BECOMES LONGER; and when the length is about 30 times the diam, 1200 lbs per sq inch will probably crush it; and the safe load will be reduced to from 150 to 350 lbs per sq inch ... ." He states, however, that the tensile strength does not change with the length of the piece. On the other hand, in discussing the average ultimate tensile or cohesive strength of metals, he states (page 179), on the basis of data from experiments by Lieut. W. S. Shock, U. S. N.: "A bar of the above No. 15, which broke at 60 tons per sq inch, when turned down for 14 ins of its length, broke with 79-1/2 tons per sq inch when turned down at one point only." His explanation of this phenomenon is incomplete, taking no account of the statistical theory of extreme values.

Chaplin (1880) presents theoretical arguments and experimental data to show that it is reasonable to believe that variations in the strength of a material will follow the normal (Gaussian) law. He then states that, according to the laws of probability, if the probability is p that the tensile strength of a bar of given length and cross section exceeds a certain value, the probability that the strength of all of n bars of the same dimensions exceeds that value is p^n. Equivalently, the probability that the strength of the

weakest of the n bars (or the strength of a single bar of the same cross section but n times as long) exceeds the specified value is p^n. Hence one should expect a decrease in the median (or mean) strength of a bar with an increase in length. Chaplin uses a table of areas under the normal probability curve to tabulate the expected decrease (in terms of the probable error) as a function of n, the factor by which the length is multiplied, for n=1(1)28. He also presents the same results graphically. He uses the results to estimate the loss of tensile strength of specimens of annealed wire of Japanese copper 4,8,12, and 16 inches long as compared with specimens 1 inch long. He finds good agreement between theoretical and observed strengths of the averages of several specimens of each length.

Chaplin (1882) presents, in somewhat greater detail, much the same material as in his earlier paper [Chaplin (1880)]. The following quotation (page 19) shows the dependence of his results on the "weakest link" concept: "A bar under tensile stress yields and breaks at one point; it is more or less extended at all points, but it breaks at only one, and it is the strength at this point which determines the strength of the bar to resist tensile stress. Suppose now that pieces a, a_1, a_2, etc., have been cut from a bar A, and tested in the usual way. The average strength of the pieces a, a_1, a_2, etc., will necessarily be greater than the strength of the weakest piece; or, as it is the weakest piece which determines the strength of A, the average tensile strength of the pieces a, a_1, a_2, etc. will be greater than the strength of A. Or <u>long bars are on the average weaker to resist tensile stress than short ones of the same material and cross-section</u>." A quantitative relation between the average strength of a bar of given length and one n times as long is worked out, assuming normality. If the former is S, the latter is S-ap, where $p=.6745\sigma \doteq .6745s$ and $a=\Phi^{-1}[\sqrt[n]{1/2}]/.6745$, p being the probable error, σ the population standard deviation, and s the usual estimate of σ from the sample, with $\Phi(x)= \int_{-\infty}^{x}\phi(x)dx$ and $\phi(x)= e^{-x^2/2}/\sqrt{2\pi}$. Values of a are tabulated for n=1(1)30(5)60(10)100(50)200(100)1000.

Baumeister (1883) finds that the smaller the diameter of an iron wire, the greater its tensile strength (per unit cross-sectional area). Auerbach (1891) reports the results of experiments which tend to show that the force on a metal sphere of radius ρ, normal to a glass plate, required to crack the plate is proportional to ρ, not to ρ^2. He also reports values of the hardness

H of glass spheres which are not independent of the radius, but are of the form $H = a + b/\rho$, where a and b are constants. He draws an analogy between this relation and the results of Baumeister (1883) [see also Karmarsch (1859)] concerning the tensile strength of wires. Voigt (1893) reports experimental results which tend to show that the torsional strength of rock salt prisms is greater for prisms of small (almost square) cross-section than for those of larger cross-section. Auerbach (1896) gives additional experimental results on the hardness of crystalline materials which tend to support the conclusions reached in his earlier paper [Auerbach (1891)]. Bach (1905) reports that strength decreases slightly with increase in diameter of specimens.

Slocum and Hancock (1906) discuss the effect of diamater and form of cross section, as well as that of length, on strength of materials. They write (pp. 13-15): "When an external force is first applied to a body the internal stress is distributed uniformly throughout the body and, consequently, all parts are equally deformed. When the stress surpasses the elastic limit this is no longer true, and certain portions of the body begin to manifest greater deformation than others. For instance, consider a bar of soft steel under tension. As the stress increases from zero to the elastic limit the bar gradually lengthens and its cross section diminishes, all parts being equally affected. When the stress passes beyond the elastic limit the cross section at some particular point of the bar, usually near the center, begins to diminish more rapidly than elsewhere. This contraction of section intensifies the unit stress at this point, and this in turn leads to a still greater reduction until finally rupture occurs. ... The contracted portion ... of the bar is called the *region of striction*. The contraction of the section at which rupture occurs is usually considerable; for soft steel its amount is from .4 to .6 of the original area of the bar. ... It has been found by experiment ... that the extent of the region of striction depends on the transverse dimensions of the bar and not on its length, the region of striction increasing in extent as the transverse dimensions of the bar increase. Consequently, if two bars are of equivalent cross section but of different lengths, the region of striction will be the same for both, and therefore the unit elongation [the ratio of the total elongation to the original length of the bar] will appear to be less for the long bar than

for the short one. On the other hand, if the two bars are of the same length, but one is thicker than the other, the region of striction will be longer for the thick bar, and therefore the unit elongation of this bar will appear to be greater than for the other. The form of cross section of test pieces subjected to tensile tests has also an important influence on their ultimate strength [the ratio of the maximum stress to the original sectional area]. If a sharp change in cross section occurs at any point, nonductile materials, such as cast iron, will break at this section under a smaller unit stress than they could otherwise carry. This is due to a greater intensity of stress at the section where the change in area occurs. For ductile materials, such as wrought iron and mild steel, the striction extends over a length six or eight times the width of the piece. Consequently, if the test piece has a form ... in which the length ... is less than six or eight times the width of the piece, the flow of the metal is restrained and therefore its ultimate strength is raised. This has an important bearing on the strength of riveted plates subject to tensile strain. It has been experimentally proved that such plates will stand a greater tension than plates of uniform cross section whose sectional area is equal to the sum of the sectional areas between the rivet holes. ... It is evident from what precedes ... that the unit elongation and the ultimate strength are not absolute quantities, but depend on the form of the test piece and the conditions of the test. ... The tensile strength of long rods is affected in a way different from any of the preceding. Since no material is perfectly homogeneous, the longer the rod the greater the chance that a flaw will occur in it somewhere. If, then, by numerous tests of short pieces, it has been determined how much a material lacks of being homogeneous, the strength of a rod of this material of any given length can be calculated by means of the theory of probabilities. Such a theory has been worked out by Professor Chaplin [(1880,1882)] and verified experimentally."

Moore (1918) summarizes much of the information available on the effect of size and shape of steel test specimens and presents new data. He concludes that "there is no great difference in results between tests made on round specimens and tests made on flat specimens whose width is not more than four times their thickness".

Griffith (1920) reports the results of theoretical and experimental studies of rupture in glass and other solids. The relevant results, based on the "weakest link" or "largest flaw" concept, are the following (pp. 179-181): "The general conclusion may be drawn that the weakness of isotropic solids, as ordinarily met with, is due to discontinuities, or flaws, as they may be more correctly called, whose ruling dimensions are large compared with molecular distances. The effective strength of technical materials might be increased 10 to 20 times at least if these flaws could be eliminated. ... Consideration of the consequences of the foregoing general deduction indicated that very small solids of given form, e.g., wires or fibres, might be expected to be stronger than large ones, as there must in such cases be some additional restriction on the size of the flaws. In the limit, in fact, a fibre consisting of a single line of molecules must possess the theoretical molecular tensile strength. In this connection it is, of course, well known that fine wires are stronger than thick ones, but the present view suggests that in sufficiently fine wires the effect should be enormously greater than is observed in ordinary cases. This conclusion has been verified experimentally for the glass used in the previous tests, strengths of the same order as the theoretical tenacity having been observed. Incidentally, information of interest has been obtained, somewhat unexpectedly, concerning the genesis of the flaws, and it has been found possible to prepare quite thick fibres in an unstable condition in which they have the theoretical strength. Fibres of glass, about 2 inches long and of various diameters, were prepared. ... For a few seconds after preparation, the strength of a properly treated fibre, whatever its diameter, was found to be extremely high. Tensile strengths ranging from 220,000 to 900,000 lbs. per sq. inch were observed in fibres up to about 0.02 inch diameter. ... The strength diminished, however, as time went on, until after the lapse of a few hours it reached a steady value whose magnitude depended on the diameter of the fibre. Similar phenomena have been observed with other kinds of glass, and also with fused silica." Griffith also reports the results of an experimental study of the relation between diameter and strength of glass fibres in the steady state. Within the limits of experimental error, his experimental values are given by the formula $F = 22{,}400[(4.4 + d)/(0.06 + d)]$ where F is in lbs. per sq. inch and d is in

thousandths of an inch. He notes that, within the range of diameters available to Karmarsch (1859), this expression differs little from F = 22,400 + 98,600 /d, which is of the form given by Karmarsch for metal wires.

Beare & Gordon (1921) conclude, on the basis of extensive experimental data, that "the width of the test-bar ... appears to have no appreciable influence upon the elastic strength of mild steel or rolled copper" and that "the tenacity, or commercial maximum stress, also appears to be but little affected".

Oberhoffer & Poensgen (1922) report the results of an experimental study of the hardness, the tensile strength and the bending strength of sand cast iron in bars 5,10,15,20,25 and 30 mm. in diameter. Unlike earlier authors, they find that both tensile strength and bending strength increase with increasing diameter up to 20-25 mm., after which they remain essentially constant. They attribute the discrepancy to the fact that the strength is also affected by the number and size of the graphite flakes present in sand castings. For test rods of small diameter, the ratio of the length of the graphite flakes to the diameter of the rod is large, and quasi-isotropy is attained only for rods of diameter 20-25 mm.

Griffith (1924) describes an experiment whose results tend to verify the "weakest link" or "largest flaw" theory which he advanced in his earlier paper [Griffith (1920)] to explain the much larger observed tensile strength (per unit cross-sectional area) of thin wires or fibers as compared with those of greater diameter. He reports (pp. 60-61): "A fairly hard glass containing much potash and alumina was chosen. By the usual mechanical tests, its Young's modulus was found to be 6,430 kg./sq. mm., and its tensile strength 18.3 kg./sq. mm. ... The glass was heated to a temperature somewhat below the temperature at which boiling commenced and was then drawn down to thin rods. Immediately after preparation, these rods were found to possess strengths as high as 630 kg./sq. mm. or about half the probable value of the theoretical strength [1300 kg./sq. mm. (found from cracked tube experiments described by Griffith)]. This strength diminished spontaneously with time, however, until after the lapse of a few hours a steady value was reached which depended on the diameter of the rod. Rods of 1 mm. dia. or over had a strength equal to the strength of the glass as ordinarily determined, while the permanent

strength of the finest fibres which could be made (about 1/300th. mm. dia.) was about 350 kg./sq. mm., or more than a quarter of the theoretical strength. By extrapolation, it was inferred that the strength of fibres of infinitesimal diameter would be about 1130 kg./sq. mm., which is the same for all practical purposes as the estimated breaking stress in the cracked tube experiments."

Peirce (1926) states: "It is a truism, of which the mathematical implications are of no little interest, that the strength of a chain is that of its weakest link. It is equally true that the strength of a test specimen is that of its weakest element of length, whether it be a metal rod, a thread of yarn, or a cotton hair. ... Tensile strength thus decreases with the length of the specimen in a way which is definitely calculated from the distribution of strength of short specimens." In his excellent summary of Peirce's paper, Epstein (1948a) writes (pp. 140-141): "Peirce was led to this problem in connection with some researches on the construction of reliable procedures for the testing of yarn. In order to accomplish this primary aim it was essential to study the effect of certain variables which one might expect on a priori grounds to be capable of greatly influencing the results unless properly controlled. Among the factors considered was the length of the specimen. It was observed, for example, that if specimens of some fixed length broke under normally distributed loads, then specimens of length $n\ell$, where n is an integer greater than one, will break under loads whose distribution is negatively skewed the larger the value of n. [Peirce determines the decrease in the mode and the median of the tensile strength corresponding to an n-fold increase in length, and estimates the corresponding decrease in the mean by making use of the fact that the difference between the mean and the mode is approximately three times that between the mean and the median.] Peirce wished to account for this phenomenon, which is incidentally a characteristic of breaking strength and thus was led to the formulation of his 'chain' model and to the equivalent statistical problem. It is interesting that the paper by Peirce was published at the time when the first basic statistical researches on the distribution of extreme values were carried out by the British statistician, Tippett [(1925)]. Peirce mentions explicitly in his paper that the theoretical work of Tippett is of importance in obtaining quantitative results concerning the distribution of breaking strengths." Epstein states that, as far as is

known to him, Peirce was the first worker to realize the close connection between the strength of a specimen and the distribution of smallest values; apparently neither Peirce nor Epstein was aware of the work of Chaplin (1880, 1882).

Templin (1926) reports experimental results which lead to the conclusion that the dimensions of a tension test specimen for sheet metals from 0.005 to 0.250 inch thick may vary between considerable limits without affecting the tensile strength.

Tucker (1927) reports the results of an investigation concerned mainly with the dispersion of the compressive strength of materials, based on experiments with columns of Portland cement mortar and concrete. He concludes that the coefficient of variation (standard deviation/mean) of the compressive strength varies inversely as the square root of the cross-sectional area of the specimen. He also includes a section on the variation in strength with length for columns. He presents the "weakest link" theory, according to which the strength of a column n units long is the strength of the weakest of the n individual units, so that the average strength decreases with the length. He points out, however, that this theory applies only when the unit length is chosen so that the failure of a specimen of unit length will occur in exactly the same manner as the failure of the column. Since failure of a concrete column normally occurs by skew in a plane inclined at approximately 35° to the column axis, the theory is invalidated if length/diameter $< K \doteq 1.5$.

Peterson (1930) describes a small fatigue testing machine weighing ten pounds and taking a test specimen fifty thousandths of an inch in diameter at the critical section. He compares the results of fatigue tests on this machine with those of similar tests on larger machines using larger specimens. For ordinary steels he finds, contrary to what one would expect from the theory developed by Griffith (1920), that there is no appreciable size effect up to two-inch diameter. For cast iron specimens, he finds, however, that there is a considerable size effect; in the discussion on Peterson's paper, R. L. Templin reports a similar result for cast aluminum.

Logan & Grodsky (1931) report the results of examinations of 1,300 specimens of ferrous pipe materials removed from 70 test locations during 1930 by the National Bureau of Standards after having been buried in various

soils for varying lengths of time. They state (p.5): "Differences in size interfere with the comparison of materials. When small areas are concerned, other things being equal, it is probable that the deepest pit will be found on the largest area. This is illustrated in [a] table ... which gives the average rates of penetration of the deepest pits on two materials for which both 1-1/2-inch and 3-inch specimens were included. The averages cover specimens removed at the close of ... 2,4,6, and 8 year periods, respectively. ... In each case the 3-inch specimens showed deeper pits than the corresponding 1-1/2-inch specimens". [Comment: The larger the area, the greater will be the number of pits (the sample size) and hence also the expected depth of the deepest pit (largest order statistic).]

Reagel & Willis(1931) report the results of an experiment, conducted cooperatively by four state highway laboratories, to determine the effect of variation in the dimensions of a test specimen (concrete beam) on the modulus of rupture when a load is applied at the one-third points of the span. The experiment covered wide variation in test specimens, with four values each of width (4,6,8, and 10 inches), depth (4,6,8, and 10 inches), and length (20,26,32, and 38 inches). Each laboratory made and tested three series of 64 beams, each series consisting of one beam for every possible combination of the dimensions. Great care was taken to insure uniformity of materials, manufacturing methods, test apparatus and methods, etc., and thus minimize the effects of extraneous variables. The conclusions reached by the authors are that the effect of variation in depth of the specimen is significant (linear to within sampling error), with the average modulus of rupture decreasing from 862 lb./in.^{2} for beams of depth 4 in. to 763 lb./in.^{2} for those of depth 10 in.; the effects of variation in the width and the length of the specimen are negligible; and different laboratories using similar materials and methods of test can check each other to within 5%. They do not attempt to give a theoretical explanation of their results, but note that they agree qualitatively with earlier results reported by the Portland Cement Association.

Reinkober (1931) reports that, in the systematic investigation of the tensile strength of thin quartz glass threads ranging in diameter from 100 to 1 μ, a strong increase in strength [per unit cross-sectional area] with decreasing diameter becomes evident. He states that an accumulation of quantitative measurements points to a decisive influence of surface tension on the

increase in strength. The strength of a single thread increases with decreasing thread length, which is explained by the fact that there are present in the structure of the material numerous derangements, of which the most marked ["weakest link"] tends to break first. Reinkober also reports [cf. Griffith (1920,1924)] that freshly produced threads show an appreciably greater tensile strength than old ones. He explains the influence of surface tension by the fact that materials softened by drawing first solidify undisturbed and therefore possess greater strength than the later solidifying and disturbed-spot permeated interior of the thread.

Alexandrov & Žurkov (1933) offer a qualitative explanation of the fact that the average value of the brittle strength does not remain constant, but depends in a regular way upon the size of the specimens tested, diminishing as the latter increases; namely, that the dependence of the strength of a fiber upon its diameter is due to the same reason as the ordinary scattering in the values of strength, i.e. to the inhomogeneous structure of the material. Kontorova & Frenkel (1941) credit these authors with being the first to remark that the observed mechanism [see Karmarsch (1859) and Griffith (1920)] can be interpreted in a purely statistical way: the larger the size of the specimen, the more frequently will particularly "dangerous" defects [surface cracks] be encountered in it.

Faulhaber (1933) reports the results of an investigation of the influence of the test-rod diameter on the vibratory bending strength of rods with various surface defects, which he summarizes as follows (p. 170, as translated by the present author): "For tests of polished rods a smaller vibratory strength is found with increasing diameter, and so much the more so, the greater the tensile strength is. The ratio of the vibratory bending strength to the tensile strength, established at about 0.5 for the hitherto usual measurements of test rods under 13 mm. in diameter,falls as low as 0.35 for tests of eutectic carbon steel rods of 27 mm. diameter. All values hitherto established with small rods for the vibratory strength under bending and clearly also under torsion are therefore to be regarded only as comparative values, in no case as absolute indices of the material, and used for measurement only with caution. For tests of bars with notched surfaces, graphs show the bending strength in relation to the kernel diameter, the depth of the notch on

ordinary and logarithmic scales, and the ratios of the notch depth to the kernel diameter or to the outer diameter with constant kernel diameter for each particular curve. The vibratory bending strength for large notch depths and sharp notches tends to decrease asymptotically to a minimum value clearly characteristic of the material." In a related paper, Faulhaber, Buchholtz & Schulz (1933) report (p. 1108, as translated by the present author): "The vibratory bending strength [of steel] is found to have smaller values with increasing diameter of the test specimen; thus no absolute constant of the material is exhibited. For notched specimens it tends with increasing diameter for all notch forms to the same minimum value, evidently characteristic of the steel." The results reported in these two papers are in accordance with the "weakest link" (or "largest flaw") concept.

Pinsl (1933) notes that Oberhoffer & Poensgen (1922) found that both the bending strength and the tensile strength of cast iron rods increase with increasing diameter of the test specimen up to about 20 mm., and then remain practically constant, the drop in both bending strength and tensile strength for rods of 5-mm. diameter as compared with those of diameter 20 mm. or more being about 30%. Pinsl reports, however, that the compressive strength decreases with increasing diameter of the test specimen up to about 25 mm. and then apparently remains almost constant, and that the resistance to vibratory shock and the deflection also increase with the diameter.

Lyse & Keyser (1934) summarize the results of earlier studies by Bach (1905) and by Moore (1918) on the effect of size and shape of test specimens on tensile strength and other physical properties of steel. They then present the results of their own experimental investigation, using specimens machined from 1/4-in., 1/2-in., and 1-in. thick plates, the size and shape of the test specimens varying from 1/4-in. to 1-in. diameter bars and from 1/4 by 1/4-in. to 1 by 4-in. rectangular sections. They report that their results show that the size and shape of the test specimens have practically no effect on the tensile strength, and assert that these results agree well with previously available data.

Mailänder & Bauersfeld (1934) describe a new machine used to test the influence of the diameter of the test specimen on the resistance of steel to oscillatory torsion. They report that the resistance (strength) dropped by from 21% to 29% as the diameter increased from 14 mm. to 45 mm.; for still

thicker specimens, the durability appears to decrease only very little and to approach a limiting value. They compare their results with those of other authors, including Peterson (1930) and Faulhaber (1933).

Nadai & MacGregor (1934) note that size effects and other discrepancies have been observed in a number of investigations on the various effects of stress concentration produced by notches. They attempt an analysis which will make it possible to compare the strength of specimens independently of their size and of other non-essential considerations. They report the results of some tests on the strength of notched steel and aluminum bars which do not indicate a size effect such as other authors have suspected.

Ruettgers (1934) reports the results of an investigation which shows that the strength of a large concrete specimen is considerably less than that of a small one, a 36 by 72-in. cylinder showing only from 80 to 85 per cent of the strength of a 6 by 12-in. cylinder.

Logan (1936) reports the final results of soil-corrosion studies of ferrous specimens on which Logan & Grodsky (1931) gave a preliminary report. The investigation, which began in 1922, continued for 12 years, the last of the original specimens being removed in 1934. Logan reports that the results of the examination of the specimens removed in 1934 confirm the earlier conclusions, i.e. that the average maximum pit depth was less for the 1-1/2-inch specimens than for the 3-inch specimens of the same material. On p. 465 he writes: "Generally speaking, the larger the area from which the deepest pit is chosen the deeper the pit. This fact has an important bearing on the determination of the conditions of a pipe line by means of local inspections. In order to make pit-depth measurements comparable, similar methods of inspecting pipes must be used."

Peterson & Wahl (1936) report (p. A-21): "With decrease in size of specimen the reduction in fatigue strength due to a fillet or hole becomes somewhat less; and for very small fillets or holes the reduction in fatigue strength is comparatively small." They warn (p. A-22): "Sensitivity factors determined for small specimens should not be applied to the design of machine parts regardless of size." In their closure at the end of the discussion of their paper, the authors note (p. A-150): "With regard to comparison of endurance limits of specimens of various diameters without stress

concentration, more data have been obtained since the publication of the size-effect chart [by Peterson (1930)]... ." They offer no satisfactory theoretical explanation of their experimental results.

Cazaud (1937), in a book on the fatigue of metals which became a classic when it was first published and has gone through five editions and an English translation, analyzes various factors, including the size and shape of the specimens, which influence fatigue.

Graf & Weise (1938) compare the compressive strength of concrete cubes with that of concrete cylinders with various ratios of height h to diameter d. For cylinders with $h/d = 2$ the compressive strength ranges from 0.8 to 0.9 that of cubes under otherwise equal conditions. For cylinders, they report that the average compressive strength decreases as h/d increases, the strength of cylinders 20 cm. high and 10 cm. in diameter being only 0.89 that of cylinders of the same height 15 cm. in diameter. These results agree qualitatively with what one would predict on the basis of the "weakest link" theory.

Weibull (1938) reports a theoretical verification of the statement of Auerbach (1891) that the force required to crack a glass plate subjected to the pressure of a steel ball is directly proportional to the radius of the ball, not to the square of the radius of the ball as required by the classical theory.

In a comparative study of the strength under compression of concrete cubes, prisms and cylinders of various dimensions, l'Hermite (1939) finds (p. 67) that the arithmetic mean of the results of trials on cubes decreases when the edge of the cube increases.

Weibull (1939a) presents a statistical theory of the strength of materials. He summarizes his results as follows (p. 45): " ... The classical theory of strength is obviously incompatible with numerous results of experimental research. This discrepancy may be bridged over by considering as an essential element of the problem the disperison obtained in experimental measuring of the ultimate strength. Viewed from this standpoint, the ultimate strength of a material can not be expressed by a single numerical value, as has been done heretofore, and a statistical distribution function will be required for this purpose. The application of the calculus of probability leads to the fundamental law of the theory, viz. that the probability of rupture (S) at any

given distribution of stress (σ) over a volume (V) is determined by the equation $\log (1-S)= -\int_V n(\sigma)dv$ where $n(\sigma)$ is a function characteristic of each particular material. This fundamental formula allows us to compute the influence of the volume on the ultimate strength, the relation between tensile, bending, and torsional strength, etc. An experimental substantiation of the theory is provided by observations obtained from tensile, bending, and torsional tests on rods made of stearic acid and plaster-of-Paris.... A description is given of the graphic method used for the statistical treatment of the observations and of some measuring series relating to strength of materials under the action of mechanical and electrical forces. It is shown that the material function may be expressed by the formula $n(\sigma)=[(\sigma-\sigma_u)/\sigma_0]^m$ where σ_u, σ_0 and m are constants [location, scale and shape parameters, respectively] characteristic of the material. This formula applies to statistically homogeneous materials." In a related paper on the phenomenon of rupture in solids, Weibull (1939b) writes (p. 55): "Experimental investigations intended to serve as verifications of a statistical theory of strength of materials have shown that the rupture in solids may follow two fundamentally divergent courses resulting in different mathematical expressions for the probability of rupture. Formulae for these two possibilities are deduced for isotropic and anisotropic materials. With a view to facilitating the numerical computation of the distribution constants by arithmetical methods, formulae are deduced and values tabulated. The theoretical investigations deal also with series comprising two or more components. As experimental evidence for the theory, two test series are shown, which illustrate the two above-mentioned possibilities." Concerning these two papers, Epstein (1948a) writes (p.141): "The application of essentially ... [the 'weakest link' theory of Peirce (1926)] to the study of the strength of materials is found in two papers by the Swedish engineer, Weibull. Unlike Peirce who assumed a Gaussian distribution of strengths, he makes the assumption that $F_0(\sigma)$, the probability of breakage of a unit volume as a function of the stress σ is given by $F_0(\sigma)= 1-\exp [-(\sigma/\sigma_0)^m]$ where σ_0 and m are unknown parameters which may depend on the characteristics of the material under test. ... The term probability of breakage of a unit volume is purely conceptual. ... He is interested in finding out how the probability of rupturing a specimen of volume V depends

on V. Weibull shows that the strength of specimens of volume V is proportional to $\sigma_0 V^{-1/m}$." The statistical distribution which was introduced in these two papers by Weibull, and which has come to be called by his name, is also known as the third asymptotic distribution of smallest values, and presumably would be known exclusively by that name except for his influence. Nevertheless, engineering statistics owes him a great debt for pointing out its broad applicability.

SECTION II

WORLD WAR II TO SPUTNIK (1940 - 1957)

Gillett (1940) writes as follows (pp. A19, 94) on the size effect in fatigue: "In evaluation, by rotary bending, of the stress-concentration effects of a fillet of a given radius connecting a large diameter and a small diameter on a shaft, it is found that the effect does not appear to be one of mere geometry, for the larger the shaft diameter, the more serious is the stress concentration, as has been brought out by Peterson and Wahl [(1936)]. That is, a small scale model behaves better than an actual large shaft. It seems to be fundamental in fatigue that the nucleus for failure starts at the sorest spot ..., i.e., wherever the relatively highest stress gets to work on the relatively weakest spot. ... Suppose we have one big balloon of 1000 sq. ft. surface and 1000 toy balloons, each of 1 sq. ft. surface. Make a single pin-prick in each case. The big balloon has failed, one of the toy balloons has failed, but the other 999 are all right. Thus, if we have sparsely distributed weak spots in a material, we may take fatigue specimens Nos. 1-999 from it, all of which behave well, while No. 1000 will fall below the S-N curve. With weak spots occurring more commonly, a sample of a dozen fatigue specimens may run into enough of them at the highly stressed volumes in the test pieces to show a wide S-N 'scatter band' instead of an S-N line, or it may not, according to chance. ... In the testing of a large specimen, with a relatively great volume stressed to the maximum, the weak spots get in their work and make it impossible to cover up. Thus, tests on large specimens tend to give the lower limit of the scatter band that small-specimen tests would show if thousands and thousands were tested. ... The so-called 'size effect' in fatigue may be a purely statistical matter, with nothing peculiar about it, unless it is peculiar that we tend to close our eyes to what causes it, and to do wishful thinking in

the assumption that the par value of endurance limit on a set of tiny specimens is necessarily attainable in practice."

Kontorova (1940) reviews the work of Reinkober (1931) and of Alexandrov & Žurkov (1933) on the effect of size on strength. Those writers [see also Karmarsch (1859) and Griffith (1920)] developed the empirical formula $F = a+b/r$ (A), where a and b are constants, F is the strength, and r is the radius of the cross section of the item. They attribute this effect to the occurrence of surface cracks--the greater the cross-section the greater the probability of a dangerous surface crack. In the literature, there are also indications that the strength depends on length [see, e.g., Chaplin (1880, 1882)], but experimental data show that the strength decreases much more slowly with increasing length than with increasing radius. Kontorova develops a statistical theory, based on the length of the most dangerous crack, which yields a result different from formula (A). The mean of the strengths of a group of objects of size nV should be the same as that of the minimal strengths of n groups of objects of size V. Kontorova's theory is based on the fact that the larger the specimen, the greater the probability that it contains a dangerous inhomogeneity. She writes (pp. 889-890, as translated by Mark Breiter): 'The probability $P_{\ell_{max}}$ *of the presence in the crystal of an inhomogeneity whose size lies in* the interval between ℓ_{max} and $\ell_{max} + \delta\ell$ may be determined by a function of the form (1) $P_{\ell_{max}} = C \exp[-(\ell_{max}-\bar{\ell})^2/\overline{(\Delta\ell)^2}]\,\delta\ell$, where $C = 1/\sqrt{2\pi\overline{(\Delta\ell)^2}}$, $\bar{\ell}$ being the mean size of the inhomogeneity, $\Delta\ell$ the effective width of the region which includes possible values of ℓ. From this the number of inhomogeneities of size ℓ_{max} contained in the model is $n_{\ell_{max}} = n_0 P_{\ell_{max}}$ (2), where n_0 is the total number of inhomogeneities. A necessary and sufficient condition for the rupture of the object is the presence of one or more such inhomogeneities. Using this condition and replacing n_0 by $\bar{n}V$, where $\bar{n}$ is the mean number of inhomogeneities occurring in unit volume of the material and V is the size of the specimen, we obtain $\bar{n}VC \exp[-(\ell_{max}-\bar{\ell})^2/\overline{(\Delta\ell^2}]\,\delta\ell = 1$. (4). From this $(\ell_{max}-\bar{\ell})^2 = \overline{(\Delta\ell)^2}[\lg V + \lg\bar{n} + \lg\delta\ell + \lg C]$, (5), or approximately $\overline{(\Delta\ell)^2}[\lg V + \lg\bar{n} + \lg C]$. (5a). To determine the quantitative relation between the dimension of the inhomogeneity ℓ_{max} and the corresponding strength of the specimen F_{min}, we use considerations developed in the time of Griffith, which yield $W_1 = F^2\ell^3/2E$ (F = tension, E = modulus of elasticity,

W_1 = elastic energy) and $W_2 = \sigma\ell^2$(σ = surface tension). This gives us $\ell = 2E\sigma/F^2$ (8). Substituting in Eq. (5a) $\ell^2_{max} = 4E^2\sigma^2/F^2_{min}$ and $\Delta\ell = -2E\,\sigma\,\Delta F/F^3$, (8a), we find finally $F_{min} = 1/\sqrt[4]{\alpha \lg \beta V}$, (9), where $\alpha = 4(\overline{\Delta F/F^6})$, $\beta = \bar{n}C$. The strength must therefore be inversely proportional to the logarithm of the fourth root of the size V. This relation (9) has, however, no relation to the empirical formula (A)."

Kontorova & Frenkel (1941) review earlier efforts of Reinkober (1931), Alexandrov & Žurkov (1933), Weibull (1939a) and Kontorova (1940) to explain the phenomenon of dependence of unit strength of material on size, and propose to present more rigorous and general results. They draw an analogy between the most dangerous inhomogeneity in a crystal and the weakest link of a chain. Consider a chain made up of n links, each of which has been chosen at random from a finite population of s links (s>>n) whose strengths are $\phi_1 \leq \phi_2 \leq \ldots \leq \phi_k \leq \ldots \leq \phi_s$. The probability that a particular link has strength not less than ϕ_k is given by $\sum_{i=k}^{s} p_{\phi_i}$, where p_{ϕ_i} is the a priori probability of encountering a link having strength ϕ_i. The probability that the weakest link has strength ϕ_k is given by $W_{\phi_k} = np_{\phi_k} \left(\sum_{i=k}^{s} p_{\phi_i}\right)^{n-1}$. If the number of chains and the number of links in each chain are both sufficiently large, we can replace this discrete distribution by a continuous one. The probability of finding a chain whose strength lies in the interval from ϕ_0 to $\phi_0 + d\phi$ is given by $W(\phi_0)d\phi = np(\phi_0)\ [I(\phi_0)]^{n-1}\ d\phi$, where $I(\phi_0) = \int_{\phi_0}^{\infty} p(\phi)d\phi$ and $p(\phi)d\phi$ is the a priori probability that the strength of a single link of the chain lies between ϕ and $\phi + d\phi$. Similarly, the probability of finding a crystal whose strength lies in the interval (F', F'+ dF) is given by the same formula with ϕ, ϕ_0, n and $d\phi$ replaced by F, F', N and dF, respectively, where N is the number of inhomogeneities present, which is given by $N = \bar{N}V$, where $\bar{N}$ is the mean number of inhomogeneities/cm³ and V is the volume in cm³. Using this and neglecting the one in the exponent, we obtain $W(F')dF = \bar{N}V\ p(F')[I(F')]^{\bar{N}V}\ dF$. We assume that F has a Gaussian distribution with $p(F) = Ce^{-\alpha(F-F_0)^2}$, where F_0 is a priori the most probable value of the strength and the coefficients C and α are given by $C = 1/\sqrt{2\pi\overline{(F-F_0)^2}}$ and $\alpha = 1/2\overline{(F-F_0)^2}$. If we set $F_0 - F' = \Delta F$, $F - F_0 = y$, then $W(F')dF = C\bar{N}Ve^{-\alpha(\Delta F)^2}[I(\Delta F)]^{\bar{N}V}dF$, where $I(\Delta F) = C\int_{-\Delta F}^{\infty} e^{-\alpha y^2}\,dy$. In the majority of cases $F' < F_0$ or $\Delta F > 0$. We rewrite the integral for $I(\Delta F)$ in the form $I(\Delta F) = 1 - C\int_{\Delta F}^{\infty} e^{-\alpha y^2}dy = 1 - (C/\sqrt{\alpha})\int_{\sqrt{\alpha}\Delta F}^{\infty} e^{-z^2}dz \cong 1 - e^{-\alpha(\Delta F)^2}/2\sqrt{\pi\alpha}\Delta F$. The

probability of finding a specimen of volume V having the brittle strength F' can then be written in the final form $W(F')dF \cong C\bar{N}V\, e^{-\alpha(\Delta F)^2}[1-e^{-\alpha(\Delta F)^2}/2\sqrt{\pi\alpha}\,\Delta F]^{\bar{N}V}\, dF$, where ΔF represents the absolute value of the difference $(F'-F_0)$. We now find the most probable value F* of the strength of a specimen of volume V by differentiating W(F')dF with respect to F', setting the derivative equal to zero, and solving for F', calling the result F*. (The value F_0 of the a priori strength may be considered a constant of the material, while F*, that of the a posteriori strength, is a decreasing function of the volume of the specimen.) The result is $F^* \cong F_0 - \sqrt{A \lg V + B}$, where $A=1/\alpha$, $B=(1/\alpha)\lg \bar{N}/2\sqrt{\pi}$. The authors compare their results with those of Weibull. The dependence of the probability of brittle rupture of specimens upon their size for a given stress is essentially the same for both theories, but the dependence of the probability of brittle rupture of the specimens upon the stress is completely different for the two theories. Weibull treats the defects as indistinguishable from one another, but Kontorova & Frenkel consider their seriousness.

Tucker (1941) writes (p. 1072): "There is an inherent difference in the strength of duplicate test specimens no matter how carefully these specimens are made or tested. Such differences are a natural characteristic of the materials and are more pronounced in some than in others. The paper shows how the variations in the strength of small elements of volume within a specimen will affect the modulus of rupture of beams of different dimensions and beams subjected to different loading. For example, the modulus of rupture of a beam will be decreased with beam length and with beam depth and will be greater in centrally loaded beams than in similar beams loaded at third points." The discussion following the paper includes comments by Gerald Pickett and G. R. Gause and a closure by Tucker. Pickett writes (p. 1089): "Mr. Tucker ... has questioned the application of the weakest-link theory to elements in parallel and has proposed instead the strength-summation theory. ... His analysis of beams differing in width on the basis of this summation theory is excellent. However, the writer sees no reason why the theory cannot also be used for beams of varying depth ... ." Gause writes (p. 1091): "In comparing Mr. Tucker's treatment of the statistical viewpoint with the treatment used by Weibull the outstanding difference to be noted is the introduction by

Mr. Tucker of the so-called 'strength-summation theory'. As stated, this theory appears to apply to certain problems and to give results which agree better with test data. However, certain applications of this theory will lead to inconsistencies. ... " In his closure, Tucker comments at some length on these and other points raised by the discussants.

Moore & Morkovin (1942-44) report the results of tests of fatigue strength on specimens of two carbon steels and one heat-treated chromium-molybdenum steel. Five different sizes of specimens were treated for each kind of steel, with minimum test diameters of 1/8, 1/4, 1/2, 7/8 or 1 in., and 1-1/2, 1-3/4 or 2 in., respectively. The results show a maximum size effect for unnotched specimens of about 30% of the endurance limit for 2-inch specimens of S.A.E. 1020 steel, with a tendency for the endurance limit to become constant for unnotched specimens of 1/2 in. minimum diameter or larger and for notched specimens 1 in. in diameter or larger. The authors compare their results with those of Faulhaber (1933) and Faulhaber, Buchholtz & Schulz (1933). In the third progress report (1944), they consider the question of whether any interval or residual stresses present in the machined and polished specimens might be at least partially responsible for the different endurance limits of specimens of the same metal and shape but of different size, and gave a discussion of size effect based on Neuber's theory.

In a paper on the influence of size and shape of cross-sectional area on endurance with irregularly distributed stresses, von Philipp (1942) proposes a working hypothesis that make the calculation of the size and shape effects feasible, at least in simple cases. He demonstrates the usefulness of this hypothesis by comparing the theoretical results which it predicts with the experimental results of material tests by various authors, including Faulhaber (1933), Mailänder & Bauersfeld (1934), and Peterson & Wahl (1936), finding good agreement with the greatest possible simplicity. He states that the results suggest examining the test values from the point of view of whether the formulated theory fits them so well that its general validity follows. In a related paper, Buchmann (1943) gives additional theoretical and experimental results concerning the size effect on the endurance of steel and light metals. He considers size effects on tensile strength, bending strength, and torsional strength. In each case he finds that the strength decreases with increasing

diameter, but tends to level off for diameters above 25-30 mm.

Johnson (1943) reports the results of an experimental study to determine the effect of height of test specimens on compressive strength of concrete. He writes (p. 19): "The results reported indicate that the standard correction factors [1939 Book of A.S.T.M. Standards, Part II, p. 320] are reliable for all values of h/d except 1.00, and those in excess of 2.00. They also show some indication that loss of compressive strength of tall cast specimens made from concrete of wetter consistencies may be due to water gain."

Wing, Price & Douglass (1944) report the results of an experimental study of precision indices for compression tests of companion concrete cylinders. Among other things, the study concerns the effect, on mean strength and on coefficient of variation, of the ratio of cylinder diameter C to maximum aggregate diameter A. The mean strength tends to increase and the coefficient of variation to decrease as A decreases for a fixed C. No data are given to indicate the effect of changes in C for a fixed A or the effect of the ratio of cylinder height to cylinder diamater.

Fowler (1945) offers the following statistical explanation of size effect (pp. 214-215): "Let $P(\sigma_p)d\sigma_p$ = probability of a yield point after pth period between σ_p and $\sigma_p + d\sigma_p$, ...$P(\sigma_f)d\sigma_f$ = probability of a true ultimate strength between σ_f and $\sigma_f + d\sigma_f$ .... Now the probability that the true ultimate strength is less than σ_f is given by the formula $P_f(\sigma_f) = \int_0^{\sigma_f} P(\sigma_f)d\sigma_f$. The probability that the final yield point lies between σ_p and $\sigma_p + d\sigma_p$, and that such a final yield point will result in failure, is given by the expression $P_f(\sigma_p)P(\sigma_p)d\sigma_p$ and the probability of failure in a unit volume is given by the formula $P_v = \int_0^{\infty} P_f(\sigma_p)P(\sigma_p)d\sigma_p$. The next step is to consider the final problem of whether there will be a failure in the unit. $1-P_v$ = probability of no failure in a unit volume; q = probability of no failure in a volume dv, or the probability of no failure at a given point; 1/dv = number of volumes dv per unit volume. As the probability of no failure in a unit volume is the probability of no failure in all the volumes dv comprising it, or the product of the probabilities of no failure in all of these elementary volumes $(q)^{1/dv} = 1-P_v$; $q = (1-P_v)^{dv}$. Therefore $(1-P_v)^{dv}$ = probability of no failure at a given point having a volume dv. As P_v is now considered known for every point in the solid, the probability of there being a point in this solid at which a

failure will occur can be calculated as follows: Let P = probability of a failure in the entire unit; then $1-P = \Pi_{i=1}^{v/dv} (1-P_v)^{dv_i}$ and $\ln(1-P) = \sum \ln(1-P_v)^{dv} = \sum \ln(1-P_v)dv$; $P=1-e^{\int \ln(1-P_v)dv}$ [6]. This statistical procedure has been used and checked experimentally by Weibull [(1939b)] for brittle materials. In place of Equation [6], Weibull uses the approximate formula $1-e^{-\int P_v dv}$. ... If two pieces of different size are geometrically similar and subjected to the same stress distributions, the probability distribution of failure is changed. In Equation [6] let $B_0 = e^{\int \ln(1-P_v)dV_0}$, where V_0 = volume of small piece, for the small piece. Note that $1-P_v$ is always less than 1, hence $\ln(1-P_v)$ is always negative and B_0 is never more than 1. Let $B_1 = e^{\int \ln(1-P_v)dV_1} = B_0^{V_1/V_0}$, where V_1 = volume of large piece; as $V_1 > V_0$, $B_1 < B_0$. Hence, for the large piece, P is always greater than for the small piece unless B_0 is 0 or 1. If the endurance limit is regarded as the fluctuating stress for which a fatigue failure and no fatigue failure are equiprobable, then the endurance limit becomes, for geometrically similar pieces, a function of the size of the piece."

Tucker (1945a,b,c), in a series of three papers, makes a study of the size effect on material strength. He synopsizes the first paper as follows (p. 952): "Statistical theories on the strength of materials are applied in estimating the effects of change in a dimension of the specimen on the mean strength and on the dispersion of the strength within a group of like specimens. The weakest-link theory predicts the effects of an increase in the length of specimens subjected to tension, compression, flexure, or torsion and in the depth of specimens subjected to flexure. The strength-summation theory appears to predict reasonably well the effects of an increase in the cross-sectional area of specimens subjected to tension or compression and of an increase in width of the flexural specimen. ..." In the second paper, he writes (p. 962): "It is not apparent which particular statistical theory ... should be used to predict the changes in strength and in scatter in strength, concomitant with change in dimensions. In our present analysis, the tensile strength characteristics of elementary fibers, of which a tension test cylinder is assumed to be composed, must be determined from the strength characteristics of the cylinder. Test data ... indicated that for a change in cross-sectional area the summation theory applied. If we apply this theory to our analysis in the

present paper, the predicted modulus of rupture is but little more than the tensile strength of the material, a result discordant with fact. If we apply the weakest link theory [Tucker (1941)] we obtain a predicted modulus of rupture greater than the tensile strength, but not so great as experiment shows. A combination theory is offered, which has the merit of predicting values for the modulus of rupture which are adequately large." Tucker summarizes the third paper as follows (pp. 983-984): "The variation of strength in the shortest compression test specimen may be separated into two regions: (a) The restrained failure region, where the length of the specimen is less than that to permit the failure to occur at its natural inclination to the axis. In this region the increase in strength with decrease in length differs markedly with the strength of the concrete under test. (b)The short-column region, where the reduction in strength with increase in length is a function of the standard deviation of the strength of the concrete, which is a function primarily of the size of the largest aggregate. ... Although the numerical values which are given apply to concrete, the principles are applicable to other brittle materials".

Freudenthal (1946), in reporting on a study of the statistical aspect of fatigue of materials, writes (p.422): "For samples of different size and, consequently, of different number of bonds a definite size effect must be expected; as the probability of rupture under a definite load amplitude S increases with increasing number of bonds, the fatigue strength is bound to decrease with increasing sample size, a conclusion which is supported by experimental evidence ... . The size effect in the stage of the initiation of fatigue cracks is modified by an opposing size effect which pertains to the stage of crack propagation: at equal rate of propagation the smaller cross-section will be destroyed more rapidly. The resulting effect depends, therefore, on the relative importance of the stages of crack initiation and of crack propagation in the process of rupture in fatigue."

Davidenkov, Shevandin & Wittmann (1947), in a paper on the brittle strength of steel, state (p. A-63): "Experiments on static tension and bending of cylindrical specimens of brittle phosphorous steel in liquid air reveal the statistical nature of the size effect and give a good qualitative verification of Weibull's theory." In the section on statistical theory of size

effect, they write (pp. A-64-A-65):" Considering now the statistical theory of the size effect, it may be recalled that this approach was the (qualitative) explanation, proposed long ago by Alexandrov and Jurkov [(1933)] in their investigation of the great strength of thin glass threads. ... The statistical theory of strength, whose mathematical interpretation was given independently by W. Weibull [(1939a)] and T. Kontorova and J. Frenkel [(1941)] is based upon the assumption that brittle failure is determined not by the value of the average, but of the local stress in the piece, at the locus where the most dangerous structural defect is located. The specimen can be represented as consisting of a set of volume elements of various strengths connected in series, the strength being distributed among them according to the probability law. The larger the piece, the lower the strength of its weakest element and therefore the lower the strength of the piece itself. According to Weibull the distribution of local strength of the material is arbitrarily assumed to be described by a power law. In the expression for the 'risk of rupture' $B = \int n(\sigma)\, dv$, the function $n(\sigma)$ is written as $(\sigma/\sigma_0)^m$ where m and σ_0 are constants of the material, m being a characteristic of the homogeneity of the material and is larger the higher the homogeneity. Both constants must be determined experimentally, and it is only necessary to know a set of strength values for a number of specimens. ... According to the theory of T. Kontorova and J. Frenkel, the distribution of the faults is assumed to follow the normal Gaussian law, while the number of constants characterizing the material increases to three. In spite of its advantage over Weibull's theory in the sense of stricter physical foundation, this theory is, however, less convenient for practical purposes. ..." In their conclusion, the authors state (p. A-67): "The experimental data ... lead to the conclusion that the statistical theory of strength explains satisfactorily and without inner contradictions the influence of size on the brittle strength of steel. ... The brittle strength varies with increase of size more and more slowly, so it can be assumed that this tends to some definite limit. The theoretical meaning of this limit consists in the fact that starting with a sufficiently large specimen, a complete set of all possible nonhomogeneities will be present. The larger the specimen, the closer this limit will be approached."

Fisher & Hollomon (1947) endeavor to construct a consistent theory to explain the size effect in solids, the scatter of fatigue-stress values, and the dependence of fracture stress upon strain, and to suggest a quantitative relation between the structure and the fracture stress. They assume that the material has an idealized structure, that of an elastic solid containing many cracks which are oriented at random. They assume an exponential distribution of crack sizes (widths), and work out the probability of failure, when subjected to a specified stress, of a specimen containing a single crack and of one containing many cracks. One of their conclusions is that there is a marked decrease in strength with increasing number of cracks, with no marked size effect to be expected in the case of materials containing many cracks. They point out that the theory of Kontorova & Frenkel (1941), which is based upon the assumption of a normal (Gaussian) distribution of flaw strengths, breaks down (for different reasons) when the number of cracks is either too large or too small.

Gensamer, Saibel & Ransom (1947), in a section on size effects (pp. 453s-454s), write: "The effect of stress and strain gradients has not received much direct study. Most of the work has involved gradients produced by notches, and is therefore open to question. ... A phenomenon intimately connected with gradients is the 'size effect'. This question is of considerable practical importance since small scale tests often predict safe structures which subsequently fail when made full scale. The size effect can apparently be resolved into three separate effects: (1) the smaller stress gradients in large sizes, (2) the greater statistical possibility of finding a defect in large sections and (3) the induced stresses due to restriction of strain by large sizes. Most of the work on size effect has been done by bending notched bars with little attempt to separate these factors. The general rule of size has been that as long as geometrical similarity is observed, tensile properties will be the same. However, in the presence of gradients, scaling the external dimensions usually changes the gradient in some irregular way so that the properties naturally change. ... The true effects of size and gradients have received practically no fundamental treatment. The experimental program in this survey outlines an attack to study these variables with reference to both the flow and fracture strength. ..." In a section on microcrack theory

and size effect, the authors state: "A statistical analysis of crack distribution and density has been developed in an effort to explain the various aspects of size effect. Ruark and associates carried out experimental and theoretical work on size effect. They were concerned with specimen sizes of practical interest, not the rise in strength occurring as one passes from the single crystal to the polycrystalline condition. In both tensile and bend tests, it was found that no appreciable size effect was encountered during the period of plastic flow. The size effect manifested itself through earlier onset of cracking in the larger specimens, and in a reduction of the work per unit volume required to propagate the crack. Inclusion counts were made, the total inclusions and the number of sharp or angular ones, per square inch of etched surface were determined. The bend tests showed larger size effect with increase of total inclusions and of angular ones. The separate effects of these two classes of stress raisers could not be determined from the specimens available. They point out that this matter calls for further investigation. ..."

Gensamer, Saibel & Lowrie (1947), in a section on nonuniform stress and strain, write (pp. 476s-477s): "The effects of stress and strain gradients have been studied considerably in connection with the so-called 'size effect'. However, as was stated in the initial survey [Gensamer, Saibel & Ransom (1947)], 'size effect' probably consists of three factors which must be separated from one another, namely, the statistically greater possibility of finding a given defect in a larger section, the smaller stress gradients in large sizes, and induced stresses in large sizes due to restriction of strain. Size effects have supposedly been found for flow strengths. ... Morrison (1940) [see Additional References, p. 409] found that a decrease in the size of torsion and flexure specimens increased their proportional limit. ... This size effect could parallel an increase in tensile flow strength for decreasing tensile specimen size. Thus, a size effect for flow strength appears possible, although it is probably not caused by stress or strain gradients. In general, little experimental evidence has been found of size effects in tension tests of metals. However, the preparation of homogeneous, metallurgically identical large and small specimens is very difficult. Thus, the gathering of really significant data requires very exacting care. ... A number of attempts have been made to account for size effect in fracture strength. ..."

In particular, the authors summarize the earlier results of Kontorova (1940), Kontorova & Frenkel (1941), and Fisher & Hollomon (1947). In their final section, in outlining a recommended research program, the authors write (p. 483s): "In reference to the effect of specimen volume on the fracture strength, there are at least two methods of approach which seem worth while. The 'size effect' itself may be investigated directly, which would further serve to check the Fisher-Hollomon conclusions on size effect. However, such an investigation faces the difficult task of obtaining large specimens of constant properties over the entire cross section or of devising a method of compensation for any variation. A second method, which would yield results directly comparable to past notched tensile tests, would use specimens machined from quite large bars such that all the specimens had the same minimum diameter although being of variable notch depth. In such tests the volume of material subject to the maximum stresses would remain nearly constant for any series of notch shapes, although the volume would increase in going from a shallow notched bar, whose locus of fracture is limited, to an unnotched specimen which could break anywhere within the gage length."

Gurney (1947) reports the results of sets of strength tests on specimens 1 in. long and 240 in. long, cut from the same cord of steel wire. He compares the results of three sets of tests with those predicted by theory, assuming that a normal distribution of strength would be obtained for each length, and that the strength of wire is statistically homogeneous. The differences in the distributions of strength for the long and the short wires are small, but the trend of the results is in agreement with the theory that average strength and scatter both decrease with increasing length, and skewness tends to become more negative. However, it is clear that other influences not covered by the theory also have significant effects. In a related report, Gurney & Pearson (1947) describe methods of estimating size effect from the scatter of test results. A very large number of tests are necessary to obtain a good estimate of the effect of size upon the strength. The authors conclude that a size effect of practical importance is to be expected in materials with large variation in strength of ostensibly independent pieces. A brittle material may be expected to fail at the breaking stress of the weakest of n elements of which the material may be considered to be composed. The theory applies

directly only to materials which are statistically homogeneous, which most engineering materials are not, and hence for them the theory can be expected to give only trends.

Holmberg (1947) gives the results of a study of the size effect of the cross-sectional diameter of hot-rolled steel rods on properties of importance in structural engineering. He reports that the size effect is especially pronounced in connection with variations of the yield point and the ultimate stress and the relation between them. His conclusions are in agreement with those previously published by other research workers.

Miklowitz (1947) presents the results of several tests with flat tension bars of mild steel under centric and eccentric loading. He reports (p. A-30): "Under the centric loading the yield stress was found to increase with an increase in the gage length of the specimen up to a length-to-width ratio of 3. Beyond this value there was no appreciable change. In the shorter specimens the plastic zone is initiated at the fillets under a higher stress concentration than in the longer ones. This high stress concentration is due to the restriction to lateral contraction of the gage length produced by the adjacent heads of the specimen. This restriction has a greater effect as the gage length becomes shorter. Once yielding has started, its propagation is independent of the length of the specimen."

Newman & Curtiss (1947) report the results of a study of the relationships of circumference C (sixteenths of an inch), weight W (lb./linear ft.), strength S (lb.), and nominal diameter D (sixteenths of an inch) of 863 samples of 3-strand manila rope ranging in size (nominal diameter) from 3/16 in. to 3 in. The following fitted regression equations are presented: $C = 3.119D$, $W = 0.001447D^{1.8827}$, $S = 70.481D^{1.82819}$, and $S = 40278W^{0.96894}$. The first exponent is less than 2 because the actual diameter tends to be slightly less than the nominal diameter, especially for the larger sizes. The fact that the last two exponents are less than 1.8827 and 1, respectively, shows that the strength per unit cross-sectional area decreases as the size increases, since (if uniform density is assumed) the cross-sectional area is proportional to the weight or to the 1.8827 power of the nominal diameter (the square of the true diameter). No theoretical explanation of this size effect is given.

Pope (1947) writes (pp. 284-285): "Cook [(1932)] [see Additional References, p.409] was among the first to draw attention to the effect of size of specimen upon the yield strength of a metal, and states 'the surface material may possess a greater resistance to the dislocation by virtue of its being at the surface, as distinct from any mechanical or heat treatment it may receive.' Morrison [(1940)] [see Additional References, p. 409] points out that yield cannot occur in an individual crystal surrounded by unyielding material but only in a number of crystals which occupy sufficient volume to allow for the readjustment. Therefore, yield due to non-uniformly distributed stress cannot occur until the stress, some distance below the surface of the specimen, has reached the critical value. This seems to the author to be perfectly sound reasoning, and the high values obtained for the yield stress in torsion can probably be explained by this simple mechanical process. Morrison, however, when accounting for the size effect in torsion, assumed that the thickness of the lamination t was a constant for the material and independent of the specimen size. It has been indicated already by Cook that it depends upon both the type of stress and the size of the specimen, and there seems little reason to doubt that t depends upon these variables. It follows, then, that in all tests having non-uniform distribution of stress, two variables are present: (1) the criterion governing failure, and (2) size and stress-distribution effect. To analyse test results the law governing one must be assumed and then the variation of the other deduced from the test results. In the past, with few exceptions, the size effect has been ignored, and the test results taken as a direct indication of the laws governing failure. ... The author proposes to reverse the procedure; that is, to assume the criterion of failure and from the various test results deduce the laws governing size and stress-distribution effect. ... The following assumptions will be made as they seem the most reasonable, and will form the basis of all subsequent work in this paper. (1) The criterion of failure for normalised carbon steel is the maximum shear stress theory. The true shear yield strength of a metal (q_0) is equal to half the tensile yield strength of the metal (i.e. $f_0/2$). (2) With non-uniformly distributed stress, yield will not occur until the maximum shear strength at some distance t below the surface of the specimen has reached a value equal to the true shear stress ($f_0/2$). (3) The

variation of stress across this lamination t is proportional to the variation of strain. (4) The value of t depends upon the condition of the metal, type of stress and size of specimen." On p. 288 the author states the following conclusions: "(A) If the upper yield strength is used as a basis of design, then for tests in pure torsion the specimens should have a diameter of not less than 1 in. if the actual structure is greater than 1 in. in diameter. If a specimen of smaller diameter is used, then correction should be made for size effect; otherwise the test results will give an apparent strength which will be superior to that which will be realised in the actual structure. In tests in pure bending for parts of a structure greater than 1/2 in. in diameter, the diameter of the specimen should not be less than 1/2 in. unless a correction factor for size effect is used. In combined bending and torsion, or combined direct stress and torsion, tests relative to machine parts greater than 1 in. in diameter should use specimens not less than 1 in. diameter unless a correction factor is used. (B) If the lower yield stress is used as a basis of design, then size effect is small, but there is a large stress-distribution effect (i.e., t is large but not greatly affected by size of specimen). ..."

Siebel & Pfender (1947) present experimental evidence in support of a *new fatigue-strength theory* based on *relative stress gradient* $\chi = (1/\sigma)d\sigma/dx$, where $d\sigma/dx$ is the stress gradient. They determine the influence of χ on the fatigue strength of a material from bending tests on specimens of different diameters and plot curves showing the relation between fatigue strength and χ, which is inversely proportional to the diameter. Bending-fatigue tests for values of χ varying from 0.3 to 2.0 mm^{-1} show that the fatigue strength remains nearly constant with variations in χ when χ is above 1.0 mm^{-1}.

Aphanasiev (1948) notes (p. 132) that "it has been established by numerous investigations that an increase in the dimensions (diameter) of test pieces lowers their fatigue limit, and that this phenomenon becomes more apparent when stress concentrations occur" (as in notched test pieces). After reviewing the literature on the subject, including contributions by Peterson & Wahl (1936), Weibull (1939a), and Kontorova & Frenkel (1941), he develops a theory to explain the greater size effect on fatigue strength for notched specimens as compared with unnotched ones.

Bagsar (1948) describes a new test for the determination of the tensile breaking loads of edge-notched rectangular steel sections and examines the conditions under which failure occurs by cleavage fracturing. He reports the results of a study of the dependence of the results on various properties of the specimen, including section depth and thickness and notch geometry. The effects of coupon or section depth and thickness on the unit breaking loads were investigated by using coupons of depth varying from 1 to 24 in. with a thickness range of 1/2 to 2-1/4 in. As the depth of the coupon decreases from the value of 6 in. the unit breaking load decreases, but increasing the coupon depth beyond about 6 in. has practically no effect on the unit breaking load. Within the range covered by the tests, thickness of the coupon appears to have little or no effect on the unit breaking load. Cleavage strength decreases appreciably with increases in notch depth up to about 5/32 in. and then levels off for further increases. A notch of 5/32 in. or greater is sufficient to reduce the breaking strength of steel to its cleavage strength if the section is 6 in. or greater in depth and of sufficient thickness to support the development of cleavage cracks.

Boodberg, Davis, Parker & Troxell (1948) report the results of tests of notched steel plates 3/4-in. thick and from 3 to 108 in. wide for several special steels. The average strength of the plates decreased markedly as the width was increased from 3 to 24 in., but further increases in width made little difference.

Brueggeman & Mayer (1948) report the results of completely reversed axial load fatigue tests, on 24S-T and 75S-T aluminum-alloy strips with a central circular hole, in eccentric crank fatigue machines. Specimen widths 1/4, 1/2, 1 and 2-in., plain and with hole diameter to specimen width ratios from 0.01 to 0.95 were used. The authors give S-N diagrams for all tests. A size effect is indicated in the plain specimens, the curve for 1/4-in. specimens being from 5,000 to 8,000 psi above the curve for the 2-in. width. Fatigue stress concentration factors are given for the specimens with holes.

Cox (1948) proposes a kinetic approach to the theory of the strength of glass which postulates spontaneous formation of a flaw in the event that at least some critical number w of neighboring atoms become activated. He writes (pp. 947-948): " ... An expression can be derived for the probability of such

an event in terms of the temperature and stress. Assuming that each such event will lead to rupture, the tensile strength is given by setting this probability equal to one half--in accordance with the usual definition of the breaking point as that stress at which the chances of rupture and survival are equal: $(\nu Nt/\tau)\{A[\exp(S/kT)-C]\exp(-B/kT)\}^{w} = 1/2$. In this expression νN is the number of different ways in which the w neighbouring atoms constituting a flaw-nucleus may be chosen out of the N atoms in the sample under a stress proportional to S; t is the duration of test, and τ is the vibrational period of the atoms. A and C are constants depending upon the composition and temperature of the glass, and B is the activation energy for the configurational-change involved. It will be noticed that there is a minimum tension corresponding to $S=kT \log_e C$ below which the glass should not break, however large the sample or prolonged the test. The relations between strength, duration of test and sample size given by this expression are in good agreement with experiments [including those of Griffith (1920)]... ." The author summarizes his results as follows (pp. 948-949): "...If the strength of glass does depend on the spontaneous development of flaws under the applied stress in the way described, then breakage can only occur if the stress exceeds a certain minimum. Above this minimum stress the probability of breakage increases with sample size and with time for the same reason, namely, that such increases provide increased opportunity for the rare event which is automatically recorded by the fracture of the specimen."

Davis (1948) describes internal pressure tests on tubular steel specimens of three sizes (outside diameter approximately 1, 2, and 4 in. and wall thickness approximately 1/16, 1/8, and 1/4 in., respectively). He reports that the size effect on the pressure required to rupture the test specimens was negligible.

Dorey (1948) presents the results of a number of tests carried out on 9-3/4-in.-diameter mild-steel shafts, and also on Meehanite cast-iron specimens of 6 in. diameter, and makes the following remarks (p. 405) on the size effect, based on results from 1/2-in.- diameter specimens subjected to reversed torsional fatigue tests: "The results so far obtained ... indicate clearly the overall reduction in fatigue strength in large shafts due to stress concentration set up by the geometrical form of the stressed parts, to lack

of homogeneity of the material, and to any size effect, if present. So far as the size effect is concerned, the results ... show that the effect may be very appreciably less than that given by other investigators when comparing test results over a considerably smaller range of diameters. In the case of the Meehanite cast iron, the limited evidence provided by the tests indicates that the overall effect is more pronounced than with mild steel."

Epstein (1948a), writing on statistical aspects of fracture problems, states (p.140): "In recent years there has been an increasing interest in the development of statistical theories of strength. A main aim of these theories is to explain in a reasonable way such things as the dependence of the strength of specimens on their volume or length. In this paper it is pointed out that the problems posed by these models are equivalent to an important problem in mathematical statistics, namely, the distribution of the smallest value in samples of size n drawn from a population having some probability function f(x). The calculations made by mathematical statisticians give a far more complete description of the results to be expected than do the estimates to be found up to now in the technical literature." In his concluding remarks, Epstein writes (pp. 146-147): "When one looks at the problem of fracture in its most general setting it is clear that the 'weakest link' concept, when applicable, may be the key to the occurrence of certain observed relationships between the strength of specimens and their size. ... From a statistical point of view, certain phenomena, which may on the surface appear to be different, are really equivalent since they lead in each instance to the same problem, namely, the distribution of the smallest value in large samples. Now, it is not unreasonable to suppose that the distribution of strengths due to flaws is Gaussian or perhaps more generally of the form $A \exp(-B|x-\mu|^p)$ for large values of $|x-\mu|$ (where A, B, μ, and p are positive constants). Under such hypotheses statistical theory leads to the prediction that the most probable value of the smallest value in samples of size n must decrease as $(\log n)^{1/p}$ and that the distribution of smallest values in samples of size n must be negatively skewed. ... One other important point that we wish to emphasize is that one should not expect the 'weakest link' hypothesis, attractive as it may be, to explain all phenomena relating to the strength of materials. In the first place it is implicitly assumed in this paper that the flaws are all independent

and do not in any way influence each other. There are physical situations [for example, notched specimens] where such an assumption is not justified. ... In the second place there have been attempts (see, for example, A. M. Freudenthal [(1946)]) to apply essentially the weakest link concept to the study of the fatigue of materials. We believe that such an attack is basically incorrect even if it leads to the prediction of relationships which agree reasonably well with what is found in nature. For example, the relationship between the length of time required to break a specimen and the applied load is in its very essence a time-dependent problem. The specimen is changing in time and its distribution of strengths due to flaws is also changing and, therefore, any essentially static approach which uses the 'weakest link' concept without modification leaves out certain basic features of the process. ..." In a related paper, Epstein (1948b) shows that the theory of extreme values is pertinent to the treatment of certain aspects of the fracture or breakdown of materials used in modern technology and makes an attempt to integrate some of the results scattered through the statistical literature. He writes (pp. 403-404): "In essence, all the statistical models proposed in the study of fracture take as a starting point Griffith's theory [Griffith (1920,1924)] which states that the difference between the calculated strengths of materials and those actually observed resides in the fact that there exist flaws in the body which weaken it. Accepting this point of view is equivalent to assuming that there will be a distribution of strengths in a given specimen in the sense that a different amount of force will be needed to fracture a specimen at one or another point. There are many physical situations where failure at one point means failure of the entire specimen. But this simply means statistically that it is the worst flaw among N flaws (where N is the number of flaws in the specimen) which determines the strength of a specimen and therefore the theory of extreme values is immediately applicable. Clearly N increases as the specimen size increases and, therefore, the dependence of strength on specimen size is equivalent statistically to the problem of how the distribution of smallest (or largest) values depends on N. Generally speaking, asymptotic theory is applicable since N, the number of flaws, is large in most practical problems." Epstein proceeds to summarize the work on the strength of materials of Peirce (1926), Weibull (1939a,b), Kontorova

(1940), Kontorova & Frenkel (1941) [Frenkel & Kontorova (1943)], Davidenkov, Shevandin & Wittmann (1947), and other authors, as well as the related work on the theory of extreme values of Tippett (1925), Fisher & Tippett (1928), Gumbel (1935) and others. He gives the asymptotic distributions of smallest values in samples of size n (n large) for the distributions of Laplace, Gauss, and Weibull and the mode, the mean, and the variance for each of these asymptotic distributions.

Epstein & Brooks (1948) apply the theory of extreme values to the study of the dielectric strength of paper capacitors and its dependence on capacitor size. The statistical theory presented is essentially the same as in the two papers by Epstein (1948a,b), except that interest now centers in the distribution of largest (rather than smallest) values. The reason for this is given in the following statement (p. 545): "Since the dielectric strength of a capacitor in the immediate neighborhood of a conducting particle is determined by the size of the particle, it is immaterial from a statistical point of view whether one considers, on the one hand, the distribution of dielectric strengths in the neighborhood of flaws, or on the other, the distribution of the sizes of the conducting particles. In this paper it will prove convenient to study the distribution of dielectric strengths of capacitors by considering the distribution of the largest values in samples of size n drawn from a population of conducting particles following some size distribution law described by a continuous probability density function f(x)." The authors write down the basic formulas for such a distribution. In applying the results to the dielectric strength of capacitors, they assume, on the basis of empirical evidence, that the size distribution of conducting particles, f(x), is of exponential type, i.e. $f(x) = \lambda e^{-\lambda x}$, $x>0$.

Frankel (1948) compares the results of applying the theory of Tucker (1927,1941), based on the normal distribution, and that of Weibull (1939a, b), based on the Weibull distribution, to the relative strengths of mortar in bending. According to Weibull's theory, the probability P of rupture is given by $P=1-e^{-kV[(S-S_\ell)/S_0]^m}$, where k is constant depending on the method of loading, V is the volume of the specimen, S is the applied stress, S_ℓ is the minimum stress (threshold parameter) below which no rupture will occur, S_0 is a particular stress constant, and m is a parameter to be determined. Tucker's

theory is essentially a special case of Weibull's with m=2. On the basis of tests performed on 99 small mortar beams under sixth- , third- and center-point loading, Frankel finds that m = 2.9 and that there is excellent agreement between Weibull's theory and the experimental findings. Further light is shed on the relative strengths and weaknesses of the two theories (Weibull's and Tucker's) by the discussion, which consists of comments by Tucker and a closure by Frankel.

Frenkel' (1948) relates both the hardness and the brittle strength of metal specimens to the presence of defects in their crystalline structure, and thus (since the larger the specimen, the greater the probability of a defect) to their dimensions. For a rod of diameter d, both hardness and strength may be expressed in the form a+b/d, where a and b are constants of the material, a relation proposed much earlier by Karmarsch (1859) in the case of strength and by Auerbach (1891) in the case of hardness.

Hill & Schmidt (1948) report that in practice it is found that large areas of insulation have a breakdown voltage with lower average and less variability than small areas of the same insulation. They apply probability theory (extreme value theory) to give a theoretical explanation of this phenomenon on the basis of insulation variability, and derive general expressions from which they calculate tables and plot curves to convert the distribution of breakdown voltage for an elemental area to that for n such areas in parallel.

McAdam, Geil, Woodard & Jenkins (1948) report the results of a study of the applicability to ductile metals of the statistical theory of fracture and the size effect (based on the assumption that fracture is determined by the largest defect). After reviewing the work of Griffith (1920), Weibull (1939a), Kontorova & Frenkel (1941), Davidenkov, Shevandin & Wittmann (1947), and Fisher & Hollomon (1947), they present the results of their own experimental investigation on notched and unnotched specimens of oxygen-free copper and of carbon steel. They reach the following conclusions (p.11): "The statistical theory of fracture is not applicable to the fracture of metals after even slight plastic deformation. Since the use of notched specimens for the investigation of the influence of the stress system on fracture requires enough ductility to relieve most of the stress concentration, the statistical theory is not applicable to such fractures. The results of this investigation,

in which notched and unnotched specimens varying greatly in size were tested to fracture, show conclusively that the increase in the fracture stress with increase in the sharpness of the notch is due to the increase in the ratio of transverse to longitudinal stress, not to a size effect."

Miklowitz (1948) reports the results of a study of dimensional factors on the mode of yielding and fracture in flat tensile bars of medium-carbon steel. Most of his results deal with local strains, but he does give data on the ultimate-load stress and the fracture-load stress of specimens 3/8 in. and 3/16 in. thick and width to thickness ratios 10, 7, 6, 5, 3, 1. He also reviews the work of earlier authors, including Beare & Gordon (1921), Templin (1926), and Lyse & Keyser (1934).

Moore (1948) gives the results of tests made to evaluate the bearing strength of aluminum-alloy sand castings and their relation to tensile properties. Comparisons between individually cast bars and specimens machined from slabs show the effect of size and form of casting on mechanical properties. Moore reports bearing-strength characteristics of 1/8 to 1/4-in. thick specimens with respect to edge distance and ratios of specimen width to pin diameter which are similar to those for wrought-aluminum alloys.

Shearin, Ruark & Trimble (1948) present the results of notch-bend tests on geometrically similar specimens cut from a 2.5-in. plate of Ni-Cr steel, with nine different heat-treatments. They report that the work per unit volume required for failure was found to decrease markedly with increase of size, for all the heat-treatments, and give a brief explanation.

Siebel & Bussmann (1948) present a method, which takes account of the stress gradient, to evaluate the effect of notches on the behavior of structures under cyclic loading. The significance of the stress gradient in the case of smooth bars is shown by the observation that the endurance strength is a minimum for push-pull tests (zero stress gradient) and increases with decreasing bar size in reversed-bending tests, since the stress gradient increases as the diamter decreases. For notched specimens, the stress gradient varies sharply across the cross section; however, since failure is initiated in a narrow zone at the notch surface, only the local gradient there is significant.

Wills (1948) discusses a size effect on the fatigue strength of aircraft which differs from the conventional one. He writes (p.146): "The increase in

size of aircraft has reduced the resonant frequency of the wing structure to something approaching the gust frequency, and it is probable that as the size of aircraft increases the wing oscillations will increase both in amplitude and in persistence [thus decreasing the fatigue strength and hence the life of the wing structure]."

Wilson, Hechtman & Bruckner (1948) report the results of tests of 3/4-inch ship plates from 12 in. to 72 in. wide containing a severe stress raiser of the jeweler's saw cut type and with a length of stress raiser to width of plate ratio of 0.25. The tests were conducted for rimmed steel and killed steels both as rolled and normalized, at temperatures ranging from -73°F. to +160°F. It was observed that the average strength decreased with an increase in width of plate up to 72 in., with a tendency to decrease still further for widths above 72 in.

Beams (1949) describes experiments in which steel spherical rotors of various sizes are spun to speeds where explosion occurs. It was found that the maximum peripheral speed obtained was approximately the same (roughly 10^5cm/sec) for the spherical rotors of various diameters. However, the probability of any particular rotor reaching this maximum speed increased as the size of the rotor decreased. This can be explained by the "weakest link" theory: the smaller the rotor, the less likely it is to contain a defect which will cause it to explode before reaching the theoretical maximum speed.

Results are reported by l'Hermite (1949) of a study of the influence of the dimension (edge) of a cube of molded concrete on its resistance to crushing. More than a hundred tests were made on cubes of concrete in which the largest aggregate did not exceed 2 cm. in diameter. Taking the resistance of a 10 cm. cube as unity, the author reports the following average resistances: 5 cm. cubes, 0.95; 10 cm. cubes, 1.00; 15 cm. cubes, 1.08; 20 cm. cubes, 1.09; 30 cm. cubes, 1.00; 40 cm. cubes, 0.80. The coefficients of variation decreased from 10% for 5 cm. cubes to 3% for 40 cm. cubes. The author notes that according to the theory of probability ("weakest link" theory), both the average and the dispersion of resistance should decrease as the edge of the cube increases. He theorizes that the observed departure from theory was due to the fact that the relative size of the defects (aggregates) increased as the edge of the cube decreased.

Kontorova & Timoshenko (1949) extend the results on the size effect in brittle fracture, given in several earlier papers by Kontorova, to the bending of bars of rectangular cross section and the torsion of rods of circular cross section. They assume a Gaussian distribution of strengths in an assembly of specimens of equal size, and a proportionality between the number of flaws and the volume of the specimen, though the reviewer (E. Orowan) believes that, while these assumptions are justified for materials like plaster of Paris, the number of flaws for vitreous materials would be proportional to the surface area rather than to the volume. The results show that, with the authors' assumptions, the mean fracture strength of specimens of given volume would be lower in tension than in bending or torsion, which conclusion agrees with earlier measurements on specimens of plaster of Paris.

Laurent (1949) remarks that fatigue fracture is certainly one of the most curious phenomena of the physics of metals. He points out (a) that fatigue fracture occurs in general for stresses less than the elastic limit and that the metal apparently undergoes no modification, (b) that fatigue resistance does not follow the law of similitude, the fatigue limit decreasing when the dimensions of the test specimen increase, and (c) that the state of the surface plays a much more important role than in plasticity. He offers the opinion that, while various theories of fatigue give in general a qualitative view of the phenomenon, they do not sufficiently associate fatigue and plasticity. He endeavors to show that the interaction between the two must be taken into account in order to understand fatigue strength and its relation to the size and shape of the test specimen.

Le Fevre (1949) reports the results of tests to determine the effect of length on the average torque at failure for steel tubes (ultimate tensile strength about 140,000 psi) with an outside diameter of 0.75 in. and a wall thickness of 0.035 in. in lengths ranging from 0.08 in. to 23.54 in. The tubes with length/diameter ratio (L/D) greater than 3.5 failed by plastic two-lobe buckling under a torque which was independent of the length. The tubes with L/D less than 1 failed by shearing at a torque which increased with decreasing length. Le Fevre compares these results with published data and empirical formulas.

Lévi (1949) points out that, just as the strength of a chain is that of its weakest link, the strength of a long bar is that of the weakest of the

pieces into which it could be divided by cross-sectional cuts. Since the distribution of the smallest of n values ($n \geq 2$) drawn at random from a normal distribution is not normally distributed, Lévi concludes that if the strength of the links of a chain (or of short segments of a long bar) is normally distributed, that of the chain (or bar) is not. He proposes the use of the lognormal distribution as an approximation to the distribution of the strength of bars or chains.

Ludwig (1949) describes a series of experiments to study the effects of length and cross sectional area of test specimens and diameter and depth of notches on the impact strength of steel, but the experiments are not properly designed to assess the individual effects of these factors. (Statisticians would say that the effects are confounded.)

A report by van Meer & Plantema (1949) on a survey of the literature concerning fatigue of materials and structures contains a section on the statistical theory of fatigue failure in which the authors discuss the "weakest link theory" [Weibull (1939a,b)] and the "strength summation theory" [Tucker (1941)] and the combination of these two theories proposed by Gause in his contribution to the discussion of Tucker's paper.

Oberg & Rooney (1949) report the results of fatigue tests on specimens of an aluminum alloy with cross sections of the same area but different shapes. It was found that the fatigue strength is greater for round specimens than for square ones, and greater for square one than for rectangular ones; i.e., the smaller the surface area for a given cross sectional area, the greater the fatigue strength.

Peterson (1949) notes that in a number of recent papers [Weibull (1939b), Fowler (1945), Freudenthal (1946), Davidenkov, Shevandin & Wittmann (1947), Hill & Schmidt (1948), and Epstein (1948a)], size effect is considered to be a consequence of probability relations based on the weakest link concept, and that in most of these analyses the standard deviation σ is a criterion of the magnitude of the size effect.

Prot (1949a,b) points out that if one performs traction tests on specimens of wire of length 3ℓ, one should not expect the strength to be the same as for specimens of length ℓ, but rather the same as the minimum strength of groups of three specimens of length ℓ. When the test specimens are homothetic, the influence of the dimensions is less evident, but it still exists; small

specimens tend to have slightly higher average strength and considerably higher dispersion than larger specimens. Prot states that the problem of the distribution of the smallest of n values can be solved completely for various forms (including normal) of the initial distribution.

Voellmy (1949) reports the results of a study of the interdependence of compression, bending and buckling strength of concrete and reinforced concrete. He expresses in algebraic form an assumed relation between stress and deformation of concrete in compression, and uses it to derive relations between strength and slenderness ratio for plain and reinforced concrete columns.

Weibull (1949) writes as follows (pp. 31-37) on the size effect: "The original source of the statistical theory of strength is to be found in the Author's effort [Weibull (1939a,b)] to explain the fact, known for a long time, that the ultimate strength of a specimen increases with reduced dimensions and that it is greater in bending than in tension. As far as he knows it was previously unknown that these two phenomena were intimately connected and could be explained by the same principle based on statistical considerations. This idea will now be briefly recapitulated. Suppose that a specimen of the length L_1 and its distributions [sic] function F_1 are given for an arbitrary life N [in cycles] lying anywhere between 1 and ∞. Suppose further, that two such, nominally equal, specimens are coupled end to end, thus forming a specimen with the length $2L_1$ with an unchanged cross-section. The probability that failure will not occur in one of the two halves is obviously equal to $1-F_1(S)$ [S = load] and the probability that failure will not occur either in one or in the other half-part is equal to the product of the two independent events. If $F_2(S)$ denotes the distribution function of a failure in the specimen with the length $2L_1$ it is easy to see that $1-F_2(S)=[1-F_1(S)]^2$ (30) or $F_2(S)= 1-[1-F_1(S)]^2$ (31). For a specimen with the arbitrary length L its distribution function F_L is determined by $F_L(S) = 1-[1-F_1(S)]^{L/L_1}$ (32). This is evidently an application of the weakest link principle and it may be found in a previous work by the Author [Weibull (1939a)]. ... We now take a distribution according to (25) [$P=1-e^{-k(S-U)^m}$, where U=S(0,N) is the load for which there is no probability of failure in N cycles]. Then $F_L(S) = 1-e^{-(L/L_1)k(S-U)^m}$ (36) and there is no change in the values of the parameters. This rule holds good even in the general expression (19) [$F(S) = 1-e^{-\phi(S)}$], which gives $F_L(S) = 1-e^{-(L/L_1)\phi(S)}$ (37) or $\log \log [1/(1-P)] = \log \phi + \log L-\log L_1$ (38).

If we then take log log $[1/(1-P)]$ as ordinate and S as abscissa, the change of the length L means a displacement of the curve without any change of the form ... . In the case just discussed, where the length only, and not the cross-sectional area, was supposed to be changed, it is quite correct to put in the volume V instead of the length L in (36) thus obtaining $F_V(S) = 1-e^{-(V/V_1)k(S-U)^m}$ (39). The question now arises, if (39) has the same general validity as (32). In order to elucidate this problem, we suppose that we couple the two specimens side by side, thus having a specimen with the original length L_1 but an area twice as great. Even in this case (39) is valid, because, if failure occurs in one of the specimens, the load on the other one will be doubled. The probability that it will endure more than a few cycles is extremely small. If the two bars are allowed to melt together, so to speak, i.e. if we have a single specimen with a doubled cross-section, the validity of (39) is not at all self-evident. It depends on the manner in which the fracture is spread over the area, but it depends also on the homogeneity of the material. ... In any case, it is no trivial mathematical affair to determine the influence of changing the diameter of the specimen. The relation (39) is based on the assumption that the stress is uniformly distributed over the entire volume. This condition is not satisfied, for instance, at bending and torsional loads. As has already been demonstrated in another place [Weibull (1939a,b)], the actual volume then has to be reduced. ... Now, the effectively tested volume is much greater in torsion than in reversed bending. For this reason it is not quite correct to use the medians (or still worse the means) when evaluating the results without introducing corrections for the volume. As a better method, I should propose the use of the U-values, which are ... independent of the tested volume."

Werren (1949) reports the results of tests in bending, compression, and tension of three thicknesses (1/16, 1/4, and 1/2 inch) of glass-fabric-base plastic laminate (7,25, and 50 plies, respectively), which were conducted in order to investigate the effect of span-depth ratio and thickness on mechanical properties. He reaches the following conclusions: (1) As the span-depth ratio for a given thickness of laminate is increased, the resultant value of modulus of rupture is decreased. (2) The rate of decrease of modulus of rupture resulting from an increase in the span-depth ratio appears to become

less as the thickness of the laminate is increased. (3) For a given span-depth ratio, a thin laminate has a higher modulus of rupture than a thick one. Additional studies are needed to investigate the cause of the variation of mechanical properties of laminates with thickness. (4) The modulus of elasticity appears to be independent of the thickness of the laminate, and increases slightly with an increase in span-depth ratio, probably due to decreasing effects of shear. (5) On the average, the fiber stress at proportional limit for a given thickness of laminate decreases with an increase in span-depth ratio.

Bollenrath & Troost (1950) consider scale effects in plastic bending of a uniform beam. They give formulas for the mechanical damage produced by a given plastic strain, which is reduced by a strain gradient, and for the decrease which occurs as the size increases in the fraction of the cross section which remains elastic when a given degree of mechanical damage has occurred at the surface. The reviewer (F. R. N. Nabarro) notes that the authors' claims that these formulas are logically established and that one follows from the other are not substantiated.

Clark & Wood (1950) report the results of an experimental study of the influence of length and cross-section of specimen on detection of a "critical" impact velocity at which total energy absorbed to fracture reaches a maximum. The authors conclude (p. 584): "In making tensile impact tests it is necessary to utilize a ratio of length to diameter of specimen of approximately 13 to properly establish the critical velocity. ... The shape of the specimen cross-section does not influence the effect of velocity on the tensile properties of the metal within the dimensions studied."

Hill (1950) applies the theory of plane plastic strain to calculation of the minimum dimensions that a specimen should have for a valid hardness test. He determines critical widths and thicknesses for rectangular specimens indented by flat dies and wedges of any angle.

Huisman & Wight (1950) note that radome designers have found divergent and conflicting results in mechanical test data from flexural tests of sandwich materials, due, in part, to wide differences in specimen size and in testing procedures. They discuss the importance of specimen width and reliable face thickness data, and suggest the desirability of a statistical approach.

Hussey (1950) reports the results of more than 2800 readings made on the flat and curved surfaces of carefully selected steel specimens of small diameter to determine the deviations of hardness readings taken on curved surfaces from the "true" values for flat areas. He states (p. 1176):"The resultant deviations, when plotted, indicate line relationships of such slight curvature that straight line averages, obtained by the method of least squares, were utilized and considered sufficiently accurate for all practical purposes. Correlation curves are presented which may be used to obtain actual Rockwell C hardness values from measurements made on cylindrical surfaces of the following diameters: 1/4, 3/8, 1/2, 5/8, 3/4, 7/8, 1, and 1-1/4 in." In response to comments by V. E. Lysaght concerning his 1/4 in. curve, the author recommends caution in the use of his data for the 1/4-in. diameter, and additional tests on this size of specimen.

Miklowitz (1950) reports that cylindrical test specimens of medium-carbon steel with diameter ranging from 3/16 in. to 3 in. were found to have ductility values that depended on the size of the specimen, an effect not observed in previous investigations, and that conventional strain at the neck was about half as much for the largest specimen as for the smallest. He attempts to give a metallurgical explanation of these phenomena.

Prot (1950a), in writing on the safety factor R/P of structures, where R is the load which a structure could support and P the load which it actually supports, notes that if one determines R by trials on a number of test specimens, the dimensions of the test specimens play a critical role. He repeats the statement made in two earlier papers [Prot (1949a, b)] that the dispersion of the strength is much greater for small specimens than for larger ones, the flaws being more evenly distributed for large specimens. He does not mention the somewhat smaller effect of the size of the specimens on the mean strength, noted in one of his earlier papers [Prot (1949b)] and in works by other authors.

Prot (1950b) points out that the dimensions of a test specimen have an effect on both the average strength and the variation about that average. The strength of a specimen of wire of length 3ℓ is the strength of the weakest of three specimens of length ℓ into which it might be cut, and hence the average strength of specimens of length 3ℓ is less than that of specimens of length

ℓ cut from the same reel. The mean deviation is also smaller for specimens of length 3ℓ than for specimens of length ℓ.

Templin & Aber (1950) state (p. 1188): "In developing and using small tension test specimens, it must be recognized that results from tests of metals having very coarse grains, that is, grains that are large in relation to the cross-sections of the specimen, in all probability will not check closely the results obtained from tests of the same material when using large size specimens. If, however, both small and large specimens are used for testing material having relatively fine grains and the specimens are geometrically similar, results from the different sizes of specimens should be in reasonable agreement, provided the material is homogeneous or that the specimens are similarly located and oriented with respect to the sample from which they are taken, and similar technique is used in testing the specimens." They describe in detail the equipment and the procedure used for testing miniature specimens (reduced section 0.05 in. in diameter by 1/4 in. long, with overall length of 3/4 in.). From their test results, they reach the conclusions that (1) miniature size tension test specimens can be used satisfactorily for the determination of tensile strength and yield strength of at least some metals provided the material has a relatively fine grain structure, (2) the errors need not be greater than those encountered when using standard 1/2-inch diameter tension specimens, provided proper attention is given to the details of the testing procedure, and (3) the cost of making tension tests using miniature specimens may be appreciably greater than for standard size specimens when determining yield strength, but not when determining only tensile strength.

Bollenrath & Troost (1951) apply the methods of their earlier (1950) paper to bending and torsion of cylinders of various symmetrical sections. They develop analogous methods to analyze the dependence of fatigue strength on scale, and apply them to experimental data.

Futagami (1951) presents data on experimental determinations of the bending strength, the tensile strength, and other physical characteristics of glass rods and plates. He finds that both bending strength and tensile strength show a size effect, strengths increasing with decrease in cross-section size, and that the same results hold when the specimens have been previously notched or cracked. He concludes that a crack is simply a notch

of zero width and that the cause of the higher strength of smaller test pieces is that the breaking strength is closely connected with the migration of the aggregation units, whose dimension he estimates to be of the order of 1000 Å.

George (1951) presents evidence for the existence in filaments of small domains of size intermediate between atomic and macroscopic dimensions. He gives the mathematical relation, based on the "weakest link" theory of Peirce (1926), between the strength distribution $f(x)$ of one domain and the strength distribution $g(n)$ of a specimen of length n. A study of the distribution of strength data allows the inference that the domains are quite numerous, but does not allow the order of magnitude of their size to be determined. On the other hand, the order of magnitude of the dimension can be determined from studies of x-ray broadness as a function of plastic strain. These dimensions seem to correlate with strength properties.

Hempel (1951) gives a critical review of the literature which leads to the conclusion that, in most fatigue tests, geometrical influences are overshadowed by technological variables, such as residual manufacturing stresses and testing conditions. He reports the results of new tests which show (1) only slight differences between fatigue strengths of small specimens (4.6 to 10-mm. diameter) in rotating bending and in tension-compression tests, (2) no influence of specimen size in pulsating tension and tension-compression tests of plain specimens and specimens with holes (diameter 0.2 to 0.25 of specimen width), (3) appreciable effects of surface treatment and heat treatment on residual stresses and rotating bending fatigue strength, and (4) noticeable effects of testing conditions. He concludes that the usual stress-gradient hypothesis to explain size effects is not correct and that the problem is primarily a technological one.

Kinzey (1951) reports the results of a study of wood box columns. He states (p. 109): "Except for short columns, the strength of solid or spaced timber columns of a given wood species and grade is a function of the L/d value or ratio of length to least lateral dimension. For box columns this ratio must be revised to include the variable, plank thickness. ... For a square cross-section with a hollow core having an outside dimension d_1 and inside or core dimension d_2, ... the strength of box columns varies according to an $L/\sqrt{d_1^2 + d_2^2}$ ratio."

Meyersberg (1951) applies statistical theory to an investigation of the influence of size on the static strength of materials. He discusses the meaning of the constants in Weibull's formula for the failure probability at stress σ, 1-exp $V[(\sigma-\sigma_u)/\sigma_0]^m$. He applies the formula to data from bending and torsion experiments on cast-iron test specimens.

Morlier, Orr & Grant (1951) report the results of an empirical study of the relation of length to other physical characteristics of American Upland cottons. They conclude that the average breaking load and the average tenacity of single fibers increase and the coefficient of variation for breaking load decreases with increasing fiber length. On the basis of single-fiber tests on only three length groups, they describe a method for calculating a tenacity "index" which shows a good correlation with the single-fiber tenacity calculated on the basis of all the fibers in the sample.

Phillips & Fenner (1951) describe fatigue tests on aluminum-alloy and mild-steel sheet, with and without drilled holes. Test panels were approximately 0.1 inch in thickness and up to 12 inches in width. For both materials, the fatigue strength is reduced by increasing the width of the test piece from 3/4 inch to 9 inches, but the reduction is much less for the mild-steel sheet than for the aluminum-alloy sheet (about 15% vs. over 50%). Further reduction in strength is caused by drilling a hole in the center of the test panel, but very small holes cause little or no reduction in fatigue strength.

Phillips & Heywood (1951) report the results of determinations of the fatigue strength under reversed direct stress for notched and unnotched steel specimens of various diameters. The fatigue strength of unnotched test pieces of mild steel (25 tons/sq. in. ultimate tensile strength) and of 2-1/2% nickel-chromium steel (65 tons/sq. in. ultimate tensile strength) was independent of the size of the test pieces for diameters of the test section ranging from 0.19 inch to 1.3 inches. For both of these materials, the fatigue strength of geometrically similar test pieces of from 0.19 inch to 2.4 inches diameter, containing a transverse hole of one-sixth the diameter, was greater for the smaller sizes of specimen, the size effect being much larger for the mild steel than for the alloy steel.

Price (1951) includes the dimensions of the test specimens among the factors influencing the compressive strength of concrete. He presents graphs showing the influence of (1) the diameter of the cylindrical specimen when

the height equals two diameters and (2) the ratio of the length (height) of the cylinder to its diameter. He attributes the former effect to faster strength gain of the smaller cylinders with age, and offers no theoretical explanation of the latter effect.

Schaal (1951) concludes, on the basis of determination by x-ray methods that the static yield stress of the surface fibers in bending of alloy steel, carbon steel, and aluminum alloy increases with decrease in thickness of the specimens, that the increase in fatigue strength in bending with decrease in thickness of the specimens is due to the steeper slope of the stress curve in thinner specimens.

Stulen (1951) writes (p. 23); "It is a common hypothesis in investigating the physical properties of a material to assume that the material is perfectly homogeneous throughout its volume in respect to these properties. This idealization of the properties of a material is one of the fundamental assumptions in the classical theory of elasticity. ... The well-known size effect in fatigue is not predictable by the classical theory." Stulen summarizes the elementary theory of extreme values as related to the size effect as follows (pp. 26-27): "The probability that the endurance limit of a specimen will be between a stress value S and a slightly higher stress value (S + dS) is equal to : $p_0\, dS$ (1) [where $p_0 = p_0(S)$ is the probability density function of the endurance limit, so that $\int_0^\infty p_0 dS=1$]. The probability that one specimen has an endurance limit <u>not less</u> than a specified value S is computed by adding up the individual probabilities that the specimen will have endurance limits in each stress increment from the specified value to infinity. This, in the limit, leads to the following relation: $P_0 = \int_S^\infty p_0 dS$ (2). The value of P_0 depends on the lower limit of the integral and is therefore a function of the stress. It is called the cumulative frequency distribution [note that it is the complement of the usual cumulative distribution function]. ... In similar manner, the cumulative frequency distribution of a specimen n times as large can be calculated. The probability, P_1, that a larger specimen composed of n small specimens has an endurance limit not less than S is equal to the product of P_0 by itself n times or $P_1 = [\int_S^\infty p_0 dS]^n$ (3). The frequency distribution of these larger specimens is calculated by taking the negative derivative of Eq 3 with respect to S. This leads to: $p_1 = -dP_1/dS = p_0 n[\int_S^\infty p_0 dS]^{n-1}$ (4). Therefore, if the original frequency distribution (or probability) curve

is accurately known for a given material it is possible to extrapolate the data for a larger or smaller size specimen, employing Eqs 3 and 4. The foregoing equations are the elementary relations in the theory of the distribution of extreme values. Epstein [(1948a)] in a recent paper showed that the size or volume effect is directly dependent on the type of distribution curve of the strengths of the reference volume. He has calculated the relations between volume and fracture strength assuming different distributions of strengths of the reference volume. It is known that the endurance limits of specimens tested in a push-pull machine are less than those tested in bending ... . This is to be expected on the basis of probability theory, since less material is being subjected to the maximum or near maximum stress in the bending test. In order to adapt the probability theory to specimens in which the stress is not uniform throughout the specimen, allowance for the fact that different volumes within the specimen are stressed at different levels must be made. Fowler [(1945)] has derived a formula for the case where the alternating stress is not uniform within the specimen or part. ..." Stulen adds (p. 32): "Since the laws governing the size effect on fatigue life have not been established, it is not possible at present to correct the complete S-N probability curves for this effect. Therefore, fatigue life data as obtained from small laboratory specimens cannot be used to predict accurately the fatigue life of full-scale machine parts." Stulen also reports the results of experimental tests of the size effect. In determining the effect of volume, he estimates the effective volume of a specimen as that portion of the volume where the oscillating stress is within 15 per cent of the maximum stress on the specimen.

Tabor (1951) does not discuss size effect, but he establishes a relationship between the hardness and the strength of metals which helps to explain the analogy, first noted by Auerbach (1893), between empirical expressions for the effect of size on hardness and on tensile strength.

Weibull (1951) writes (p. 293): "If a variable X is attributed to the individuals of a population, the distribution function (df) of X, denoted F(x), may be defined as the number of all individuals having an $X \leq x$, divided by the total number of individuals. This function also gives the probability P of choosing at random an individual having a value of X equal to or less than x, and thus we have $P(X \leq x) = F(x)$ [1]. Any distribution function may be

written in the form $F(x)= 1-e^{-\phi(x)}$ [2]. This seems to be a complication, but the advantage of this formal transformation depends on the relationship $(1-P)^n=e^{-n\phi(x)}$ [3]. The merits of this formula will be demonstrated on a simple problem. Assume that we have a chain consisting of several links. If we have found, by testing, the probability of failure P at any load applied to a 'single' link, and if we want to find the probability of failure P_n of a chain of n links, we have to base our deductions upon the proposition that the chain as a whole has failed, if any one of its parts has failed. Accordingly, the probability of nonfailure of the chain, $(1-P_n)$, is equal to the probability of the simultaneous nonfailure of all the links. Thus we have $(1-P_n)=(1-P)^n$. If then the df of a single link takes the form Equation [2], we obtain $P_n= 1-e^{-n\phi(x)}$ [4]. Equation [4] gives the appropriate mathematical expression for the principle of the weakest link in the chain, or, more generally, for the size effect on failures in solids. ... Now we have to specify the function $\phi(x)$. The only necessary general condition this function has to satisfy is to be a positive, nondecreasing function, vanishing at a value x_u, which is not of necessity equal to zero. The most simple function satisfying this condition is $[(x-x_u)/x_0]^m$, and thus we put $F(x)= 1-e^{-[(x-x_u)/x_0]^m}$ [5]. [Misprints noted in the discussion have been corrected.] The only merit of this df is to be found in the fact that it is the simplest mathematical expression of the appropriate form, Equation [2], which satisfies the necessary general conditions. Experience has shown that, in many cases, it fits the observations better than any other known distribution function."
Weibull does not claim any theoretical justification for the distribution function (Equation [5]) which has come to be called by his name, but actually, as pointed out earlier, it is the third asymptotic distribution of smallest values, which has adequate theoretical justification as the limiting form of a distribution whose range is limited $(x \geqslant x_u)$.

Wiegand (1951) reports finding a size effect in bending- and torsion-fatigue tests on Cr-Ni-W steel with unnotched, smooth specimens as well as specimens with a transverse hole. The fatigue strength decreased as the specimen diameter increased from 5 to 30 mm. for bending and from 15 to 35 mm. for torsion, with no significant further decrease for larger diameters.

Wilson, Hechtman & Bruckner (1951) report an attempt to determine the factors that influence the formation of cleavage fractures in ship plates by

fracturing (in tension) three-quarter-in. plates from 12 in. to 72 in. wide, containing centrally located geometrical stress raisers. They found that for all kinds of steel tested in the form of plates 48 in. or more in width, the average unit strength of the plate decreased (by about 6,000 to 7,000 p.s.i.) with an increase in the width of the plate from 12 in. to 72 in., with indications that the decrease in strength with increase in width would continue beyond a width of 72 in., the width of the widest plates tested.

Backer, Marshall & Shaw (1952), having found the energy required to remove a cubic inch of metal (specific energy) by grinding to be unusually high, consider the size effect and its influence on the specific energy. After reviewing results found much earlier by Karmarsch and by Griffith, they investigate a cutting process involving the formation of very small chips at high cutting speeds (micromilling), and apply the results to a study of the grinding operation. A significant increase in shear energy is observed with decrease in specimen (chip) size. The sheer stress involved in grinding metals under mild conditions is found to correspond to the theoretical strength, which is about 1.8×10^6 psi for steel.

Cazaud (1952) describes an investigation of the influence of shape, size, and type of loading on the fatigue limits of several types of steel. In axial alternating tension-compression loading of cylindrical bars having a certain length (20 mm.) of constant diameter (minimum cross section), the fatigue limit was independent of the size, but bars with a concave throat and the same minimum cross section as the straight bars were found to have higher fatigue limits. Rotation-bend loading gave higher fatigue limits than axial tension-compression, the difference increasing with decreasing diameter of the bend specimen, in accordance with the general experience that, for nonuniform stress distributions, the fatigue limit increases when the size is decreased, because of an increase in the stress gradient. A statistical analysis shows that the data follow a normal distribution, with standard deviation twice as large for the smallest bend specimen (diameter 2.58 mm.) as for the largest one (diameter 9.6 mm.).

Epremian & Mehl (1952) review the literature on the statistical theory of fatigue and on the size effect, including the work of Weibull (1939a), Moore & Morkovin (1942-44), von Philipp (1942), Buchmann (1943), Freudenthal (1946), and Fisher & Hollomon (1947). They write (pp. 66-68): "The size effect

refers to the observation that the endurance limit in rotating-cantilever tests increases with decrease in specimen diameter. ... The usual explanation of this effect is based on the 'weakest link' theory. According to this view, the probability of obtaining a serious internal flaw or stress-raiser in the specimen increases with increase in the volume of the specimen and therefore lower fatigue properties are obtained with larger-diameter specimens. Some of the work in the present investigation has substantiated this concept, but there is an additional factor in this problem which has been overlooked by all but a few investigators. The rotating-cantilever fatigue test, with which most of the size-effect studies have been made, produces a stress gradient across the diameter of the specimen since one side is in pure tension while the opposite side is in pure compression. Thus tests of this kind on specimens of different diameters involve correspondingly different stress gradients. Phillips [von Philipp] ... and Buchmann have studied the effect of this factor on the size effect and concluded that with a steeper stress gradient (smaller-diameter specimen) the underlying fibers have a greater opportunity to aid the outermost fibers in supporting the stress. Buchmann has conducted axial push-pull fatigue tests (which eliminate the stress gradient across the section) and found only a small increase in the fatigue strength of steels and aluminum with decrease in specimen diameter. He also showed that the fatigue properties under rotating bending of specimens with increased diameter approach the strength obtained under reversed axial loading as a lower limit. ... The first and obvious point to be made is that the endurance limit for a material with many imperfections is lower than that for the same material with extremely few imperfections. Thus the endurance limit of a material is strongly influenced by the presence or absence of imperfections. In the axial push-pull test the maximum nominal stress is uniform along the gage length and the cross section and therefore any point in the volume of the specimen can be the locus of crack initiation. Changes in the diameter of this type of specimen have little influence on the probability of occurrence of an effective imperfection in the test volume, because in this case the critical volume is vast at all diameters and the probability of encountering an active imperfection is very high. Consequently, the endurance limit in push-pull tests does not increase appreciably with decrease in specimen diameter and further

the endurance limit obtained is relatively low because it is certain that effective imperfections are encountered. On the other hand, in the R. R. Moore [rotating-cantilever] test, only a small region is subjected to maximum applied stress. Aside from the differences in stress gradient, the smaller-diameter specimens have a smaller critical-stress volume which decreases the probability of the existence of a severe imperfection in this volume. Thus a decrease in diameter produces an increase in endurance limit. Upon increasing the diameter, the critical volume increases (as does the probability of imperfection occurrence) to a size effectively the same as that existing in push-pull tests and the same endurance limits are obtained in the two types of tests."

Gensamer (1952), in reviewing the results of tests of large ship plates [see Boodberg et al (1948)] states (pp. 10-11): "The objectives were to learn (1) how large specimens ... behaved, and (2) how one could predict the behavior of large specimens from that of small specimens, economically tested. ... Below the transition temperature, an energy balance indicates that the square of the stress should vary as the reciprocal of the width; this it does. As a result, an infinite plate should not break at a stress much below the stress at which a 6-foot plate breaks, which is very comforting."

Hempel (1952) lists the factors responsible for fatigue fracture, and groups them into influence of size, of shape, and of the characteristic properties of the test specimen. The actual work which he reports deals with the influence of factors other than specimen size.

Isibasi (1952) reports the results of tests, in rotating beam machines, of hollow cylindrical specimens of gray cast iron with outside diameter of 12 or 23 mm., with transverse holes whose diameters varied from 0.05 to 0.48 of the specimen diameter. The strength-reduction factor of the notch was nearly unity as long as the diameter of the hole was small compared with the specimen diameter. The strength reduction increased with increased size of specimen and greater diameter of hole. The author proposes an explanation for this size effect based on the stress-raising effect of the presence of graphite flakes, superposed on the localized stress at the hole. Values computed from an empirical formula based on the stress at the root of the graphite flake show good agreement with observed values of strength reduction.

MacGregor & Grossman (1952), in a paper dealing primarily with the dimensional effect on the transition temperature from ductile to brittle fracture of circular disks and unnotched rectangular plates of 0.95C steel, report finding a definite increase in brittle fracture strength with a decrease in size.

Meyersberg (1952) surveys the literature on the size effect, with particular attention to the theories of Griffith and Weibull. He accepts the theory of Weibull, and gives a new interpretation of the scale parameter m as a "sensitivity factor" whose value is a function of the stress state as well as a material constant. For uniform tension, it is essentially a material constant and reflects the nature, number, and distribution of the flaws in the material. It can be determined experimentally, by testing specimens of various volumes, as the slope of the logarithmic plot of the breaking stress against the volume. The author gives corrections needed in order to apply the values of standardized tests to various loading conditions. His discussion of the size effect deals primarily with the breaking strength of brittle materials like cast iron, but he assumes that the same theory could also be applied to the yield strength of ductile materials.

Muller (1952) discusses the "weakest link" theory of size effect on fracture strength, following the development of Kontorova & Frenkel (1941). If F_1 and F_2 are the cumulative distribution functions of strengths of specimens of volumes V_1 and V_2 respectively, the weakest link theory gives the relation $1-F_2= (1-F_1)^x$, with $x=V_2/V_1$. The author gives a procedure and a numerical table for estimating the mean and the variance of F_2, given the mean and the variance of F_1. The table is based on the normal distribution, and the procedure is based on the assumption that both F_1 and F_2 are normal. The reviewer (William Fuller Brown, Jr.) points out that simultaneous normality of F_1 and F_2 is inconsistent with the relation between them; hence he believes that the author's procedure (if valid) requires more elaborate justification.

Palm (1952) reports the results of an experimental investigation of the true fracture strength in static tensile loading of sharply notched cylindrical test bars of 24ST and 51SW aluminum alloys. Factors varied included length and size (diameter) of test bar, notch angle, and notch depth at constant diameter of notched section. He concludes from the test data that the fracture strength

(1) has a slight tendency to decrease with increasing notch length and size of the test bar, (2) first decreases with decreasing notch angle and eventually increases again, and (3) at first decreases sharply with increasing notch depth and then increases again. He attributes the size effect to a change in relative notch sharpness.

Petersen (1952) surveys ideas, hypotheses, and mathematical formulas on the relation between fatigue limit and various factors, including internal notches and the size and shape of the structural member. The fatigue limit of a simple straight bar with an external notch can be calculated from these formulas. For every more complicated structural member, it is possible to find a simple externally notched reference bar which will behave exactly the same under the same conditions of fatigue stress. The relation between the geometry of the reference bar and the size and shape of the structural member has been determined experimentally for several types of structural members.

Peterson (1952), in reviewing the fatigue of materials field, discusses, among other topics, the statistical aspects and the notch effect. He writes (pp. 1-2): "Apart from improving accuracy of data, there is another reason for studying the statistical aspects of fatigue, *namely the consideration of* 'size effect' [Weibull (1939b)]. This is important because large members are weaker in fatigue than tests on small specimens would indicate. While a complete theory has not as yet been established, it is safe to say that the concept of statistical behavior is a necessary part of the explanation of 'size effect'. ... It seems certain that in the next decade the statistical aspects of fatigue will be emphasized in numerous publications. ... Usually the effect of a notch (using the term generically to denote a geometrical variation causing stress concentration) is less than would be obtained if the stress concentration factor were fully effective. This is particularly true as the absolute size of the notch is decreased to small values. ... Since the critical section of a machine part nearly always involves some degree of stress concentration, it is clear that the work on notch effect in fatigue will be of considerable interest. Fatigue tests of various cross-sectional shapes show that lower mean values are obtained for larger volumes of material at the peak-stress region. This indicates that statistical theory can at least partly explain the above-mentioned variation of the fatigue notch factor."

Puttick & Thring (1952) give a treatment of the effect of specimen length on the strength of a material with random flaws which they synopsize as follows (p. 56): "The statistical strength properties of materials are discussed for tests, such as the tensile test, in which it may be assumed that the specimen fails at its weakest point. The test piece thus behaves like a chain, and the variation of mean and standard deviation of the results of tests with length of specimen depends on the distribution of strengths of individual links. The consequences of two distributions are considered: first, that the distribution is normal, and second, that there is a very small number of links, randomly spaced, whose strength is much smaller than the rest. These correspond respectively to a material homogeneous on the scale of the test and one in which there are defects whose size is small compared with the test length but whose mean distance apart is comparable with it. The former model predicts a continuous decrease in the mean test result and standard deviation with increase in specimen length; the latter predicts a continuous decrease in the mean but a maximum for the standard deviation for a specimen length for which the probability of including at least one flaw is 1/2. Experiments on cold-drawn phosphor-bronze wire show that this model gives a better description of the variation of standard deviation and mean with test length, the mean spacing of the flaws being about 3 in. For such a material it is desirable to take large samples ($\not<$20) for at least three different lengths." The authors also write (p. 56): "In varying the lateral dimensions of the specimen the appropriate model is not obvious; even if the discussion is restricted to purely statistical effects, the theoretical variation of strength with diameter depends on whether the flaws responsible for failure are at the surface or distributed throughout the volume of the material. The question of varying the diameter of the test piece when the weakest-link condition does not hold is discussed in the Appendix." The discussion in the Appendix of the effect of varying the lateral dimensions of the specimen is based on the strength-summation (or strength-averaging) theory of Tucker (1945a).

Shaw (1952) in discussing a yield criterion for ductile metals based on atomic structure, writes (Abstract, p. 109): "A strength theory for ductile materials is presented that takes into account the short range inhomogeneities that are present in all commercial metals. Examples are presented to illustrate the shortcomings of the conventional maximum shear stress and distortion energy

theories that result from the failure of these theories to recognize the inhomogeneous nature of metals. The new theory predicts a variation in yield stress with specimen size and it is shown why such a size effect is observed with torsion specimens but not with tensile specimens of usual size. The size effect in fatigue testing and hardness testing by means of micro techniques is discussed to illustrate further the concepts presented. Finally, the relation of the new theory for ductile metals to the statistical theory of Weibull for brittle materials is considered." In his concluding remarks (p. 125), the author states: "The size effect for ductile metals might be referred to as a surface effect as opposed to the volume-type size effect that arises from Weibull's theory for brittle materials."

Weibull (1952a), writing on the statistical design of fatigue experiments, makes the following statements (pp. 109-111) relevant to the size effect: "It is quite impossible to decide experimentally whether or not the size, or any other factor, has any effect on the endurance limit, without knowing how accurately this limit has been determined. Furthermore, if such a decision is to have any significance at all, the accuracy must be sufficiently high, in proportion to the observed effect. ... The scatter in fatigue lives consists of two parts, the first caused by the specimen its material, surface conditions, and so forth, the other caused by errors in the nominal loads of the testing machine. ... When studying the size effect, which statistically depends entirely upon the scatter, it is absolutely necessary to separate the two parts of the scatter, as the second part does not have any influence on the size effect".

Weibull (1952b), writes (pp. 449-450): "This [size] effect has been proved in a great many static tests with specimens of brittle materials such as rock salt [Voigt(1893)], plaster of Paris [Roark & Hartenberg (1935); Roark, Hartenberg & Williams (1938)] [see Additional References, p.409], glass [Griffith (1920); Auerbach (1891); Weibull (1938)], crystalline minerals [Auerbach (1891, 1896)], steel at low temperatures [Davidenkov, Shevandin & Wittmann (1947)], cast iron [Oberhoffer & Poensgen (1922); Pinsl (1933); Meyersberg (1952)], and porcelain [Weibull (1939b)]. It may be mentioned as most remarkable that thin glass fibers have a tensile strength much higher than steel [Griffith (1920)], that the breaking load on a ball pressed against

a plate is proportional to the diameter of the ball [Auerbach (1891, 1896); Weibull (1938)] which is in sharp contrast with all classical rupture criteria [Weibull (1939a), p. 20], and that cast iron shows a surface effect which overlaps the volume effect [Meyersberg (1952)]. The volume and the surface effects may be separated by statistical mean[s] as demonstrated on porcelain rods [Weibull (1939b), pp. 47-49]. The earliest observations of size effects in ductile materials were made on wires of many different metals by varying the diameter [Karmarsch (1859); Baumeister (1883)]. The influence of the length of the wire [Gurney (1947)], of the diameter of cylindrical test specimens [Miklowitz (1950)], and of the width of ship plates [Wilson, Hechtman & Bruckner (1948)] have also been demonstrated. ... The law of similitude [Weibull (1939b), p. 12] has been verified by tensile tests on steel specimens of complicated shape [Weibull (1939a), pp. 31,42] and by bursting spherical rotors by centrifugal forces [Beams (1949)]. Size effects in fatigue have been studied in many investigations [Mailänder & Bauersfeld (1934); Gillett (1940); von Philipp (1942); Buchmann (1943); Siebel & Pfender (1947); Aphanasiev (1948); Weibull (1949, 1952a); Phillips & Heywood (1951); Hempel (1951); Wiegand (1951)]. ... As the scatter of fatigue life, in general, is very large, one must be warned against basing conclusions about size effects on too small a number of observations [Weibull (1952a)]. Evidently, the relation between size effect and scatter holds good only in so far as the scatter is due to the material, and not to the measuring technique or other irrelevant factors". Weibull also includes a brief section on notch effects and their relation to size effects, with 14 references, including Nadai & Mc Gregor (1934); Shearin, Ruark & Trimble (1948); Peterson & Wahl (1936); Laurent (1949); and Brueggeman & Mayer (1948).

Wellinger & Gimmel (1952) report that fatigue tests with nitrided 0.34%C, 1.4% Cr, 1.1% Al steel of 70 kg/mm^2 tensile strength showed bending fatigue strength (at 10^7 cycles) of core ± 34kg/mm^2 independent of test-piece diameter, with surface fatigue strength (at 10^7 cycles) decreasing from ± 45kg/mm^2 for 5-mm diameter test pieces to ± 41 kg/mm^2 for 6.5-mm diameter test pieces and ± 40 kg/mm^2 for 8-mm diameter test pieces. They conclude that for test pieces of diameter greater than 20 mm, the nitrided layer has little or no effect on the bending-fatigue strength, which will fall to that of the core material.

The reviewer (R. Weck) suggests that the value of the investigation would have been greatly enhanced by including tests on non-nitrided test pieces in order to determine whether the effect observed is size effect or is definitely attributable to the nitrided layer, and by extending the range of diameters up to (say) 25mm.

Wright & Garwood (1952) report a study involving the size effect on the flexural strength of concrete which they summarize as follows (p. 67): "As a basis for standardizing a method of test for the modulus of rupture of concrete, flexure tests have been carried out on beams of various sizes using central and third-point methods of loading and various rates of loading. Tests were also carried out on small beams sawn from larger ones in order to isolate the effects of specimen size. A reduction of approximately 30 per cent was observed in the modulus of rupture when the depth of the beam was increased from 3 in. to 8 in. for a span-depth ratio of 3, but other effects of size were small. Increasing the rate of increase of stress from 20 to 1,140 lb per sq. in. per min. resulted in an increase of about 15 per cent in the modulus of rupture. Central loading gave results about 20 to 25 per cent higher than third-point loading but the results were less uniform. The effects of the size of the specimen have been largely accounted for by variations in the quality of the concrete in beams of different sizes together with a statistical aspect termed the 'weakest link' theory, and an effect due to changes in the rate of increase of stress. The effect of the method of loading can largely be explained by the 'weakest link' theory and by considerations of the stress distributions in the comparatively short beams used." The body of the paper is the work of Wright. In an appendix, Garwood discusses the statistical basis of the "weakest link" theory advanced by Weibull (1939a) and Tucker (1941).

Yokobori (1952) discusses discontinuous phenomena in plasticity of metals, such as yielding, ultimate strength, and brittle fracture, which he regards as a kind of Markoff process and studies from the standpoint of nucleation theory. He gives a unified interpretation of size effect and other features of these types of failure. He develops two approximations to the most probable yield stress S_m as a function of the specimen volume V, namely $S_m \doteq V^{-\gamma}$ (where γ is an unspecified constant) and $S_m \doteq$ constant $- \log V$. He reports that the first of these is well in accordance with the experimental results of Davidenkov et al (1947).

Afanas'ev (1953) explains the statistical theory of fatigue stability of metals developed and previously published by him, and relates it to the influence of various factors, including the shape and dimensions of the test specimen. His theory is not the "weakest link" theory, but one based on stresses in the crystalline structure of the metal.

Findley (1953), in a review of the state-of-the-art in the mechanical behavior and testing of plastics, states that, up to that time, little or nothing had been done on the size effect on the strength of plastics, although some work had been done on the effect of notches. Among his 119 references, there are none on size effect and five on notch effect. He reports that fatigue notch factors observed ranged from 2.1 to 0.8 and depended markedly on the number of cycles at which the fatigue strengths were determined.

Freudenthal & Gumbel (1953) present a statistical interpretation of fatigue tests, based on extreme-value theory. The size effect is not their primary concern, but the following remarks (pp. 310-311) are relevant: "Assuming constant crack density, the number of cracks increases with the volume of the specimen, and so does the probability of encountering a crack of given severity. A relation can therefore be assumed to exist between the strength of a specimen and the volume of the specimen if the distribution, in the specimen, of cracks of different size is known, and if it is assumed that a condition of instability at the end of the largest crack, which causes the most severe stress concentration, is sufficient to ensure the rapid propagation of this crack throughout the entire volume of the specimen [Griffith (1920)] ('weakest link' concept). The problem of the distribution of the strength of specimens determined by the largest crack is thus reduced to one of the distribution of largest cracks ('extreme value' theory). This problem is solved by the derivation of the distribution function of the smallest value of strength associated with the largest crack, as a function of the total number of randomly distributed cracks in the specimen, for any given distribution function of crack size or severity. Extreme value distributions are well known in applied statistics, and their importance has been established in various fields of science and technology. Their theory has been developed by Fisher & Tippett (1928), von Mises (1936) and Gumbel (1935) [see Bibliography on Extreme-value Theory, p. 409]. Different forms of distribution functions of

cracks can be assumed and different resultant relations between the strength and the volume of the specimen obtained [Epstein (1948a)]."

Johnson (1953) reports the results of an investigation on the effects produced by the volume and the stress distribution on the strength of structures. In Section 13(pp. 58-76) he reviews the literature on the size effect on material strength and works out the theory for variable volume with uniform or variable stress distribution and brittle or plastic materials. In Section 17(pp.94-125) he compares the results of the theoretical study with test results.

Kase (1953) gives a mathematical treatment of the distributions of tensile strength of natural rubber and GR-S, based on the "largest flaw" or "weakest link" theory. He shows that the distributions are of the so-called doubly exponential type [Type I extreme-value], and that the tensile strength S of rubber is given by $S = S_0(1-\alpha'x/A)$, where S_0 is the theoretical tensile strength of rubber when free of flaws, x and A are the original cross-sectional areas of a flaw and the specimen, respectively, and α' is a constant.

Moszynski (1953) notes that the strength properties of a machine element under static conditions are chance variables which appear to obey the lognormal distribution, and he assumes that the fatigue limit follows the same distribution. Although the safety factor under static conditions can be calculated from probability considerations by assuming as criterion the simultaneous occurrence of a number of independent chance variables, he points out that the assumption of independence does not hold for fatigue applications, since geometry (dimensions, size, and notches) will affect strength variables through variation in stress gradients, as will surface condition and environment. The author attempts to account for the most important interactions of chance variables for a machine element under cyclic loading, and illustrates the method by a numerical example.

Plum (1953) shows that if the homogeneity of concrete is improved, the average strength may be lowered without impairing the actual reliability of the structure. He gives a simplified example based on certain assumptions, one of which is that the frequency distribution of compressive strength in the completed structure is independent of the dimensions of the structural member. He states (Appendix, pp.335-336): "From experiences with the testing of materials, it is, however, well known that the form and parameters of the

frequency distribution vary with different shapes and dimensions of the test specimen. Further references on this point are given in the Bibliography ... . So far as the Author knows, it has not been possible to establish a relation between the frequency distribution of compressive strength of concrete test specimens with given dimensions, and the frequency distribution for structural members made of identical concrete but with other dimensions."

Timoshenko (1953) traces the history of the strength of materials from the times of the ancient Egyptians, Greeks, and Romans. Knowledge of the size effect goes back at least as far as Leonardo da Vinci, who made experimental studies of the dependence of the strength of iron wires on their length, of beams on their length and width (but apparently not on their depth), and of columns on their length and cross section (see pp.3-6). Leonardo da Vinci did not publish his work, but left notebooks containing much information on his great discoveries in various branches of science. Probably the first to publish work on the size effect was Galileo Galilei (1638) [see Additional References, p.409]. Timoshenko notes that Galileo states that if we make structures geometrically similar, then, with increase of their dimensions, they become weaker and weaker; he reports in some detail (pp. 11-15) on Galileo's elaboration of this principle. From these beginnings, Timoshenko traces the development of theoretical and experimental work on the strength of materials up to the middle of the twentieth century. Work relevant to the size effect which he discusses includes that of Karmarsch in the mid-nineteenth century on metal wires of various diameters, of Griffith (1920) on the tensile strength of thin fibers of glass, and of Peterson [& Wahl] (1936) on stress concentration, also the theoretical explanation of the size effect on a statistical basis ["weakest link" theory] by Weibull (1939a), related experimental results of Davidenkov [et al] (1947), and an attempt by Peterson to explain the size effect in fatigue on the basis of Weibull's theory.

Chechulin (1954) credits Alexandrov & Žurkov (1933) with origination of the idea of the statistical nature of brittle fracture, and notes that two formal statistical theories of brittle strength have been developed, the theory of Weibull (1939a) and the theory of Kontorova & Frenkel (1941). He gives a critical analysis of the underlying assumptions of these two theories. He points out that the Kontorova-Frenkel theory, based on the assumption of a normal distribution of local strength, admits negative values of brittle

strength, which are theoretically wrong and practically absurd, for sufficiently large volumes and infinitely large values of brittle strength for vanishingly small volumes. He criticizes the Weibull theory, based on the Weibull distribution [third asymptotic distribution of smallest values] for its lack of a rigorous physical basis, but admits that, when compared with experimental results, it gives not bad qualitative agreement with the basic conclusions of the theory. He proposes a new theory, based on the Pearson Type III distribution, which he claims is more rigorous than the previous theories. He shows that Weibull's theory is a special case of his own when the number of defects in the body is large.

Colner & Francis (1954) report the results of a study of the "area effect", which they define as the phenomenon whereby small areas show long times to failure while large areas show short times, in 24S aluminum alloy specimens prepared from rolled sheet. The variable measured was not strength as such, but the related variable time to failure (deflection due to cracking), at various stress levels. The effects of stress level, degree of sensitivity of the alloy, and hydrogen peroxide concentration in the corrosion medium were studied. The following conclusions were drawn: (1) The area effect is quite critically dependent on the hydrogen peroxide concentration, with the maximum area effect, under the conditions of the experiment, at about 2.0 grams/liter of hydrogen peroxide; (2) A stress level in excess of about 60 percent of the yield strength does not appreciably influence the area effect; (3) The area effect seemed most pronounced with specimens having maximum sensitization (quenched and aged 6 hours at 325^{o} to 375^{o}F.);(4) The area effect in the 24S alloy is an electrochemical phenomenon and not a mechanical one. No mention is made of the role of extreme-value theory.

Corten, Dimoff & Dolan (1954) appraise the Prot progressively increasing load method of fatigue testing by comparing the experimental results for three ferrous metals and an aluminum alloy with conventional (constant load) fatigue data. They conclude that the Prot method appears most promising for rapid estimation of the endurance limit of ferrous metals. On page 875 they state: " ... after the endurance limit is established, the direct application of this information to the design of complex load-resisting members is somewhat uncertain due to such phenomena as the effect of the state of stress,

notch sensitivity, and size and shape effects. Consequently, testing of expensive complex assemblies is often necessary to determine endurance limit loads. Thus the desirability of a reliable short-time method for determining the endurance limit and estimating the statistical variation using only a relatively few specimens or complete assemblies has long been evident. The Prot method of determining the endurance limit appears to incorporate many desirable features from the point of view of reducing the number of specimens required and the time for each test."

Derman, Kwo & Gumbel (1954) denote the probability that a metallic specimen will survive at least N stress cycles, where the stress load is S, by the function $\ell_s(N)$, which they call the survivorship function. They note that Frendenthal & Gumbel (1953) used the relation $\ell_s(N) = e^{-(N/V_s)^{\alpha_s}}$, where α_s and V_s are unknown parameters dependent upon S, which was proposed by Weibull (1949) in considering the distribution of S for a fixed N, and which provides a good fit to some observed data. However, Freudenthal & Gumbel found it desirable in other cases, especially when the stress level was low, to assume $\ell_s(N)=1$ for $0<N<N_{0,s}$ and $\ell_s(N)=e^{-[(N-N_{0,s})/(V_s-N_{0,s})]^{\alpha_s}}$ for $N>N_{0,s}$ (3), where $N_{0,s}$ (the "minimum life"), α_s, and V_s are unknown parameters dependent upon S. The present authors point out that there is some theoretical justification for the latter assumption (of which the former is a special case, with $N_{0,s} = 0$) other than the fact that a good fit is obtained, namely, that it is a discrete approximation (since N is an integer) to the (continuous) extreme value distribution. They also present (pp.3-4) another argument to justify the appropriateness of (3): "Weibull [(1949)] considered fatigue trials on metal bars subjected to bending stresses. He reasoned that if $\ell_s(N)$ is the probability that a bar of length L will survive at least N cycles, then $\ell_s^2(N)$ is the probability that a bar of length 2L, all other dimensions remaining constant, will survive at least N cycles. This follows from elementary notions in the theory of probability if one assumes that the bar will break whenever the first or second half of the bar breaks, and that the probability of either half breaking is independent of the other. In the data considered by Weibull, this size effect relationship is shown to hold true. Now, if one assumes this relationship true for any positive length L, a consequence is the formula (3). This follows from the same mathematical argument used to derive (3) as an extreme value distribution. Thus, at least for certain experimental conditions

(in this case a bar and bending stress) the formula (3) seems to be a reasonable parametric form of $\ell_s(N)$." The authors develop the necessary theory for the statistical analysis of fatigue data under the assumption that (3) holds.

Grover, Gordon & Jackson (1954), in their book on the fatigue of metals and structures, devote Chapter VII (pp.108-121) to the effect of size and shape upon the fatigue strength of a part. They review some of the accumulated evidence concerning the apparent effects of size and shape on the fatigue strength of a member, pointing out several complications which make it difficult to draw general conclusions concerning the theoretical implications of reported investigations of size effect. They summarize reported data on the size effect in rotating-bending tests [Peterson (1930), Faulhaber et al (1933), Peterson & Wahl (1936), von Philipp (1942), Moore & Morkovin (1942-44), Buchmann (1934), Siebel & Pfender (1947)], repeated-torsion tests [Mailänder & Bauersfeld (1943), Dorey (1948), Wiegand (1951)], and axial-loading tests [Hempel (1951), Philipps & Heywood (1951)]. They give reasons why it is difficult to draw valid conclusions concerning size effects for stress levels above the fatigue limit. They also review reported data on the effect of cross-section shape [Buchmann (1943), Oberg & Rooney (1949), Wiegand (1951)]. In discussing the apparent size and shape effects, they note that one point of view that has been advanced [Weibull (1939a), Fowler (1945), Afanasiev (1948)] is that a test piece in which a large volume is subjected to high stress has more chance of having some weak spot that determines failure than does a smaller piece (the "weakest link" theory). They also discuss size effect in notched specimens, and close with some simple rules for making allowance for size and shape effects in design.

Gumbel (1954) summarizes the statistical theory of extreme values and discusses some practical applications, one of which is to the size effect on the breaking strength of materials. He writes (p.6): "Most authors concerned with these problems do not realize that they are of a <u>statistical</u> nature. To find a solution, A. A. Griffith (1920) assumes the fact that the flaws are distributed at random with a certain average density per unit volume. This means that a different amount of force will be needed to fracture it at one or another point. Then the strength of a given specimen is determined by the weakest point. <u>No chain is stronger than its weakest link</u>. This experimental fact is the basis for the application of statistical theories. The problem

of establishing the dependency of the strength on length or volume is equivalent to studying the distribution of the smallest value in a given sample size and for a given distribution. The relation of this truism to the strength of specimens has been introduced by Peirce [(1926)] in his studies of the strength of yarn. A good exposition of these methods to fracture problems and the breakdown of capacitors has been given by Epstein and Brooks [(1948)]." More details of the application of extreme-value theory to the breaking strength of materials are given later in the monograph (pp.40-42).

Gumbel & Lieblein (1954) discuss some applications of extreme-value methods, including fracture and fatigue. They write (pp.14-15): "In these instances, the observed strength of a specimen often differs from the calculated strength, and depends, among other things, upon the length and volume. An explanation is to be found in the existence of weakening flaws assumed to be distributed at random in the body and assumed not to influence one another in any way. The observed strength is determined by that of the weakest region."

Huber & Beedle (1954) report that the residual stress level of a full cross section of hot-rolled steel members tested in compression is as much as 10% lower than the value obtained from compression coupons. Since the same behavior is observed in annealed material, they reason that it cannot be attributed to residual stress, and suggest that it is possibly due to an influence of size and shape of the specimen. The reviewer (F. Garofalo) believes the differences in size and shape have an effect on strain distribution or state of stress.

Hyler, Lewis & Grover (1954) note that, despite some concern as to proper allowance for the effect of size on the fatigue behavior of materials, little definite information along this line is available for the aluminum alloys of major interest in aircraft design. They report on an investigation initiated to study the influence of size, particularly the notch-size effect on extruded 75S-T6 aluminum-alloy test specimens under rotating bending. They state (Summary, p.1): "Unnotched and notched specimens, with minimum section diameter of 1/8 inch, 1/4 inch, 1/2 inch, 1 inch, and 1-3/4 inches were tested. For each size, a semicircular groove having a theoretical stress-concentration factor of 2.0 was used. In the largest diameter specimen, a 60^{o} V-notch having a stress-concentration factor of about 19 was tested also.

... Within the large (but not exceptional) scatter of fatigue strengths observed, no general size effect could be concluded for either unnotched or notched specimens. One exception was the fact that the sharp notch in the large-diameter specimen did not reduce fatigue strengths as much as might have been predicted in view of its high value of theoretical stress-concentration factor."

Jacobs & Hartman (1954) report the results of fatigue tests carried out on Redux bonded 75S-T clad simple lap joints to obtain data on the relation between sheet thickness and overlap and the fatigue strength. They state (Summary, p. ii): "Lapjoints were made from 0.8; 1.2 and 2.0 mm sheet with an overlap varying from 12.5 to 50 mm. The fatigue load was repeated tension (stress ratio $R=0$). The static tensile strength of the joints was also determined. The results show that the fatigue limit ($n=50 \cdot 10^6$), calculated as shear stress in the glue, decreases if the overlap increases and if lap joints with the same overlap and various sheet thicknesses are compared, the thickest sheet gives the highest fatigue limit. If the fatigue strength of the joints is calculated as tensile stress in the sheet, the fatigue strength increases if the overlap increases. If joints with the same overlap and different sheet thicknesses are compared, the smaller the sheet thickness the higher the fatigue strength. ..."

Lieblein (1954) calls attention to two early papers by Chaplin on the relation between extreme values and tensile strength. On page 559 he states: "Practically every writer (e.g. Epstein, 1948[b]; Gumbel, 1954; Johnson, 1953; Epstein & Brooks, 1948) who makes a historical reference to the use of extreme values in tensile strength testing dates such application from Peirce's paper in 1926, the statistical model for which is based on Griffith's theory of flaws enunciated in 1920. In the interests of historical accuracy, it therefore seems important to report any material that comes to light which would significantly affect the widely held belief concerning priority. The purpose of this note is to bring to light two apparently forgotten articles written by W. S. Chaplin [(1880), (1882)], Professor of Civil Engineering, University of Tokyo, that appeared in American engineering journals over 70 years ago and recently came to the attention of the writer. ... The second ... was essentially an elaboration of the first. It will be sufficient to confine

attention to the first of the two articles. ..." There follows a summary of the paper by Chaplin (1880), including a two-paragraph quotation therefrom, and an interpretation in modern symbols and terminology. In a footnote, Lieblein acknowledges indebtedness to Dr. Churchill Eisenhart for calling his attention to an old engineering text [(Slocum & Hancock (1906)] in which these articles were cited.

Mackey (1954) reports the results of investigations into the strength of short plain and reinforced concrete columns. He summarizes his conclusions relevant to the size effect as follows (pp. 948,1085): "At low values of the slenderness ratio, the effect of friction developed at the end faces influences the strength of plain concrete columns. The column length over which this friction can act is a function of the least lateral dimension of the column and of the concrete cube strength. With concretes having a unit strength of 7,600 lb/in^2 ($530kg/cm^2$) local failure of the concrete is resisted by the adjacent concrete over a column length equal to 3D approximately, measured from each side of the incipient failure along a column axis. With increase in cube strength the influence of the adjacent concrete is felt over a proportionately longer length and conversely for a reduction in cube strength. From the present writer's investigations it is found that with cube strengths of 7,600 and 10,500 lb/in^2 ($740kg/cm^2$), this influence is still perceptible in columns having respective L/D ratios of 6 and 9 respectively. ... A marked increase in the strength of reinforced concrete columns is obtained with decrease in the slenderness ratio within the 'short column' range. This increase is not very significant as the L/D value is reduced from 15 to 12 but thereafter any further reduction yields a noticeable improvement in column carrying capacity. Determination of the extent of the 'short column' range from tests on plain columns leads to conservative design ... ."

Majors (1954), after reviewing the literature on size effect in wrought metals and cast materials, reports the results of rotating-beam endurance tests of modular cast iron specimens at room temperature, with special attention given to the size effect. He states the following conclusions (pp.214-215):"... For castings 0.75 in. thick in the annealed condition and unnotched test sections 0.30 in. diam and larger ... there is a trend for a small size effect, whereas for sections less than 0.30 in. diam there is a noticeable size effect upon the endurance limit. ... For unnotched specimens

in the annealed or as-cast condition, there is no significant size effect on the endurance limit for test sections ranging from 1.25 in. to 0.30 in. diam when specimens are removed from a 2.0-in-diam casting. A comparison of data from 4.0-in-diam with those from 2.0-in-diam castings did not reveal a significant size effect. No data were obtained for sections smaller than 0.30 in. ... Notched fatigue specimens showed a size effect in both the annealed and as-cast condition. Notches in large fatigue specimens (D=1.50 in.) ... have an endurance limit of 18,000 psi while notches in small specimens (D=0.38) have an endurance linit of 21,000 psi. ... A size effect was discernible for tension specimens machined from 2.0-in-diam castings. In the as-cast condition, the tensile strength and true strain at fracture were higher for a 0.19-in-diam section than for a 0.75-in. test section. ... For tension specimens in the annealed condition, the proportional limit and true strength at fracture were larger for 0.19-in. than 0.75-in. sections. ... The endurance curves were identical for specimens removed from 4-in and 2-in-diam castings. Also, no difference of any magnitude was observed in testing tension specimens at room temperature."

Mann (1954) [1970] lists over 4,000 publications on the fatigue of materials, components and structures, of which about 40 deal with the size effect.

Peterson (1954), writing on the relation between stress analysis and fatigue of metals, states (page 206): "It should be noted that there are areas of agreement between K_f[the fatigue notch factor] and K_t [the stress concentration factor], and that these are associated with the larger 'notches'. If, for example, one obtains data from small fatigue specimens and applies the notch factors to the design of a large member, it should be realized that the error will be on the 'unsafe' side and could therefore have serious consequences. For a quite large member, such as a steam turbine spindle, it is better to use a theoretical factor (as determined from mathematical or experimental stress analysis) than a factor obtained from small test pieces. This is because (a) it is impractical to make large tests; (b) small tests would give misleading results in an unsafe direction; and (c) the theoretical factors for fillets and groove details are well established by photoelastic and other means."

Richards (1954) synopsizes his conclusions from theoretical and experimental work on the size effect in tension testing of mild steel as follows (page 995): "In 1931 Fujio Nakanishi made his well-known report of a yield point in bending raised by 50 per cent above that in tension, using rectangular beams of mild steel. The presence of a size effect seems to offer the only satisfactory explanation for this and subsequent widely divergent results that have been reported over the past twenty-odd years. The investigation described herein has for its objects the development of an adequate theoretical basis for such a size effect and the testing of its validity by experiment. Due to a similarity between the discontinuous yielding of mild steel and brittle fracture, Weibull's Statistical Theory of the Strength of Materials has been adapted to fit the new problem. Because of its statistical nature, large numbers of tests are required to check the theory. The present report covers the first series of tests, which were made in tension. The experimental results obtained demonstrate a definite dependence of the upper yield point of mild steel on specimen size. Thus the conclusion is reached that in the tension test of mild steel the specimen size is one of the important variables."

Sidebottom & Clark (1954) write (summary): "When mild-steel members are subjected to dead loads, catastrophic yielding occurs in the member at a stress level less than the conventional yield point, thus lowering the load-carrying capacity of the member below the expected value. The present investigation was undertaken to determine if this type of action was dependent on the size of the specimen. A total of 18 geometrically similar rectangular mild-steel beams having depth of 3, 1, 1/2, and 1/4 in. were subjected to dead loads. The experimental hinge moment for the three largest sizes was independent of the beam depth and the average value was found to be 9.3% below the theoretical fully plastic moment; the average hinge moment for the 1/4-in. beams was 1.3% above the theoretical. The increased load-carrying capacity is believed to be associated with the steeper stress gradient, which appears to alter the spread pattern of yielding in the shallow beams. A study also was made of the behavior under dead loading of beams made of a high-strength, low-alloy steel (Mayara R). Like mild steel, this material exhibits a definite yield point; therefore, the behavior of these beams when subjected to dead loads was also similar to that observed for mild-steel beams. The

average experimental hinge moment was 10.7% below the theoretical value."

A thesis (in Portuguese) by da Silva Leme (1954) includes the following English summary and conclusions (pp. 140-141):"In this thesis an attempt is made at studying the statistical theory of the extremes of random samples, as well as of its applications in the field of engineering. The literature is reviewed up to 1953, (some 1954 papers included) and our personal contribution added to the individual subjects. The matter is expounded in seven chapters, as follows: 1) Statistical aspects of the problem: moments, central tendency measurements and limit forms of the distribution of extremes. 2) Statistical and other non-engineering applications of the theory of extremes .... 3) Prevision [prediction] of floods. ... 4) Influence of dimensions and interval distribution of deformations on strength of materials; in this chapter are reviewed and criticized the studies of Weibull, Daniels and Tucker. 5) As our personal contribution to the problem outlined above, a mathematical model is proposed for ideal (plastic ductil[e] materials, showing qualitatively what corrections should be made in order to apply our model to actual (plastic-brittle) materials. This model furthermore affords a rational explanation of an empirical law proposed by Weibull. 6) A review of the concept of 'Safety factor' as heretofore analysed (Prot, Lévi, etc.). 7) Application of the theory of extremes to the problem of the safety factor. ... From the analysis of the above-mentioned subjects we are led to stress the importance of the distribution $F(x) = \exp [-(\alpha(x_0-x))^k]$ which determines, among others, the populations of strengths, loads and floods. In the field of engineering this law is as important as or more important than the normal law, and deserves further investigation, especially in reference to significance tests." Chapters 4 and 5 are particularly relevant to the size effect on material strength.

Winkler (1954) recalls that, given the distribution of breaking strength for single fibers or yarns at a given test length, the distribution at other test lengths can be calculated by the theory of Peirce (1926) or of Weibull (1939a). The theory can also be applied to bundle tests by taking the total length as the product of the number of fibers or yarns and the test length. There is close agreement between the theory and the author's experimental

results on multiple lengths of cotton thread and on bundles of cotton threads and of acetate filaments.

Chechulin (1955) establishes a formula for the dependence of the critical temperature of brittleness (the temperature at which crystalline particles appear in the fracture) on size for similar notched specimens, which has the form $1/T_{xp} = K\ell g\ M + B$, where M is some arbitrary measure of specimen size and K and B are material constants. He gives a theoretical foundation for this formula on the basis of a general scheme of A. F. Joffe and the statistical nature of brittle strength. He notes that the dependence of brittle strength on volume was given by Weibull (1939a) and by Kontorova [& Frenkel](1941). He uses the formulation of Weibull, who (he says) gave by far the simplest analytic formula, which finds good experimental support, whereas the formula of Kontorova is considerably more complicated and is difficult to apply to the analysis of a given experiment. He suggests a new characteristic, called the size coefficient of the critical temperature of brittleness, which reflects the tendency of steel to the size effect. He shows that the physical-metallurgical nature of brittleness, individual peculiarities of melt and type of steel have but little effect on the size coefficient, although the latter, i.e. the influence of type of steel and melt, requires further verification. He makes apparent the significant influence, on the size coefficient, of the annealing temperature after hardening, with regard to which he establishes that the greater the annealing temperature, the larger the absolute magnitude of the size coefficient of the critical temperature of brittleness.

Duncan (1955) discusses the technical aspects of two 1954 accidents of Comet airplanes, and outlines several of the known concepts relating to fatigue of metals and their implication in design. Concerning the size effect, he writes (p. 198): "One of the unexplained phenomena of fatigue in metals is 'size effect'. It is found that the fatigue strengths of geometrically similar specimens are not proportional to their cross-sectional areas; the safe stress falls as the linear dimensions increase. The metal itself has several inherent linear dimensions which might conceivably be significant, for example, the average diameter of the crystal grains, the size of the crystal lattice itself or some linear dimension associated with the intercrystalline material.

The point of practical importance is that specimens used in fatigue tests should have dimensions as nearly as possible the same as those of the actual stressed members."

Hunt (1955) summarizes his results on elastic-plastic instability caused by the size effect and its influence on rubbing wear as follows (p. 850): "The yield strength of many materials is much higher for minute specimens than for bulk samples. The region around the point of highest shear stress in a solid undergoing deformation by a small plastic spherical indenter can be regarded as such a minute specimen which may be 'protected' by the size effect against plastic yielding if it is small enough. Formulating the effective yield strength and elastic stress under the indenter in terms of a common parameter provides a basis for assessing the influence of size scale on the plastic yield threshold. Four size categories are identified, including a critical case for which a small change of loading may cause a discontinuous transition from the elastic to the plastic regime throughout the region of contact, and another, more frequently encountered, in which the supportable pre-yield elastic stress is materially enhanced. The latter effect may exert an important influence on the rate of rubbing wear since it can make available a wider range of loading for which a low wear rate prevails. Reported wear tests on steel riders and on sapphire phonograph styli confirm these predictions qualitatively". The author also writes (p. 850): "On phenomenological grounds, and without the need for specifying the causal mechanism, the size effect can be characterized by expressing the yield strength of a solid material in the form $Y(V) = Y_0 + Y_1 e^{-KV}$, in which Y_0 is the critical yield strength of the material in bulk, $Y_0 + Y_1$ is the enhanced value of the yield strength approached in the limit as the volume V of the stressed specimen is decreased, and K is the average number of weakening flaws or imperfections per unit volume."

McClintock (1955) summarizes his results on the statistical theory of size and shape effects in fatigue as follows (p.421): "Fatigue specimens are considered in which the stress amplitude is constant with respect to time but falls off parabolically along the length of the specimen from the point of maximum stress. From assumptions regarding the local variability in the strength of the material, equations are derived relating the scatter in cycles

to failure to the scatter in position of failure. It is found that the shape of the distribution function does not affect seriously the relation between scatter in cycles to failure and scatter in position of failure. The size effect, however, is markedly influenced by the shape of the distribution function. A modification is suggested to make the results applicable to tests to determine the endurance limit, where the stress amplitude is a variable."

Muckle (1955) describes experiments carried out on composite model structures consisting of a steel main structure to which aluminum alloy superstructures were attached. He writes (Summary, p. 453): "Superstructures of three different breadths, and of two different thicknesses were tested in a simple bending frame, central loads being applied to the structures. A series of experiments in which the length was systematically reduced was carried out for each superstructure tested, thus enabling the influence of the ratio length of superstructure ÷ total length of structure to be investigated. ..." He states the following conclusions (p.480):"(1) In partial superstructures the length of the superstructure is the major factor which affects the stress distribution in the sides and deck at the centre of length. When the superstructure length is less than some 60 per cent of the main structure the load carried is less than would be forecast by the ordinary theory of bending. Broad confirmation of this result has been obtained by comparison with other investigations. (2)The breadth of the superstructure does not have a great influence on the efficiency except at the shorter lengths. (3) Thickness of side and deck plating do not affect the efficiency so long as both thicknesses are the same. ..."

Otto (1955) presents experimental evidence which (he claims) shows that, contrary to generally accepted belief, the strength of a fiber does not depend on fiber diameter. He summarizes his results as follows (p.124): "(1) The properties of glass fibers, including strength, are associated with the special circumstances attending their formation. (2) In the past, fibers of different diameters were inevitably made under different forming conditions; it was found necessary in hand drawing to heat the glass to higher temperatures and to pull faster to obtain fibers of small diameter. (3) In the past, therefore, fibers of different diameters were reported as having different strengths (i.e., breaking stresses, not breaking loads), and this difference was ascribed to different forming conditions. (4) When fibers of different diameter are

formed under controlled, nearly identical conditions, the breaking strengths are identical within the experimental limits and there is no significant effect of diameter as such."

Wells (1955) writes (p. 277): "In structural strength investigations there has always existed a difficulty in that the behaviour of a large prototype can never quite accurately be predicted from tests on a smaller scale model. The discrepancy between the stress and strain curves for each component at the two extremes of scale is often defined as a scale effect. On a closer examination, scale effects may be subdivided into metallurgical and geometrical, depending on whether or not they are associated with significant changes of material on the two scales. A size effect can only be described as truly geometrical when any metallurgical difference between the material at the two sizes has first been eliminated and, of course, vice versa. In discussing structural scale effects, care must also be taken to distinguish the influence of change of shape in comparing the behavior at one size with that at another. ..." After examining data found in the literature and resulting from new experimentation, Wells states the following conclusions (p.286): "1. The geometical size effect, first shown by Docherty, of suppression of notch-brittle fracture in small, notched, slow-bend specimens, at a given temperature and strain rate, is fully supported by the experiments described above. 2. The suppression of notch-brittle fracture in small specimens is due to absence of sufficient available elastic energy for propagation. ... 4. The geometrical size effect as defined above offers a reasonable explanation as to why spontaneous notch-brittle fracture from cracked butt welds may be prevented by residual stress relief, and so possesses a practical as well as academic importance."

Blatherwick & Lazan (1956), after reviewing the literature on size and shape effects in fatigue tests, develop the theory of fatigue testing for materials showing idealized stress-strain behavior. The assumed stress-strain loops are formed of linear segments, and show work-hardening or softening depending on the number of cycles and the amplitude of the present cycle. The reviewer (F. R. N. Nabarro) believes the model is acceptable for tests at constant strain amplitude but not under other conditions where the position of the plastic boundary depends on the number of cycles. The analysis is carried

out for direct axial loading and for bending tests under conditions of (1) constant strain, (2) constant maximum stress, and (3) constant bending moment for a variety of cross sections.

Brenner (1956a) reports the results of a study of the growth of whiskers of copper and various other metals by reduction of their halides. He grew whiskers with a large variety of shapes and perfection. He states that the near-perfect whiskers which exhibit high strength and are bounded by smooth planes are the ones of greatest interest. Whiskers larger than 8μ are rarely near-perfect. Overgrown and distorted copper whiskers exhibited little elastic strength, and are probably no stronger than bulk copper. In his conclusions, the author states (p. 74): "The thin, smoothly bound[ed] whiskers exhibit bending strains in excess of 1.5 per cent indicating near-perfection. There seems to be a size-dependence on the strength which will be investigated further [see Brenner (1956b)]."

Brenner (1956b) reports the results of tensile tests performed on metal whiskers 1.2 to 15μ in diameter, which he summarizes as follows (pp. 1490-1491): "It has been shown that the strongest iron, copper, and silver whiskers exhibit strengths which are either close to or above the lower estimate of the strength of perfect crystals. The stress-strain curves of two of the strongest iron whiskers indicate that within the experimental accuracy Hooke's law is obeyed up to about two percent strain. Beyond two percent strain, considerable deviation from linearity occurs. The stress-strain behavior is taken as a sign that the theoretical strength of perfect crystals has been closely approached. The strongest whiskers were also the smallest in size. As the diameter and length is increased, the strength of the whiskers decreases with considerable scatter. It is postulated that this decrease in strength is due to defects which are formed accidentally during the growth of the whiskers."

Doman & Schwartz (1956) summarize the results of a study of size effect in sheet-stringer panels as follows (p. 1): "The object of this study was to determine whether there are significant size effects in compressive strength of large Z-stiffened sheet-stringer panels as compared with geometrically similar smaller models and thus to ascertain whether the prediction of the strength of large panels by model tests is reliable. The specimens for the study were manufactured from 7075-T6 aluminum alloy. There were four representative types of panel designs, with full-scale and one-quarter-scale

panels of each type. A comparison of the average failing stresses shows that there is no significant effect due to the panel size. For the panels tested, which failed by general instability, there was no significant compressive-strength size effect between the large Z-stiffened prototype and geometrically similar model panels." The authors conclude (p. 4): "Therefore, the compressive strength of large-scale panels failing by general instability may be predicted from model tests or accepted design data presented as nondimensional parameters."

Endicott & Weber (1956) draw the following conclusions (p. 10) from a study of the effect of sample size (area) on dielectric strength: "1. It has been shown that the breakdown of transformer oil follows extreme-value theory (Gumbel's distribution I) rather than the normal law. Limited data are presented indicating that this is also true for solids. 2. Extreme-value theory yields a precise equation for predicting the minimum breakdown voltage for any large number of breakdowns taken under identical conditions, once the modal breakdown voltage and the standard deviation are known for this condition. 3. This equation is $V = V_m - (\ell n\ N)/\alpha$ [where V_m is the modal voltage and $1/\alpha$ is a measure of dispersion of the observed voltage]. 4. Extreme-value theory also yields a precise equation for predicting the breakdown voltage for electrodes of any area for a uniform field, once the modal breakdown voltage and the standard deviation are known for any one area for this oil. 5. This theoretical relationship is $V_{A1} - V_{A2} = 1.80 s_v \log_{10} A_2/A_1$ [where s_v is the standard deviation obtained from the observations and N is the number of tests]. 6. Neither of these equations contains an arbitrary constant."

Kuhn (1956) points out that it has been known for a long time that the stress concentration produced by a notch on a fatigue specimen is less severe than predicted by the theory of elasticity and that the factor of stress concentration increases with the absolute size of the specimen. He presents a method for estimating the effect of varying geometric size of notch for steels and for strong aluminum alloys. He uses the term "geometric size effect" to emphasize the assumption that no metallurgical size effects are present. He shows that the effect of changes in geometric size can be predicted with fair accuracy.

Landers & Hardrath (1956) summarize the results of a study of notch-size effect in the following concluding remarks (p. 4): "Axial-load fatigue data

on electropolished 2024-T3 and 7075-T6 aluminum-alloy-sheet specimens with central holes have been presented. The specimen widths and hole diameters were varied in order to provide data suitable for study of notch-size effect. The data are compared with data from tests of unnotched electropolished specimens made from the same lot of material. The fatigue stress-concentration factors K_F are plotted against the ratio of hole diameter d to specimen width W. From these plots it is evident that K_F decreased with decreasing width for the same value of d/W; the difference between K_F and the elastic stress-concentration factor K_T increased with decreasing d/W. Definite evidence of a notch-size effect was thus indicated."

Martinaglia (1956) reports the results of a study of the effect of rounding in the thread base on the fatigue limit under tension of bolts of diameters 38, 30, 16 and 4 mm. His graphs also show a definite size effect of the bolt diameter, the tensile fatigue limit (kg/mm^2) for bolts of diameter 4 mm. being over twice that for bolts of diameter 38 mm. with the same thread base rounding.

Miller & Albert (1956) summarize the portion of their results on mechanical tests on specimens from large aluminum-alloy forgings which is relevant to the size effect as follows (p. 1): "Results of ... bend tests on specimens in the T6 condition from 12- by 12- by 24-inch hand forgings of 7075 (75S) and 2014 (14S) aluminum alloy are presented in this report. Thirty-six notched bend specimens were tested as cantilever beams ... . Two-thirds of the bend specimens were 1/4 inch wide. In these, the critical extreme fiber stresses were unidirectional. The rest of the bend specimens were 6 inches wide. In these, there were biaxial stress conditions at the critical extreme fiber. ... Comparison of the results for the 1/4-inch and 6-inch bend specimens showed that on the average the biaxial stress conditions in the 6-inch specimens caused the strain at failure to be about 0.3 that of the 1/4-inch specimens. ..." It is not clear how much of the difference represents true size effect and how much is due simply to the different stress conditions.

Muckle (1956) gives a condensed version of the contents of his earlier paper with the same title [Muckle (1955)], minus references, appendices, and discussion. The conclusions are identical with those stated in the earlier paper.

Muvdi, Sachs & Klier (1956) present the results of tension and notch-tension tests performed on hot rolled sections from commercial, electric furnace heats of several high-strength steels. They also consider and evaluate information from the literature pertaining to the effects of as-processed section size. In general, the tensile strength was found to decrease gradually with increase in the specimen size. The notch strength decreased with increase in stress concentration, specimen diameter, and as-processed section size, these effects being pronounced at the highest strength levels but insignificant at strength levels below 200,000 psi.

Nowinski (1956) summarizes the results of a series of experiments concerning the scale effect on the rupture of steel wires as follows (p.572): "The scale effect or, in other words, the influence of absolute dimensions of structural elements on their strength has, together with the dispersion of the strength, a common backgound attributed to the non-homogeneity of materials. This probabilistic phenomenon was explained for brittle materials by W. Weibull, J. I. Frenkel and T. A. Kontorova. This note based on the author's experiments on a wire of medium ductility is a contribution to this problem. Samples of wire (subjected to no previous treatment) 5 cm. and 50 cm. long were tensile tested. From the curves of strength distribution assumed to be Gaussian curves it follows ... that the strength of short samples ... is greater than that of long ones ..., which seems to confirm the scale effect and its connection with the non-homogeneity of the material."

Shevandin, Razov & Serpeninov (1956) introduce a method for investigating kinetics of plastic failure which permits one to appraise the size effect of metals and also the influence of yielding of the loaded system on their limiting plasticity. With the aid of the method developed, they give indications of a relation of size effect of metals to elastic energy, accummulated under loading by the sample-testing machine system, and of absence of dependence of this effect on statistical factors.

Sopwith (1956) writes on recent research on the size effect in fatigue as follows (pp. 260-262): "The acquisition ... of some 60 Ton direct-stress machines enabled comprehensive investigations to be carried out to determine the effect of size of specimen on fatigue strength of materials in various forms. It was shown [Phillips & Heywood (1951)] that while no intrinsic size

effect in fatigue was observed with plain specimens of a mild steel and a nickel-chrome steel, a pronounced size effect was exhibited by notched specimens. Indeed, this particular work suggests that in sections of mild steel of sufficient size, it might well be the case that the full theoretical stress concentration effects due to notches might be realized. It has long been accepted that with high-strength materials strength reduction factors much more closely approach the theoretical values than is the case for softer materials. The importance of size when mild steel is used is now demonstrated. Arising from this work, it was suggested that the importance of stress distribution in relation to fatigue strength was often overlooked, and that, in fact, most so-called size effects can be explained in terms of stress gradient. An investigation [Phillips & Fenner (1951)] carried out in parallel with the one described above demonstrated the effect of holes on the fatigue strength of wide thin sheets of aluminium alloy and steel. The reduction of fatigue strength with increase in hole diameter from 1/144 to 1/3 of the net sheet width was over 25 per cent, but while, in general, the presence of a hole caused a marked reduction in fatigue strength, the smallest holes resulted in only a slight reduction, and occasionally none at all. One still unresolved anomaly in this work was the difference in fatigue strength between unperforated test-pieces 3/4 inch (19 mm) wide and 1/10 inch (2.5 mm) thick, and panels of the same material 9 inches (229 mm) wide and also 1/10 inch thick. This difference was of the order of 50 per cent, and is of obvious interest to aircraft designers. Very careful studies were made of the precise stress distributions occurring in each form of test-piece, but only one-half of the difference could be accounted for. [One figure] shows the effect of size on transversely-drilled specimens, and [another figure] shows the fatigue strength of aluminium alloy panels 9 inches (229 mm) wide, drilled with holes of various sizes."

Weber & Endicott (1956), writing on the area effect in the electric breakdown of transformer oil and its extremal basis, conclude (p.378): "STATISTICAL NATURE OF BREAKDOWN: 1. It has been shown that the breakdown of transformer oil follows extreme-value theory rather than normal probability. 2. The distributions of breakdown voltages for gaps of different area are similar exponential distributions shifted successively toward zero for successively larger areas. In general, the specification of the dielectric strength

of a given oil is not complete unless this area effect is taken into account. FORM AND MAGNITUDE OF THE AREA EFFECT: 1. Extreme-value theory yields a precise equation for predicting the breakdown voltage for electrodes of any area for a uniform field, once the modal breakdown voltage and the standard deviation are known for any one area for the oil used. 2. This theoretical relationship of voltage to area is $V_{A_1} - V_{A_2} = 1.80\ S_v \log_{10} (A_2/A_1)$ [the notation here is essentially the same as that in the authors' related paper {Endicott & Weber (1956)} summarized above]. 3. For the particular oil used, a 10-to-1 increase in area results in a 16% decrease in dielectric strength from the higher value."

Weibull (1956), writing on basic aspects of fatigue, divides the fatigue process into two different stages, crack initiation and crack propagation, and during each stage applies the concept of cumulative damage to a single point of the specimen, that point during the first stage being the point x_0 where the crack is suspected to start. He denotes by N_0 the number of stress cycles at which the damage reaches the value 100% at x_0 and the crack starts propagating. He notes that the probability of encountering heavy stress raisers increases with the surface area, so that a considerable size effect is to be expected on N_0, and he states that this conclusion has been verified experimentally. He denotes by N_{10} the number of cycles at which a crack starting from a notched central hole has developed to 10% of the width of the specimen, and gives a table showing the size effect on N_{10} in 24S-T, Alclad.

Endicott & Weber (1957) report the results of a test of the electrode area effect on breakdown of transformer oil for impulse voltage, and compare these results with those for 60-cycle tests presented in their earlier paper [Weber & Endicott (1956)]. They state the following conclusions (p. 397): "Front-of-wave impulse-voltage breakdown of transformer oil follows an extreme-value pattern rather than that of normal probability: (1) the experimentally found decrease in breakdown voltage with increase in area is accurately described by equation 1 $[V_1 - V_2 = (s_v/\sigma_N) \ln(A_2/A_1)]$ for the present impulse tests as well as for the previous 60-cycle tests; and (2) any differences in breakdown mechanism which may exist between impulse and 60-cycle tests do not influence this area effect."

Klier, Muvdi & Sachs (1957) state that, with an increase in specimen size, susceptibility of high-strength steels to sustained-load failures can be expected to decrease, and report that limited tests that have been completed verify that the expected decrease does occur. They report also that if notch ductility is high, size effect is not observed, but if notch ductility is low, size effect is encountered. They make use of this fact to explore the size effect indirectly by controlling those factors (testing temperature, testing speed, and notch geometry) which influence notch ductility. They reason that if testing temperature is sufficiently lowered; if testing speed is sufficiently increased; and if notch acuity is sufficiently increased, notch ductility will be reduced to a minimum value, so that the notch strength then measured will correspond to the notch strength of a very large specimen and thus will constitute a limiting value. They examine these predictions in the light of the available data.

Shanley (1957a), in his textbook on the strength of materials, deals briefly with size effects in connection with local and lateral buckling and in connection with stress concentration due to notches. In Chapter 25, he considers the effect of the diameter-thickness ratio (D/t) for round tubes under compression and of the length for constant D/t. He states that size effects are properly accounted for when the optimum column stress or the optimum diameter-thickness ratio is plotted against the structural index (P/L_e^2), where P is the applied force and L_e is the effective length. In Chapter 27, he deals with the relation between stress concentration and fatigue and the effects on the former of notch depth, notch angle, and notch radius, noting that the highest concentration will be obtained when the notch depth is large and the notch radius and notch angle are small.

Shanley (1957b) writes (Summary, p. iii): "A theory for the exceptionally high tensile strength of very fine wires (or fibers) is developed on the basis that the 'core' material slips at nominal stress, while the 'skin' material must develop a very high tensile stress in order to completely disrupt the atomic bonds. The resulting equation gives excellent correlation with test data. The physical significance of the theory is discussed as applied to glass and metal fibers. ..." Shanley notes that he developed this theory in 1953 and presented it in a seminar at UCLA, but delayed publishing it because an article [Otto (1955)] appeared in which it was apparently shown, by test data, that the

high strength of glass fibers was not a function of diameter <u>per se</u>, but rather a consequence of unequal cooling rates, etc. However, he now says, the original theory correlates quite well with new test data, as well as with the much earlier experimental results of Karmarsch (1859) on metal wires and of Griffith (1920) on glass fibers.

Volkov (1957) states that tests of plaster of Paris, under conditions of plane stress, show better agreement, in experiments involving brittle fracture, with the maximum-stress theory than with the maximum-strain theory of strength. He considers a statistical specification of strength which is a natural generalization of the maximum-stress theory and which gives better agreement with experiments than does the classical theory. The probability density of microscopic normal stresses on the boundary of a macroscopic element of volume W, perpendicular to the x-axis, corresponding to the direction of principal direction 1, on which is acting a macroscopic stress σ_1, is $f(\sigma_x)=[1/(\sqrt{2\pi}\ \omega_x)]$ $\exp[(\sigma_x-\sigma_1)^2/2\omega_x^2]$. Here σ_x is the microscopic normal stress on the boundary of microscopic volume element V, perpendicular to the x-axis, ω_x is the standard deviation of microscopic stress σ_x from the mean value σ_1; σ_1, σ_2, $\sigma_3(\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3)$ are the macroscopic (mean) principal stresses. Let S_n denote *the resistance of the material to microscopic fracture* in volume V. Then volume V fractures (a microscopic crack appears) when $\sigma_x \geqslant S_n (S_n \geqslant 0)$. The relative number of microscopic cracks is $q = \int_{S_n}^{\infty} f(\sigma_x)d\sigma_x = 1/2-(1/\sqrt{\pi})\int_{\infty}^{y} e^{-t^2}dt(1)$ where $y=(S_n \mp \sigma_1)/c\sigma_1\sqrt{f(\alpha,\beta)}\,(\sigma_1 \neq 0)$, $y=S_n/c\sigma_3\sqrt{\phi(\gamma)}\,(\sigma_1=0)$, $f(\alpha,\beta)=1+\alpha^2+\beta^2-2\gamma(\alpha+\alpha\beta+\beta)$, $\phi(\gamma)=1-2\nu\gamma+\gamma^2$, $\sigma=\sigma_2/\sigma_1, \beta=\sigma_3/\sigma_1, \gamma=\sigma_2/\sigma_3 (2)$, where c is a constant of the material. The number of microcracks provoking macroscopic fracture of the material may be called "critical", and designated q_k. If we assume $q_k= 1/2$, then from (1) we get $y=0$, and then from (2) we find $\sigma_1=S_n(3)$. If we denote macroscopic stress at the moment of macroscopic rupture by R_σ, then $R_\sigma=|\sigma_1|$ for $\sigma_1 \neq 0$ and $R_\sigma\ |\sigma_3|$ for $\sigma_1=0$, so that, according to (3), $R_\sigma=S_n(4)$. However, this equality contradicts the well known phenomenon of increasing resistance to fracture with decreasing size of body (the size effect). Because of the size effect, we must have $S_n \geqslant R_p$, where R_p is the material's resistance to macroscopic fracture in uniaxial tension. But then in (3) $y \geqslant 0$ and, according to (1), $q_k \leqslant 1/2$. These conditions lead to a family of limiting surfaces of rupture, depending on two parameters, $y_\sigma= \chi/[(\chi-1)\sqrt{f(\alpha,\beta)} \pm 1](\sigma_1 \neq 0)$, $y_\sigma=$

$\chi/[(\chi-1)\sqrt{\phi(\gamma)}](\sigma_1=0)$ (5), where $\chi=S_n/R_p$, $y_\sigma=R_\sigma/R_p$. Thus strength conditions (5) constitute a simple natural generalization of the maximum-stress theory of strength.

Weber & Endicott (1957) state the following conclusions (pp. 1095-1096): "1. The reduction in breakdown strength for transformer oil with increased electrode area, described in the two previous papers in this series, has been verified for similar and larger areas and area ratios. ... 2. The extremal nature of breakdown voltage distributions, and the breakdown-strength equation derived on this basis, have been further corroborated. 3. The theoretical relation between voltage and area presented may be compared with the minima-of-groups area effect and with the experimentally found effect for electrodes of different area. In general, the minima-of-groups area effect is more consistent than the physical area effect because experimental variations among electrodes are not included ... . 4. Such experimental variations would, however, be encountered in practical applications. Hence it is useful to know the magnitude such variations are likely to have. Differences in distribution parameters for tests with identical sets of electrode pairs appear to be a large part of the variations from ideal, theoretical values."

SECTION III
THE SPACE AGE (1958-1975)

Anderson (1958) recalls that, since the classic work of Griffith (1920), very fine glass fibers have been noted for their high strength. He notes that this strength is customarily attributed to the absence of flaws, a contributing factor being that their method of production involves extremely rapid cooling, which inhibits the formation of flaws. He deduces, from the theory of heat conduction, simplified equations for the cooling time of glass fibers.

Bartenev & Tsepkov (1958) write (p. 822 of translation): "The effect of the dimensions of the load-carrying part of test pieces or products on strength, called scale effect, is well known. Scale effect appears most clearly in brittle materials, to which silicate glass belongs. Since the bulk of the glass is several times stronger than the surface layer, then, in contrast to metals, the strength of glass is determined practically by the strength of the surface. Therefore, in the statistical theory of the strength of glass, it is necessary

to consider the dimensions of the load carrying surface, and not the volume. Data regarding scale effect on the strength of glass is contradictory since, according to some, the strength of thin glasses is highly different from the strength of thick glasses, but according to others--only slightly different ... . The hypothesis was stated for glass plates ..., and confirmed for glass fibers ..., that this contradiction could be explained by two kinds of scale effect." After reviewing experimental data on the strength of glass fibers and plates, the authors close with the statement (p. 824):" ... Experimental facts indicate the presence in glasses of two scale factors--technological and statistical. The effect of the diameter of a glass fiber and the thickness of a glass plate on the strength is explained by the technological factors--plastic deformation and thermoelastic stresses arising in the process of formation of the glass. The effect on the strength of the linear dimensions--length of glass fibers, length of the edge of strips, and the dimensions of the surface of glass-- is explained by the statistical theory of strength."

Bateson (1958) recalls that in a short note [Bateson (1953)--see Additional References, p. 409] he drew attention to certain discrepancies among the observations of Griffith (1920) on the strength of fine glass fibers, and suggested that the increase in the strength of the fiber with decreasing diameter was caused by an increased rate of cooling as the diameter decreased, thus achieving a stress-free structure singularly free from internal dislocations caused by the shearing forces introduced during a more leisurely cooling program, so that one would expect the rapidly cooled fiber to be stronger. He takes note of two significant papers published during the intervening five years, those by Otto (1955) and by Deeg & Dietzel (1955) [see Additional References, p. 409], and reports that further experimental work has been carried out on the strength and elastic properties of fibers using a new single fiber tensile testing machine. The reasonably good agreement between the new strength measurements and a flaw distribution theory proposed by Greene (1956) [see Additional References, p. 409] leads to the conclusion that the flaw theory of high strength is valid.

Chang & Kesler (1958) report that the formulas developed by previous investigators for predicting the static diagonal tension cracking loads and shear-moments of simply supported reinforced concrete beams with tensile

reinforcement only were found to be inapplicable for beams of relatively small cross section. Consequently, they apply dimensional analysis to develop equations to predict these loads, both for the relatively small beams which they tested and for similar beams of large cross sectional area for which they were able to find adequate data. These equations, which include size effect, are converted into nomographs for ease of application. Specific instructions are given for using the nomographs to make the required correction for size effect. This study of static strength includes results from tests of 105 beams, 42 of which were tested by the authors. Fatigue tests were made on 39 reinforced concrete beams with tension reinforcement only.

Coleman (1958) writes (Summary, p. 60): "This paper discusses tensile strength distributions for fibers (called 'classical fibres') whose strength is independent of the rate of loading. Reasons [based on the 'weakest link' theory] are presented for expecting, in the absence of all other information, that the tensile strength of long classical fibres [a decreasing function of their length] from a common stationary source should obey the Weibull distribution. The statistical theory of the strength of bundles of classical fibres, as developed by Daniels, is applied to infinite bundles composed of fibres which obey the Weibull distribution. It is found that the ratio of the tensile strength (units of force at break per initial unit area) of a bundle to the mean tensile strength of the constituent filaments decreases montonically [sic] with increasing dispersion in the strength of the constituent filaments. In general, the tensile strength of a large bundle has the same order of magnitude, but is <u>less</u> than the mean strength of the component filaments. Previous calculations have yielded this conclusion for fibres with a special time-dependence to their tensile strength; here it is shown that the conclusion also applies to classical fibres."

Dolan (1958) writes (p. 141): "Every fatigue test is a 'specimen' test and is influenced fully as much by the shape and size of specimen as it is by the material of which it is made. True dynamic or geometric similarity does <u>not</u> apply for purposes of design, the presence of notches or other abrupt changes of section materially alter the behavior. ... Laboratory test data from samples or from small models are useful in a qualitative way but cannot be depended upon to give quantitative answers to the final operation of a part

in service. Differences in geometric size and differences in processing (particularly with respect to final finishing operations), together with differences in environment of operation, may materially lower the fatigue strength or life in a service condition as compared with that predicted from a laboratory test. Larger sizes, difficulties of fabrication of exact contours, roughness of surface, presence of decarburized layers on the surface, or mild corrosive conditions while in service all have harmful effects that may markedly reduce the fatigue limit of a part in an actual machine component as compared with its performance in a carefully controlled laboratory test."

Glikman & Kostrov (1958) review the literature on experimental investigations of the influence of the absolute dimensions of the cross-section of steel specimens on the fatigue resistance. They analyze various hypotheses on the nature of the scale factor. They express the opinion that the basic reasons for the influence of the dimensions on the fatigue resistance appear to be the stress gradient, the nonhomogeneity of the properties of the microstructure, and the related statistical nature of the process of disruption due to fatigue and also the part played by stored potential elastic energy.

Harris (1958) writes (Summary, p. 193): "An overwhelming body of experimental evidence exists to support the view that the discrepancies between theoretical and experimental values descriptive of the mechanical property behaviour of real crystalline solids can be attributed to 'imperfections' or 'defects' in the lattice structure. The fundamental role played by 'size' or 'scale' phenomena throughout the laws of nature can, it is suggested, in the case of the static and fatigue strength of metals and the existence of 'non-propagating' fatigue cracks, be attributed to the existence of characteristic 'flaw patterns'."

Lavrov (1958) points out that in the case of ice, as in the case of other materials, lower values of ultimate strength are obtained when testing large specimens, because of the scale effect. He states that none of the proposed hypotheses on the nature of the scale effect produces, when applied to ice, results which agree with experience. He considers the bending of a beam with a surface defect and shows that, under certain stated assumptions, the scale or size effect can be obtained. The reviewer (M. J. Manjoine) points out that Lavrov's assumption that the absolute elongation of the outer fiber is

concentrated at the point defect disregards the fact that the defect is parallel tc the surrounding elastic material and is constrained to deform with it. The stress gradient in the thickness direction results in a larger stress at the base of the defect for thicker specimens but does not account for the size effect observed. The reviewer believes that the Griffith crack theory is a more realistic approach to this problem and will yield good agreement with the data.

Lubahn (1958) writes (Synopsis, p. 678): " The congruency principle states that fracture will occur at the same nominal stress in two different objects if the nominal stress gradient at the notch root is the same in both objects and if the notches are congruent in the two objects. This concept was applied to the results of notch bend tests and notched disk bursting tests. The strengths in the two types of tests could be correlated within about 8 per cent. The congruency principle and another fracture concept, the Griffith-Irwin theory, were also compared as to accuracy of correlation and breadth of applicability."

Lubahn & Yukawa (1958) write (Synopsis, p. 661): "Slow notch-bend tests were performed on specimens of various sizes and notch radii cut from a large nickel-molybdenum-vanadium steel forging. High strength values were obtained for either a small specimen with a sharp notch or for a large specimen with a mild notch, but the combination of large specimen size and sharp notch resulted in nominal strengths as low as 40,000 psi. The notch-root strain at fracture decreased with increasing size of geometrically-similar specimens. A static crack appears to cause no more weakening than a sharp machined notch. The tests also serve to show the effects of notch depth, testing temperature, hydrogen content, and proximity of welds."

Sedrakyan (1958) uses elementary combinatorial probability methods to investigate the following two problems: (1) To determine the strength distribution function of chains consisting of n links, if the strength distribution functions of the links are given; (2) If a cable is braided of n wires of equal length and cross-section, with their strength distribution function given, to determine the probability that $c \leq n$ wires will break under the influence of a given external load. He points out that many questions which arise in the statistical theory of strength reduce to these two problems.

Shevandin (1958) points out that, while the problem of the effect of the scale factor on the tendency of steel to brittle fracture is of greatest theoretical interest when geometrical similarity is maintained, it is often of greater practical interest when only one of the dimensions is changed. He reports the results of an experimental study of the effect of scale on cold brittleness with and without geometrical similarity, for various steels, heat treatments, plate thicknesses, and orientations of specimen to plate surface. He shows that the data obtained from the tests described can be used to formulate safety criteria with respect to cold brittleness for the use of structural steel in buildings.

Shreiner & Pavalova (1958) point out that a reduction in the fatigue limit with increasing homogeneity of the structure is characteristic of both rocks and metals, and that the fatigue limit for both of these materials also depends on the conditions of the state of stress, the medium, and the absolute dimensions of the samples.

Sippel (1958) writes (English summary, p. 163): "The Weibull theory of the tensile strength is declined [rejected] for orientated fibres from macronuclear material owing to experimental tests and general considerations. An empirical formula for the description of the connexion between tensile strength and length of the specimen between the jaws is established. The application of this formula possibly allows to distinguish between the influence of pure building [structural] faults and the influence of disturbing impurities on the tensile strength." In a companion paper, Sippel & Hützen (1958) write (English summary, p.213): "Results from experimental tests of the Weibull theory of the tensile strength with endless fibres from acetate rayon are communicated. They show that this theory is not applicable to endless artificial fibres. The evaluation of the measurements leads to an empiric relation between the tensile strength and the length between the jaws. Thus the tensile strength may be calculated for the length between the jaws of zero and infinite values; the last constitutes a considerable fraction of the first."

Solov'ev (1958) reports the results of an experimental investigation on the change of properties of twisted cotton yarn in relation to the number of turns and the size and direction of twist of the threads. He gives formulas for designing the basic properties of twisted cotton yarn.

Vagapov, Khripina & Shishorina (1958) write (p. 17, as translated by David A. Lee): "We consider now the 'size factor' in connection with non-uniform state of stress and inhomogeneities of the metal. Naturally there will be a size effect noted in the case of cyclic bending, associated with the effects of nonuniform state of stress [Afanas'ev (1953)], which vanishes when the diameter of the specimen becomes very large; thus for large specimen diameters the fatigue limit in cyclic bending must decrease, tending asymptotically to the fatigue limit for tension-compression; the greatest reduction must be observed in steels of low strength, and the least reduction in high-strength steels... . However, in general the experimental dependence of σ_0 [the fatigue limit] is seen to be opposite for the two groups of steels, and the size effect does not come down to the size of the fatigue-limit for tension-compression ... . From this we conclude that the size effect is [not (?)] determined only by effects of the absolute dimensions, and does not agree with the experimental results [Phillips & Heywood (1951)], according to which for tension-compression it is very weak or vanishing for specimen diameters in the range 7 to 50 mm. ... ; in fact, for cyclic bending the largest reduction of strength occurs on this interval of diameters. Moreover we can say that for clarifying the nature of the size factor it is necessary to separate the influences of absolute size from those of nonuniform stress. ... We conclude that the size effect for steels of low strength is basically determined by the effects of non-uniform stress, and for high strength steels, it is chiefly determined by the effects of absolute dimensions."

Geil & Carwile (1959) report the results of a study of the combined effects of low temperatures and the geometry of a notch on the tensile behavior of specimens of annealed commercially pure titanium. They found that the strength and ductility of notched specimens were affected greatly by the test temperature and by the geometry of the notch and the accompanying stress concentration and triaxiality. Strength indices tended to increase with decrease in temperature or increase in triaxiality that accompanied an increase in notch depth or decrease in root radius. The ductility of the titanium specimens decreased greatly with decrease in temperature or root radius, or increase in notch depth.

Kudryavtsev & Naumchenkov (1959) report the results of an investigation whose purpose was to establish the resistance to failure, under alternating loads, of welded joints in cast 35L medium carbon steel, taking into account scale factor and heat treatment effect. They compare these results with previously available ones for low-carbon 22K rolled steel. For both types of steel, the endurance limit was found to be considerably lower for specimens 200 mm. in diameter than for those 20 mm. in diameter--7.5-11.5 kg/mm^2 vs. 11.5-14.5 kg/mm^2 for medium-carbon 35L cast steel and 16.5 kg/mm^2 vs. 18.5-21.5 kg/mm^2 for low-carbon 22K rolled steel.

Kushelev (1959) discusses the influence on the fatigue limit of various geometric factors, especially that of grooves in relation to their length. He gives an interpretation of the scale factor, in accordance with which "increase in the dimensions of the test samples results in a corresponding increase in the area of dangerously stressed material producing a decrease in the limit of endurance".

Mandel (1959), after giving a quick review of the theory of extreme values, writes as follows (p. 30): "The theory of extreme values is of great usefulness in many practical applications. Consider, for example, a textile fiber, and suppose that we study the breaking strength of this fiber as a function of its length. As the length increases, there is an increasing probability of finding weaker and weaker spots. If we consider the specimen as a chain that is no stronger than its weakest link, we see that the breaking strength will tend to decrease as the length increases. The law that governs this phenomenon is intimately related to the theory of extreme values. To see this, suppose that we wish to determine the fraction of samples of length L that will have a strength of less than 10 lb. A specimen will break at less than 10 lb. if its weakest spot has strength less than 10 lb. Therefore, all we have to know is the probability distribution of this weakest spot, and this is, of course, the probability distribution of an extreme value. We see that what we considered as the sample size in the case of the thickness measurements [discussed earlier in the paper] now becomes the length of the specimen, or, in other cases, such as dumbbells for tensile measurements, the area or the volume of the specimen. Thus, the questions we asked concerning the manner in which the extreme value distribution changed, in location and in shape, as the sample size

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increased, now acquire great significance with regard to the relation between the strength of materials and their size."

Mylonas (1959) discusses the effect of specimen size and other factors in static brittle-fracture initiation. Concerning the size effect he writes (pp. 423s-424s): "An inverse relation between general stress level at fracture and size of test specimen, so-called 'size-effect', has first been observed in brittle materials, and has been invariably explained by two theories ... the statistical theory ... [and] the Griffith theory. ... In a flawless homogeneous material, the size effect should be nonexistent provided exact similarity is maintained. Exactly similar specimens of various sizes of a perfectly brittle material would have the same factor of stress concentration and would fracture at the same applied-stress level. Likewise, with a ductile material, the stresses, the extent of plastic deformation, and the strain history should be identical for the same variation of the average applied-stress level, independent of the specimen size. ... When the occurrence of defects is small and the ductility is high, the material should behave more nearly like a flawless material without any size effect in exactly similar structures. A strong size effect should be expected with great flaw frequency and with reduced ductility. Furthermore, the greater probability of occurrence of a bad flaw or of a spot of reduced ductility in the larger critically-strained regions of the larger structures should by itself result in a size effect. In general, a pronounced scatter of results should exist. In addition, the appearance of early cracks may greatly affect the results. ... Thus, the term 'size effect' covers a complicated behavior. Variations of behavior from total absence to strong existence of a size effect should not be surprising. Each particular instance should be carefully studied and precise tests should be conducted for the assessment of any observed size effect."

Ouchida (1959) reports the results of a study of size effect on fatigue strength of 0.18% carbon steel (SF45), 0.35-0.40% carbon steel (SF60) and Ni-Mo steel (SNCM). He states the following conclusions (pp. 17-18): "(1) The size effect of plain and notched specimens is great in the range of the diameter smaller than 20 mm, but in the range of the larger diameter 20 to 100 mm the effect is comparatively small. (2) The endurance limit of [plain] steel specimen decreases about 10% with increase of diameter from 10 to 100 mm, regardless of materials. And the endurance limit of notched specimen

having the value of stress concentration factor equal to 3.3 decreases by about 15% for mediate steel and by about 30% for 0.18% C steel and Ni-Mo steel. This results [sic] would indicate that the decrease of endurance limit with increase in size is greater ... for notched specimens than for plain specimens. (3) The fatigue strength-reduction factor of the notched specimen of 100 mm in dia. for mild steel and mediate carbon steel is considerably less than the stress concentration factor but for Ni-Mo steel it is nearly equal to the stress concentration factor of 3.3. (4) The size effects on the fatigue strength of notched specimen were computed from Isibasi's formula and the calculated values were in fairly good agreement with the test results."

Razov, Shevandin & Yefimov (1959) state that the size effect is usually explained either from the standpoint of the statistical theory [Weibull (1939a, b), Kontorova & Frenkel (1941), Chechulin (1954)] or from that of the stored energy theory of strength [Wells (1955)]. They describe an experiment, conducted to determine which of the two theories is valid, from which they draw the following conclusion (p.117): "... the size effect as a whole represents a combination of energetic and statistical factors. The former plays the main, and the latter a subsidiary role, determining the position and form of the distribution curve respectively."

Weibull (1959) applies the theory of probability ("weakest link" theory) to determine the relation between the strength of acetate threads or metal rods of different lengths and between the strengths of notched flat metal bars with one hole and two holes. He considers both exponential and Weibull distributions of strength, and compares his theoretical results with experimental data.

Yatsenko & Dybenko (1959) report the results of a study of the effect of the scale factor on the compressive strength of the wood-pulp layer plastic DSP. Since the loading rate affects the ultimate strength, they adopted a testing method which allowed the rate of stress increase to remain constant for all dimensions of the specimens with a varying rate of loading. The application of such a method of testing made it possible to establish that a change by a factor of about 18(from 1.4 to 25 cm^2) in the cross-sectional area of the specimen has practically no effect on the ultimate compressive strength of DSP plastic.

Bartenev & Tsepkov (1960) note that Lavrov (1958) considers the existing hypotheses inapplicable in explaining the influence of the scale factor on the bending strength of ice, on the basis of the assumption that, since the physical state of the surface does not change with changes in dimensions, the absolute elongation magnitude $\Delta\ell$ of the extreme stretched layer remains constant in the breaking of ice in bending. The authors reject this assumption because: (1) The linear dimensions of the most dangerous microcrack are small in comparison with the specimen width, and, therefore, the deformation $\Delta\ell$ of the extreme stretched layer is mainly determined by elastic deformation and not by the divergence of microcrack planes; (2) The bending failure of brittle materials, among which Lavrov includes ice, starts at the surface, where the most dangerous defects are formed and the stresses are at the maximum. The authors state that, in bending tests, the working surface is the surface subjected to tension, so that, in the equation given by Weibull (1939a), the working volume V must be replaced by the working surface S, and the strength is independent of the thickness. The authors illustrate their theory by analyzing Lavrov's data on the bending strength of ice and data of their own [Bartenev & Tsepkov (1958)] on that of glass. In each case the relation between strength σ and working surface S is linear on a log-log scale.

Davidenkov (1960) writes (p. 338 of translation): "As is known, the strength of products, especially in the case of brittle fracture (due to fatigue, impact, or low temperature), greatly depends on the product dimensions; even if geometric similarity is entirely preserved, larger parts appear to be weaker than parts with smaller dimensions (for instance, laboratory specimens). Regardless of the fact that this phenomenon is of great theoretical as well as practical importance, the nature of the so-called 'scale factor' has not yet been explained. There are several competing theories, none of which has been experimentally confirmed. The following hypotheses can be cited. <u>1. The statistical theory</u>, which explains the influence of dimensions by the presence of chance inhomogeneities in a solid, where the weakest of these inhomogeneities determines the strength; the probability of its existence is not equal for solids of different dimensions. ... <u>2. The energy theory,</u> which states that brittle failure depends on the elastic energy which is stored in the specimen-machine system and contributes to the specimen failure. ... <u>3. The technological theory</u>, which ascribes strength differences to unequal treatment

conditions, for instance, to the effect of treatment by cutting ... . 4. The theory which has been recently proposed by V. V. Lavrov [(1958)], according to which--at least in bending--failure is caused by the initial microcrack on the stretched side of a beam, the dimensions and the critical deformation of which do not depend on the beam dimensions. ..." He cites references to literature resulting from theoretical and experimental work on the various theories, and notes that Lavrov's theory is convincingly confirmed by experiments on bending of ice specimens.

Kosaka (1960) writes (Abstract, p. 97): "The effect of the ratio of height to width of concrete specimen on the compressive strength was investigated by means of the theoretical analysis as the statically determined plane plastic problem. Consequently, it was found that the above mentioned effect of the compressive strength of concrete specimen could be explained quantitatively by the analytical result of plastic stress which was carried out by applying the yield criterion of cycloid type."

Shchapov (1960) writes (pp. 340-341 of translation): "The absolute dimensions of specimens or products affect in different ways their behavior in deformation and failure and the mechanical properties of the material. Therefore, it can be hardly considered that such diverse phenomena are based on a single mechanism and that a single hypothesis, which would be valid for all cases, can be used in analyzing the effect of dimensions. It is also difficult to consider only the influence of absolute dimensions separately from an analysis of the shape factor. Thus, without simultaneously studying the influence of dimensions and of shape, it is difficult to solve the problem of the effect of stress gradients in deformation and failure and to evaluate qualitatively this influence in dependence on specimen dimensions. ... The purely statistical nature of the breaking load dependence ... is very clearly manifested in a number of cases, for instance, in the plastic failure of long wires, especially when the tests are repeated on pieces of the same broken wire, i.e., when specimens of decreasing length are tested. ... In a large number of papers, in particular, in papers cited in the article by N. N. Davidenkov [(1960)] in the present issue, the effect of the stored deformation energy increase with an increase in one or several specimen dimensions or with an increase in the elastic yielding of the entire system (specimen-machine) is very clearly established. This is especially true of the case where final

break conditions are concerned. ... Since, in connection with the diversity of failure types, it is impossible to explain the effect of absolute dimensions on the basis of only one hypothesis, it is neceasary to provide not only a more detailed, but also a more systematic determination of scale factor values for different cases. ... For an accurate determination of the dimensions effect proper, one should attempt to change only the dimensions of geometrically similar specimens while keeping the other factors constant, including the law of the increase in elastic energy stored in the entire system when loading is stepped up. ... The minimum ultimate strength values for small specimens in bending (or in alternate tension and compression) are apparently close to the average values of this characteristic for very large specimens, however, we do not have at our disposal sufficient amount of experimental data for verifying this assumption."

Uryu (1960) states the following conclusions (p. 46): "On the cracked mild steel plate specimens of two different sizes [thicknesses 7.5 mm. and 1.5 mm.], the rates of fatigue-crack propagation as well as the fatigue limits of the cracked specimens were measured under pulsating tension, and the following conclusions were obtained: (1) The fatigue limits of the cracked specimens were 4.5 and 3.5 kg/mm^2 for the small- and large-sized specimens, respectively. (2) Between the crack propagation rate [$\Delta\lambda/\Delta N$] and the nominal stress [σ], the empirical relation (1) [$\Delta\lambda/\Delta N = C\ \sigma^n$] was obtained, in which the coefficient C was dependent on the specimen size but the index n was not. (3) The rate of crack propagation in geometrically similar specimens under a given nominal stress was nearly proportional to the square root of the specimen size."

Volkov (1960) states (Summary, pp. 350-351 of translation):" 1. Generally, the scale effect is determined by the action of several factors. In theoretical considerations, the statistical and the energy factors are of primary importance, and, in experimental work, the technological factor plays an important role. 2. Changes in either of the two factors--the statistical or the energy factor--is not a unique and sufficient indication of the existence of the scale effect. Under certain experimental conditions, these factors act simultaneously, and, under different conditions, they act independently. The scale effect which influences the strength is related to the energy factor by a nonuniquely defined function, and the scale effect influencing the maximum plasticity is so

related to the statistical factor. 3. The statistical theory, which is most developed in the quantitative sense, can serve as a basis for establishing a general scale effect theory which would include the action of not only the statistical, but also of the energy and, possibly, the technological factors."

Weibull (1960) writes (Summary, p. 1):" Fatigue tests on axially-loaded 24S-T and 75S-T aluminium sheet specimens of various sizes were conducted with a constant nominal stress amplitude, established by a stepwise reduction of the load in proportion to the remaining cross-sectional area. Earlier observations were confirmed, showing that, in this type of test, the crack propagates with a constant, stable rate of growth after a--generally short--transition period has been passed. This transition period was found to increase with the number of cycles necessary to initiate a visible crack. A law, deduced by means of dimensional considerations and stating that the stable rate of growth should be proportional to the size of geometrically similar specimens, was well substantiated, which implies that the relative rate of growth is, for a given stress amplitude and mean stress, a material constant. For geometrically similar specimens, the period of crack initiation was observed to depend on the size of the notch and to increase with decreasing size, which is in accordance with the statistical theory of strength."

Weiss, Sessler, Packman & Sacks (1960) report the results of a systematic study of the effect of several geometrical variables on the notch tensile strength (with stress concentration factor K held constant) of 4340 sheet steel heat treated to three strength levels (150, 210 and 260 ksi). They state (Abstract, p. iii): "For the material conditions investigated, it was found that the effect of stress gradient on notch strength was fundamentally identical with that of section width. Increasing the section width (or decreasing the stress gradient) resulted in a decrease in notch strength when all other factors were held constant. However, the effect was small when compared to the section size effect observed previously for notched cylindrical bars."

Wessel (1960) writes (Abstract, p. 277): "The applied stresses and temperatures at which brittle fracture will occur in the presence of a pre-existing crack are dependent upon section size and crack length. The fracture strength was found to decrease in proportion to the inverse square root of the crack length in specimens of constant section size. Hence, size effects must be considered in practical situations."

Williams (1960) states the following conclusions concerning size effect in sheet material (pp. 20-21):" 1. The failing stress of thin sheet under tensile forces across a crack is largely independent of its thickness whether the sheet is flat or constitutes the skin of a pressurized cylinder. ... 2. Comparison of results for similar flat sheet specimens (same planform, and crack lengths proportional to the linear dimensions) shows that there is a scale effect--different for different materials--that makes the larger specimen fail at a lower applied stress than the smaller. 3. That it is a true scale effect is indicated by the shape of the curve that has applied stress at failure for ordinate and crack length as abscissa. When the crack is still short compared with the width of the sheet, so that the stress at distances from the crack large compared with its length are relatively unaffected as the crack extends, the curve follows closely that given by equation (4) [$\sigma_x/\sigma_{\ell_0} = r^{[\log(x/\ell_0)/\log n]}$, where σ_x and σ_{ℓ_0} are the failing stresses at length x and at the initial length ℓ_0 of the crack and r is the ratio in which the failing stress is reduced when the length is increased in the ratio n]. 4. The amount of this scale effect is readily obtained by comparing the applied stresses at failure of two similar sheet specimens (with similar crack lengths) that are substantially different in size. Once this is obtained for a particular sheet material the applied stress at failure of any size of sheet (of the same shape and material) can be estimated from a formula such as (4). 5. A similar scale effect occurs in pressurized sheet cylinders with cracks. Once this is evaluated by comparing two similar cylinders (with similar cracks) the stress at failure for any size of (similar) cylinder and any length of crack can be estimated from the curve of stress at failure against crack-length for one of the original cylinders. 6. For corresponding flat sheet and cylindrical specimens the scale effects referred to ... above should theoretically be the same for the same material."

Ames (1961) reports the results of tests conducted to determine the effects of section size and testing condition on tensile properties of zircaloy 2 at 600°F. He concludes (Synopsis, p. 820):"... Yield and possibly tensile strengths appear to be dependent on the type of specimens used. Flat specimens generally give higher values than round specimens while machining of flat specimens to thinner gages also results in an improvement."

Bell (1961) writes (Abstract, p. 1260): "This paper reports the results of a study to determine the effect of glass fiber geometry [defined to include the length and cross-sectional area of the fiber as well as the orientation of the fiber in the matrix] on composite material strength. Equations are derived for the load distribution in a composite material and also for the stress distribution. To determine the effect of fiber geometry, epoxy resin composites are analyzed for composite efficiency. These same composites are analyzed for glass fiber efficiency. The highest composite efficiency determined was 58% for filament wound fibers; and the highest fiber efficiency was 68% for cross laminated fibers. In general, this type of analysis may be useful in evaluating fiber geometries in other composites. No attempt was made here to analyze other systems."

Bluhm (1961a) writes (Synopsis, p. 1324): "A simple model of the fracture phenomenon in thin sheet is suggested based essentially upon the assumption that the shear lip formation is a volume-sensitive mechanism whereas flat fracture is essentially a surface phenomenon. A critical shear lip width is hypothesized; this concept is substantiated and is shown to lead to the variations in shear lip width with thicknesses that were experimentally observed for a beta titanium alloy. Furthermore, the fracture toughness-thickness relationship constructed by the simple linear superposition of the above two fracture mechanisms predicts satisfactorily the observed fracture toughness values for this alloy. A numerical illustration indicates how two tests at different thicknesses might be used to depict the g_c [fracture toughness] versus thickness curve at a given temperature over a wide range of thicknesses." Blühm (1961b) writes (Synopsis, p. 1332):" A model previously suggested for predicting the fracture behavior of thin sheet is extended to cover laminated composites. Thus extended, this model leads to a prediction of the behavior of laminated Charpy bars which is at least qualitatively substantiated by the observed behavior." He notes that Arnold (1957) [see Additional References, p. 409] has proposed a laminated Charpy specimen for evaluating impact strength of sheet metal and has experimentally determined the effect of sheet thickness on that impact strength, while Curll (1961) has suggested a relationship governing the size effect in Charpy bars which is somewhat similar to his own [Bluhm (1961a)].

Borges (1961) writes (Summary, p.1): "The present paper deals with statistical theories of structural similitude. These theories make it possible to study how the random behaviour of models changes when considering geometrically similar prototypes, made of materials with the same properties from a statistical point of view. Similitude for rupture is discussed considering hypotheses corresponding to a behaviour ideally brittle and to a behaviour ideally ductile. Similitude taking deformation as the condition for failure is also analysed. The way ... changes of scale influence the dispersions and the mean values [and medians] is quantified for the different theories [based on normal, lognormal, and Gumbel (extreme-value) distributions]. It is concluded that the main practical interest will be in the theory of similitude for failure by deformation, which can be easily generalized so as to become a statistical theory of structural behaviour."

Bravinski & Osipov (1961) state at the outset (p. 250 of translation): "The effect of the size of an article or sample on its mechanical strength (scale effect) has often been noted in the literature. However, there is no satisfactory explanation of this phenomenon; the experiments have not satisfactorily proved the theories [Davidenkov (1960) has discussed four proposed theories]. These experiments concerned only crystalline or amorphous substances. Oxide ceramic materials, however, which are of particular interest because they include crystalline as well as amorphous phases (glass), have not been investigated. These materials are brittle and the scale effect is particularly clearly defined. The effect of the scale factor on the long range and short range strength of ceramic materials has not been previously investigated, although it is of theoretical as well as practical interest." They proceed to describe tests performed on unpolished specimens of aluminum silicate and aluminum oxide ceramic materials 5 to 20 mm. in diameter, polished specimens of the same materials 5-8 mm. in diameter, and specimens of porous magnesium silicate ceramic material 4 to 15 mm. in diameter. The strength of the unpolished specimens showed a decrease (linear on log-log paper), with increase in diameter in accordance with known theoretical equations [Weibull (1939a), Chechulin (1954)]. The strength of the polished aluminum silicate specimens showed a smaller decrease, while that of the polished aluminum oxide specimens remained constant within experimental error (about 5%), as did that

of the porous magnesium silicate ceramic specimens. The authors also report the results of experimental tests of the variation of resistance to bending of cylindrical samples of aluminum silicate and aluminum oxide materials 10 mm. in diameter as a function of the working length ℓ of the sample. In each case the resistance first decreased and then remained constant with increasing values of ℓ. Finally, they report the results of a study of the variation of the resistance to transverse bending of samples of aluminum silicate 13, 10, 7.5 and 5 mm. in diameter as a function of time. The relationships of log time to bending stress are linear and the slopes of the straight lines do not change noticeably when the diameter is decreased from 13 to 5 mm. In conclusion, the authors state (p. 251 of translation): "On the whole the experimental results make it possible to assume that the scale factor has an effect not only on the short range but also on the long range strength of ceramic materials. The increase of short range strength with the decrease of the size of the sample or its working length can be explained by the effect of the technological or statistical scale factor."

Curll (1961) writes (Abstract, p. 91): "To provide a means of impact acceptance testing of specimens that must be obtained from thin or small-diameter sections, a study was made using various specimen sizes, of the Charpy V-notch impact properties of alloy steels over a range of temperatures encompassing their transitions from ductile to brittle behavior. The test data tend to indicate a linear correlation between the energy required to fracture standard-size and subsize Charpy V-notch specimens at the normal acceptance-test temperature of -40C."

Frost & Denton (1961) report the results of tests which lead to the conclusion that, for similar nominal stresses and crack lengths, there is no significant difference in the rate of growth of fatigue cracks in 0.128-in., 0.3-in. and 1.0-in. thick specimens of mild steel (10 in. wide).

Hickey (1961) summarizes the results of an investigation of the effect of section size and other factors on the mechanical properties of annealed 6Al-4V, 6.5Al-3Mo-1V, and 6Mo-3Al titanium alloy stock ranging in size from 2 to 6 in. in diameter as follows (p.878): "... Section size proved to be a very important and sensitive variable. With a decrease in section size strength values increased, impact values decreased and, in general, the ductility decreased. The strength effect can be partially acccounted for by the finer

microstructure present in the smaller size bars. Beyond this, however, there is a hardenability effect, since maximum properties could not be developed in the larger sections. ..."

Jastrzebski (1961) writes (p. 130): "The existence of a relation between the absolute dimensions of a structural element and the ultimate stress is called the 'scale effect' and found its theoretical justification in the works of W. Weibull [(1939a, b)] and J. Frenkel and T. Kontorova [Frenkel (1948)]. However, these publications concern brittle materials only, the brittle tensile strength of metals being included. The fact that ductile materials were not considered can be explained by the circumstance that, for brittle materials, the scale effect is of relatively greatest importance." On the basis of tensile and yield tests on over a thousand steel bars, he proposes the equation $R=R_D+k/L^c$ for the tensile strength R, where $L=\ell/\ell_1$ is the ratio of the length ℓ to the length unit ℓ_1, R_D is the minimum strength (for $L\to\infty$), and k and c are material constants. This equation agrees in principle with Weibull's equation for brittle materials. For yield strength, he proposes an analogous equation $Q=Q_D+ k/L^c$. He estimates $R_D(Q_D)$, k, and c from his experimental data by the method of least squares.

Kermes (1961) reports experimental results on the brittle fracture of carbonaceous structural steel which confirm the influence of the size factor on the limiting state of steel structures, but he warns that other equally significant factors (such as stress gradients and notch geometry) must not be neglected.

Klier & Weiss (1961) write (Synopsis, p. 1307): "Cylindrical sharp-notch tension specimens of gross section sizes up to 2 in. in diameter have been tested at room temperature. Steel, titanium, and aluminum materials in both strengthened and softened structural conditions have been examined to establish the effect of test specimen size on the notch properties. The changes in notch geometry at the point of fracture initiation have been studied as function of specimen size for unalloyed titanium. Limited test data are offered to show that notch geometry changes must play an important role in the phenomenon of size effect. ..."

Kremser (1961) claims that the decrease in tensile strength with an increase in length of the specimen observed in experimental data depends not

so much on the material as on the variation of strength in specimens of the same length. He suggests that the statistical nature of strength variation should be recognized in specifications for structural materials and that several specimens of the same length should be tested and the arithmetic mean and some measure of variation (such as the standard deviation) of the test results should be reported, in order that the length effect on strength may be properly taken into account.

Ludley & Drucker (1961) write in their introduction (p. 137): "The concept employed by Griffith [(1920)] to explain the tensile fracture strength of a brittle material such as glass gives a very strong size effect. Essentially, crack growth is viewed as an equilibrium process quite similar to bubble growth in a liquid under decreasing pressure. Energy needed to maintain additional crack surface is equated to the accompanying release of the potential energy of strain and of the loading system. The result is that the nominal or average fracture stress varies inversely with the square root of the crack length. A modified Griffith hypothesis has been advanced for the brittle fracture of notched plates of normally ductile steel. ... However, the validity of the Griffith type of reasoning for the initiation and propagation of brittle fracture in ordinary structural steel seems highly doubtful." They proceed to describe tests performed on specimens of project E-steel, 3/4 in. thick and 10 in. long; four different plate widths were used: 3-1/3 in., 6-2/3 in., 10 in., and 20 in., and the notch depth on each side was always 15 per cent of the gross plate width. The results show that the size effect was nowhere near what would have been predicted by the Griffith theory; in fact, the data indicate almost complete size independence for 6-2/3 in., 10 in., and 20 in. plates.

Moe (1961) writes (Summary, p. 178): "A theory of the basic mechanism of failure of wood in bending is outlined which qualitatively accounts for the well known height effect on the strength of timber beams. The theory agrees in certain important respects with experimental findings obtained through tests of more than forty laminated timber beams. Further experimental verification is necessary."

Nemets (1961) states that the appearance of initial defects (cracks) of a given size depends on the dimensions of the body, and hence it is necessary,

for determining the effect of size of body on brittle strength, to find the dependence of the strength on the number of defects. He considers the problem of finding the actual strength of a specimen with cracks, taking account of the influence of boundary conditions and of the distribution of elastic stresses in the region of the crack. This leads him to consider the elastic energy near a crack, expressed by the function $f(\xi)$, which he plots for various values of η for plates of width B and length L, having a centrally located crack of length ℓ, where $\eta=\pi B/L$ and $\xi=\ell/B$. From his graph, it is clear that for small (short) specimens (large values of η) the growth of free elastic energy is decreased by crack growth, and therefore the energy expended on overcoming material bonds at the end of the growing crack is dominant. This impedes the development of sudden brittle fracture. For large (long) bodies (small values of η) the growth of free energy of elastic deformation is increased, which promotes the onset of sudden brittle fracture. Thus by varying the length of specimens, ductile fracture may be changed to brittle fracture. The author goes on to consider some limiting cases and to develop a quantitative expression for the brittle strength which involves $f(\xi)$ implicitly and thus embodies a kind of size effect.

Vitovec (1961) writes (Summary, p. 163): "A study was conducted on the effect of grain size, size of specimen and stress gradient on the direct stress fatigue strength of a magnesium alloy at room temperature. Specimens with three different grain sizes and seven different notch sizes were used in this investigation. The data are analyzed with regard to relationships between grain size, size of notch, relative stress gradient, stress gradient per grain, and theoretical maximum strength of notched specimens. The best correlation of the data was obtained by plotting the ratio of theoretical stress concentration factor and effective stress concentration factor as a function of the relative stress gradient."

Weibull (1961) devotes Sections 61.6 and 63.1 of his book on fatigue testing to the size effect. In Section 61.6 (p. 99), he writes: "The effect of size on fatigue strength is a complex problem. It frequently depends both upon structural changes in the material and upon the 'statistical size effects.' Only the former effect will be discussed here. The latter effect will be discussed in Section 63.1. The best known effect of size on the strength

properties of a material relates to cast iron. It is an old observation that its strength is in general much better in bars of small than of large diameter. This effect is readily explained by the difference in cooling rate. An investigation separating this effect from the statistical effect has been carried out by Meyersberg (1952) by means of different types of static tests. In the same way, the effect of heat treatment of the material may depend on the size, and fabrication processes such as rolling and wire drawing may turn out products which differ with regard to the material owing to the dimensions." In the remainder of Section 61.6, the author summarizes the literature on size effects related to structural changes. In Section 63.1 (pp. 111-113), he writes: "It has been known for a long time that dimensions have a definite influence on the static strength of the test piece and that the strength, in general, decreases with increasing size. Several investigators have presented evidence that this rule also applies to the fatigue strength ... . The explanation of size effect has been approached from two different viewpoints. According to the first concept it is assumed that fatigue failures originate at small local inhomogeneities (inclusions or the like) which are statistically distributed over the volume of the specimen. Each such nucleus has its individual endurance limit, no failure being started from it if the nominal stress is below this value. The probability of encountering a nucleus of a certain severity increases with the volume. This concept, initially intended as an explanation of size effects in brittle materials (Weibull, 1939a,b) subjected to static strength (N=0), is directly applicable to the fatigue strength (but not to the fatigue life) at an arbitrarily preassigned life N. ... Stulen (1951) ... states that ... 'the origin of failure is almost always at a microscopic non-metallic inclusion which is open to the surface or is slightly subsurface.' ... It follows that in many, if not all, cases the surface and not the volume is the appropriate 'size'. The other approach to the size-effect problem takes into consideration the structure of the material, in particular the grain size. This quantity is a relevant factor when the stress distribution is non-uniform ... . From the preceding it is safe to conclude that the size effect exists, but that it is not easily established, the reason being the many irrelevant factors which mask the result by simulating the proper size effect. There is, however, another reason of even greater significance, and

that is that the laws of size effect must be quite different for the pre-crack and for the post-crack stages of the fatigue process. In a study of crack initiation and propagation in flat specimens notched by a central hole, size effect was found (Weibull, 1956) on geometrically similar specimens only in the pre-crack stage; but the propagation time of the crack was independent of the size of the specimen. The relevant size in this case is the diameter of the hole (or perhaps, more exactly, the circumference). ..."

Yukawa & McMullin (1961) write (Summary, p. 543): "The test results of fracture tests on notched specimens of a heat-treated alloy steel indicate that an arrested cleavage or fatigue crack lowers the brittle fracture strength by about 35 per cent compared to a machined notch with approximately 0.005 in. radius. This difference is observed if the specimen size is sufficiently large; with decreasing size, the difference becomes less and may disappear altogether if relatively small specimens are tested. By nitriding the notch, it appears possible to obtain effects with machined radius notches equivalent to cracks. The concepts of the modified Griffith theory of fracture and of limiting notch strength are utilized to indicate the conditions under which notch acuity effects may or may not be observed".

Agnew & Stout (1962) state the following conclusions (p.159s): "1. The effect of plate thickness on the tendency toward brittle fracture in service is inextricably dependent upon the other purely geometrical factors acting in the structure. 2. In order to evaluate the true effect of plate thickness by means of a bend specimen, it is essential that the other dimension of the cross-section of the specimen be large enough to act as though it were 'infinite' so far as its influence on brittle fracture tendency is concerned. 3. In designing bend tests to investigate the effects of specimen geometry, the criterion specified to measure transition temperature must be taken into account. Generally, if the value of the criterion changes with the dimensions of the specimen (such as percent lateral contraction), the indicated effect of size is greater than if the criterion is invariant. 4. The ability to initiate cleavage fracture is directly related to specimen width and depth and inversely related to test span, a maximum effect being observed with each. If the limiting dimensions are not exceeded, equivalent combinations of width and depth (span constant) appear to be expressed by a product or sum relationship; the combined

effect of depth [D] and span [L],with width constant, can be expressed by the term D/L^2. ..."

Couts & Freeman (1962) write (pp.222-225): "A notch-sensitive forging was tested at 1300 F to measure the effect of machining practice and of limited variations in K_t [theoretical stress-concentration factor] and specimen size upon notch rupture life. Within the limits studied, specimen size appeared predominant. A discontinuity was observed in the dependence of notch life upon stress. ... Specimens with 0.125 or 0.350-in. diameter at the notch had longer rupture times than were obtained for the notched specimens with 0.250-in. notch diameter at equal stress. ... The extremely limited data suggest that a specimen-size effect overrode any effects from method of preparing the notches or from minor variations in K_t. One should recognize, however, that this change in rupture time with specimen size, and especially the apparent shorter rupture times for 0.250-in. notch diameter than for either smaller or larger size, was not well established."

Durelli & Parks (1962) write (Introduction, p.931): "It is well known that the tensile strength of brittle materials, determined by tension tests, is somewhat dependent on the size of the tensile specimen used. It is also known that the tensile strength of brittle materials, determined by some other means than a simple tension test (often a bending test), is almost invariably higher than the values obtained from tension tests. This second type of test can be characterized by the σ_1 stress gradient where σ_1 is the maximum principal stress at each point along the line of failure, and the σ_1 stress gradient is the gradient of these stresses at the initial point of failure, along the line of failure. Various investigators have attempted to explain and correlate these two phenomena. Weibull's statistical study [Weibull (1939a)], which evaluates both phenomena in terms of material constants, is probably the most important contribution in this area. Tucker [(1941)] also used statistical concepts, but followed a different approach. A paper was prepared by one of the authors considering the stress gradient influence only ... and in a report ... an attempt was made at an explanation using the 'finite particle' concept. It does not seem that these previous contributions explained the influence of size and gradient with sufficient generality or that they fitted the experimental data with sufficient accuracy. In the present paper a group of

twelve shapes was used to study both phenomena, and a relationship [$\sigma=c(v)^{-k}$, where c and k are constants computed from the results of two types of failure tests] is obtained to account for both the size influence and the stress gradient influence. Although the relationship is empirical and somewhat arbitrary, it is very simple and it is believed that it will fill a great need of researchers in the field. ..."

Ferguson & Thompson (1962) report the results of an experimental investigation whose main purpose was to establish the development and anchorage length required to develop the full tensile capacity of steel reinforcing bars in concrete. They found that the bond strength depends primarily on development length rather than bar size, but decreases somewhat for the largest size bars and is also a function of concrete strength, bar cover and beam width. They recommend further tests on large bars.

Hartman, Jacobs & de Rijk (1962) report the results of an experimental investigation and state the following conclusions (p.8): "Fatigue tests were made on double row riveted single lap joints of 2024-T4 alclad at fluctuating tension with a mean stress in the nett section of 9.0 kg/mm^2 and stress amplitudes ranging from 8.6 to 3.0 kg/mm^2. Tests on geometrically similar specimens with a size ratio of 1: 2.66 did not show any significant difference in endurance. A measured small, but significant difference in endurance at high stress amplitudes between the specimens with a sheet thickness of 1.0 mm. and those with thicknesses of 0.6 and 1.5 mm. was probably related to differences in mechanical properties of the sheet materials. With increase in size the fracture gradually changed from fracture in the sheet adjacent to the closing head of the rivets to fracture in the sheet adjacent to the die head. In general fretting around the holes was less severe for the small than for the large specimens."

Manjoine (1962) writes (Introduction, p.220): "Much of the evaluation of rupture properties of large components, such as rotors, is made on the basis of tests made on small specimens. The reduction of strength properties with increasing size has been demonstrated for several materials in certain temperature ranges. The size effect for brittle fracture at lower temperatures has been shown to be one which caused a decrease in strength with increasing size. The work of Davidenkov, et al. [(1947)] [and] Weibull [(1939a)] ... indicates

that the statistical theory of strength can explain this decrease in brittle strength with increasing size. The work of ... Griffith [(1920)] suggests a relationship between the stored energy and critical energy adsorption of the material which predicts a decrease in strength with increasing size. For ductile strength, a size effect has been observed by Miklowitz [(1950)] and Lyse [& Keyser (1934)]. The former attributed the decrease in strength and strain to the restraint at the neck. This effect of constraint accounts for the focus of attention in size phenomena which has been given to notched bar testing." He summarizes his results as follows (p.221): "A size effect can be demonstrated for elevated temperature notched rupture strength. As the size is increased, the rupture strength approaches that of the unnotched bar. For a material which is notch strengthened, the rupture time of a notched bar is primarily that of the initiation time which decreases as the size is increased, since the stress gradient and strengthening effect at the base of the notch is decreased. For a material which is notch sensitive or weakened, the initiation time and propagation time both increase with specimen size, therefore, the rupture time increases with size. ..."

Okushima & Kosaka (1962) report the results of an experimental investigation of the effect of specimen size on the mechanical properties of concrete under compression. They used cylindrical specimens with diameters 15, 10 and 5 cm. and lengths twice the respective diameters. They found that the mechanical properties were influenced little by the specimen size for stresses up to 70 per cent of maximum stress, regardless of the mixing ratio, but that the ultimate compressive strength increased slightly with a decrease in the size of the specimen.

Ouchida (1962) summarizes the results of rotating-bending fatigue tests on shafts 10 to 100 mm. in diameter of medium carbon steel of 0.39 per cent carbon content, with special attention given to shrink-fitted members. He states the following conclusion (p.594): "Shrink-fitted specimens hardened by induction-hardening exhibited size effect as large as untreated shrink-fitted specimens; the reduction ratio 1.8 was obtained by comparing the endurance limit for a shrink-fitted specimen 10 mm. in diameter with that of a specimen 100 mm. in diameter, and the value 1.6 was obtained for an induction-hardened specimen with a shrink-fitted hub."

Repko, Jones & Brown (1962) report the results of an experimental investigation of the influence of sheet thickness on the sharp-edge-notch properties of B 120 VCA titanium alloy, which they tested at both room and low temperatures for thicknesses between 0.010 and 0.130 in. On p. 228 they write: "The following conclusions appear substantiated by the smooth and sharp-edge-notch tensile data presented in this paper. It is to be emphasized that these conclusions apply to the β titanium alloy investigated and that generalizations regarding the thickness effect cannot be definitely established without similar data on other alloys. Such data are not yet available. 1. Large differences in sharp-edge-notch strength and fracture toughness of aged sheet are associated with inherent variations between the heats representing different thicknesses. These differences obscure the true thickness effect. These large differences are not revealed by tests on smooth specimens which show the tensile strength properties vary by not more than ± 10 per cent over the thickness range investigated. 2. The absolute thickness effect can only be established for this alloy by tests on a single heat of material. ... 3. The absolute effect of thickness on both the sharp-edge-notch strength and fracture toughness are qualitatively identical. Both these quantities decrease with increasing thickness to a nearly constant value between 0.063- and 0.130-in. thickness. 4. The plane strain fracture toughness of one heat of the aged β titanium alloy was about 46,000 psi $\sqrt{in.}$ The highest K_c value observed was 75,000 psi $\sqrt{in.}$ for 0.010-in. sheet. ... 7. While the influence of test temperature on the thickness effect was not established, it is apparent that the solution treated β titanium alloy at all thicknesses investigated maintains very high toughness down to -110 F but is quite brittle at -320 F."

Serensen & Strelyaev (1962), as part of a theoretical and experimental study of the properties of glass reinforced plastics, consider the size effect and make use of the Weibull distribution to assess this effect statistically. They note that Weibull's statistical theory of strength rests on two basic premises: 1) that the probability of fracture is a function of the stress; 2) that fracture is determined by the presence of one weak link (an elementary volume or area). On p. 14 of the translation, they state the following conclusion: "Glass reinforced plastics have greater structural heterogeneity than metals and alloys. The lack of uniformity of the properties leads to a

considerable scatter of the strength characteristics (strength variation coefficient amounting to 20%) and causes the absolute dimensions to have a considerable effect on the strength of components made from these plastics; this effect can be determined statistically by means of the homogeneity coefficient m [the shape parameter of the Weibull distribution] and the relationship between the strength and the area and volume of material in the component under load."

Sitnik (1962) reports the results of an experimental investigation of the strength and other properties of three types of concrete, with specimens of various sizes and shapes. Measurements were made after the specimens had been cured for various numbers of days. The author reports that 20-cm. cubes had from 90% to 97% of the strength of 10-cm. cubes, while the strength of 10x10x40 cm. prisms was, in most cases, between 70% and 77% of that of 10-cm. cubes. These ratios tended to reach their maxima for curing periods of from 14 to 28 days and then drop off slightly, probably because the smaller specimens cured more quickly and thus reached their maximum strength earlier than the larger ones.

Hickey & Larson (1963) report the results of a comparison of various tension specimens in the determination of notch sensitivity and fracture toughness of high-strength sheet materials. Specimens considered were 1-in. wide edge notch, 1.5-in. wide center notch, and 3-in. wide center notch specimens of several molybdenum steels and one titanium alloy. Notch sensitivity data were obtained from all specimens, whereas fracture toughness data were gathered primarily from the 3-in. wide specimens. The following conclusion concerning size effect is stated on p. 796: "Specimen size is a very important criterion in the selection of material. Results in this study indicate that most materials exhibit increased notch sensitivity as specimen size increases from 1.0 to 3.0 in. An exception to this trend is evidenced by H-11 steel which shows greater sensitivity in the 1.5-in. wide fatigue-cracked specimen than in the 3-in. wide specimen. It was also shown that the obtainable fracture toughness range increases with specimen width, thus a narrow specimen can be used only for relatively brittle materials. Therefore it has been concluded that of the specimens considered in this investigation the 3-in. wide specimens generally offer the best test to measure notch sensitivity or fracture toughness."

Hoshino & Arai (1963) report the results of (1) static stress measurements and rotating bending fatigue tests on three model crankshafts (80 mm in pin diameter, 45 mm in web thickness, 106 mm in web breadth) with pin fillet radii of 8.0, 5.0 and 3.0 mm, respectively and (2) rotating bending fatigue tests of plain cylindrical specimens of several sizes taken axially and tangentially from the crank material (0.3% carbon steel). They state the following conclusions (p. 192): "(1) The stress concentration factors of the crank specimen ... can be given approximately by the following formula: $\alpha_{K_1} = 1.22(d_p/\rho)^{0.45}$ where d_p is the diameter of crankpin, and ρ is the fillet radius of crankpin, in mm. (2) The fatigue limits represented by the stress on the parallel part of crankpin were 6.8, 5.5, and 4.3 kg/mm² for the fillet radius 8.0, 5.0 and 3.0 mm, respectively. (3) The fatigue limit of the plain cylindrical specimen of 10 mm in diameter taken from the web perpendicular to the metal flow is 21.5 to 23.1 kg/ mm², and the fatigue notch factor obtained as the quotient of these values divided by the stress on the parallel part of the crankpin coincided with the above-mentioned value of the stress concentration factor." They note, however, that the size effect on the fatigue strength of a material having a fine crack is much larger than that of a sound material, so that conclusion (3) would not be expected to hold if the material of the crankshaft includes some defects such as segregation crack.

Klier, Muvdi & Sachs (1963) write (Synopsis, p. 546); "The results of tension and notch-tension tests are presented for ... [seven] high-strength steels. The steels were heat treated to characteristic strength levels in the range from about 180,000 to 300,000 psi. Longitudinal and transverse tension and notched tension specimens to 0.9-in. diameter were tested. The different diameter notch specimens were notched to give elastic stress concentration factors of K = 3, 5, or 10. In general, the tensile strength was found to be independent of the specimen orientation but to decrease gradually with increase in specimen size. The ductility of smooth specimens was observed to depend on both specimen orientation and specimen size. The notch strength decreased with increase in stress concentration, specimen diameter, and as-processed section size. It also decreased as the specimen orientation was changed from longitudinal to transverse. As the tensile strength was reduced to less than about 200,000 psi these effects became of little significance."

Peterson (1963) enumerates various factors, including the size effect, that enter into engineering design problems. He plots some constant-load-amplitude test results, obtained by Phillips & Heywood (1951), for mild steel unnotched specimens of various diameters. He notes that crack propagation is of key importance in size effect, since failure in constant-strain-amplitude tests is often defined on the basis of a small crack of arbitrary length. He also considers the notch size effect and its relation to stress concentration.

Vorliček (1963) considers the problem of the effect of the extent of the stressed zone upon the strength of a body. Since strength is a random variable, he employs statistical methods. The stressed zone may be the entire volume V, or it may be only a portion of the volume or even a surface; for simplicity, the volume V is used in the derivations. The author reviews the literature on the subject, especially the writings of Weibull (1939a), Kontorova & Frenkel (1941), and Chechulin (1954). He analyzes three idealized cases: 1) A series-like bond between the basic elements in a body of plastic or brittle material (the principle of failure is the same for both); 2) a parallel bond of basic elements in a body of brittle material; 3) a parallel bond of basic elements in a body of plastic material. In each case, he determines the asymptotic behavior (as $V \to \infty$) of the mean, standard deviation, coefficient of variation, skewness, and least value of the strength, and tabulates statistical parameters of the strength distribution and lower probability limits on the strength. He states that for some kinds of real engineering materials it should be possible to apply directly the results for one of the three idealized materials, while for others the situation is more complex, requiring a combination of the results for the analyzed cases.

Berg & Nagevich (1964) describe tensile tests performed on smooth bars having tar content of 26% to 36% with a view toward using them as electro-corrosion-resistant reinforcement in concrete ground structures. For bars of diameter 5, 11, 22 and 28 mm, Young's modulus E was found to be 487, 384, 380 and 360T/cm^2, with ultimate strength 13.5, 10.4, 7.5 and 5.3T/cm^2 and ultimate strain 2.74, 2.67, 1.95 and 1.47%, respectively (T = metric tons). Strength under sustained loading dropped to 65% and under vibrational loading ($2 \cdot 10^6$ cycles) to 45% of the above values.

Kani (1964) writes (Summary, p. 441): "Under increasing load a reinforced concrete beam transforms into a comb-like structure. In the tensile zone the

flexural cracks create more or less vertical concrete teeth, while the compressive zone represents the backbone of the concrete comb. The analysis of this structural system has revealed that two rather different mechanisms are possible: as long as the capacity of the concrete teeth is not exceeded the beam-like behavior governs; after the resistance of the concrete teeth has been destroyed a tied arch, having quite different properties, remains." For shear arm ratio $\alpha = a/d$ (the ratio of shear span a to effective depth d) less than α_{min}, the strength of the beam is governed by the strength of the concrete arch and decreases with increasing value of α, reaching a minimum at $\alpha = \alpha_{min}$. For $\alpha_{min} < \alpha < \alpha_{TR}$, the strength of the beam is governed by the capacity of the concrete teeth and increases with increasing value of α. When $\alpha \geq \alpha_{TR}$, the full flexural strength of the beam is reached. The author gives formulas for determining α_{min} and α_{TR}. The reviewer (K.-H.Chu) questions some of the author's conclusions, especially the prediction that poorer bond will result in an increase of load-carrying capacity, which, the reviewer asserts, is contradicted by test results.

Karapetyan (1964) reports the results of a study of the effect of the scale factor on creep, during contraction and expansion, of prismatic concrete specimens with cross-sections of 7 × 7, 10 × 10, 15 × 15 and 20 × 20 cm., all of height 60 cm. As the title implies, his paper deals primarily with size effect on creep deformation, but it also considers the size effect on prismatic strength and on ultimate strength. Before the specimens were loaded with a prolonged load, 12 prisms were tested to determine the prismatic strength, the ultimate strength during axial expansion, and the modulus of deformation of concrete as a function of specimen dimensions. Cubes with sides measuring 7, 10, 15 and 20 cm. were also tested. It was found that, despite the appreciable variation of the factor h/a (where h is the height of the prism and a is the side of the cross-section), the prismatic strength of the different dimensions of specimens was essentially identical. The ultimate strength of concrete during axial expansion was found to depend to a greater extent on the cross-sectional dimensions of the specimen. The greater the cross-section of the specimen, the less the tensile strength of the concrete. The ratio of the ultimate tensile strength of concrete to cubic strength decreases as the cross-section of the specimen increases. The author presents

a theoretical explanation of the size effect on creep deformation, but not on strength.

Kogaev (1964) concludes, as a result of a theoretical and empirical study of the effect of stress concentration and of the scale factor on fatigue strength, that the use of the statistical theory of "weakest link" strength and the Weibull distribution satisfactorily describes the effect under study. He gives equations, obtained from the indicated theory, which may be used as the basis of a method of constructing probability diagrams of the fatigue of machine parts, which characterize the dependence of lifetime and failure stresses on specimen diameter d, stress gradient $\overline{G}$ and failure probability P. It follows from the theory, and is confirmed by experimental data, that round specimens of different dimensions and geometry, but with identical $d/\overline{G}$ ratio during plane bending, have coincident lifetime distribution functions and failure stress functions. These functions should therefore be determined from fatigue tests of specimens having two or more different $d/\overline{G}$ ratios.

Low, Campbell et al (1964) discuss critical areas where modifications or additions to the recommendations in earlier reports of the ASTM Committee on Fracture Testing of High-Strength Materials are needed. One of these areas is specimen and crack size effects. The authors point out that a specimen must fracture prior to general yielding for valid measurement of K_c (the crack or failure toughness value of the material) and K_{Ic} (the plane-strain crack toughness of the material, i.e., the limiting value toward which K_c decreases with increase of plate thickness). They write (p. 111): "As the crack lengths become shorter, and the failure stress approaches and exceeds the yield strength of a material, the experimental data deviate from the theory. At zero crack size the failure stress is merely the unnotched tensile strength.··· In addition to short crack length, inadequate width in sheet-type specimens and too small a diameter in sharply-notched round-bar tests can also cause general yielding prior to fracture. Specimen sizes that are too small give data of little significance for most structural applications." Several graphs are given showing the effect of crack length, specimen width, and specimen size (shank diameter) on measured K_c values for 2219-T87 aluminum, 4330M steel and/or 2014-T6 and 7075-T6 aluminum.

Misiolek (1964) reports the results of 24 long and 24 short ($L/\sqrt{A}$ = 11.3 and 5.65, respectively, where L is the length and A is the cross-sectional

area) ¼-inch thick sheet specimens of each of three materials: refined copper, 70/30 α brass, and 0.15C mild steel. By means of an analysis of variance, he shows that the effect of length on ultimate strength is not significant, but that the interaction between length and material is significant (the average strength was slightly greater for the long specimens than for the short ones of refined copper, but the difference was somewhat larger and in the opposite direction for the other two materials). Specimen length does have a significant effect on elongation and a very significant one on the ratio of elongation to initial length. The coefficient of variation of ultimate strength was the same for the two lengths of refined copper specimens, considerably larger for short specimens of 70/30 α brass, and somewhat larger for long specimens of 0.15C mild steel. The author advocates the use of short specimens (common in Poland) rather than long ones (common in the United States), largely on the basis of their smaller coefficient of variation for the other variables measured (elongation, elongation/initial length, and especially position of fracture along the gage length).

Schneeweiss (1964) assumes that the dependence of compressive strength on specimen volume found earlier for wood applies in analogous form to concrete. He also examines the surface effect. He plots the compressive strength against the volume (both on logarithmic scales) for a number of test series in the literature, and finds a relatively uniform volume effect. He also considers the relation of size effect to the type of concrete and the degree of hardening.

Schuerch (1964) writes (Introduction, p. 569): "The use of thin fibers in composite materials is of considerable interest for a variety of applications. Examples for the potential value of filamentary composites are the remarkable mechanical properties of filament-wound structures made from endless, thin glass fibers bonded with various organic resins. These properties result, in part, from the size effect (increase in strength observed in thin glass fibers as compared to the bulk strength of glass), combined with an effective crack barrier function of the bonding material.···If the size effect in those materials can be controlled and exploited, thin fibers made of refractory 'glasses' [berillia, borides, etc.] may be expected to provide significant advances in the state of the art of structural materials.···A study of the size effects in glass fibers upon elastic (structure-dependent) properties appears worthwhile from two points of view: it may lead to a better understanding of the

mechanisms that manifest themselves in the observed size effects on strength and temperature resistance in thin fibers, and the results of such a study may lead to useful guidelines in orienting the search for improved filamentary materials." After reviewing the literature, starting with the work of Griffith (1920), the author uses data resulting from tests by Reinkober (1932) [see Additional References, p. 409] to demonstrate validity of the theory that the increasing strength of thinner fibers is due to a "skin effect". He states the following conclusions (p. 571): "The foregoing analysis of a limited set of experimental data tends to substantiate a hypothesis that the remarkable physical properties of thin glass fibers are associated with a skin layer of modified structure, possibly exhibiting properties similar to those found in crystalline modifications of the same material. Such a mechanism, if found effective in refractory materials capable of forming glass compounds, would enhance the utility of glass-fiber-forming processes in the refractory materials field."

Shimizu, Matino and Matumoto (1964) describe a plane bending fatigue machine, using a moving-coil-type shaker, which they developed for small-size sheet and wire specimens, and its use in investigating the size effect on fatigue strengths of a nickel silver, a phosphor bronze, and an 18-8 stainless sheet, cold-drawn or rolled. Specimen sizes varied from 0.40 to 1.00-mm diameter for wire specimens and from 0.10 to 0.55-mm thickness for sheet specimens. The authors report that an appreciable decrease in fatigue strength with increasing diameter or thickness was observed on 18-8 stainless steel, but that, in the case of nickel silver, the size effect on the fatigue strength was found to be small.

Stefanski and Marcinkowski (1964) note that construction laboratories often receive elements which were not made properly, elements made of concrete of reduced strength. Samples of regular shapes are cut out of these elements and subsequently subjected to tests. However, the results obtained on the basis of destructive tests of such samples do not determine the conclusive strength. In order to determine it, it is necessary to apply a correction coefficient. The authors describe tests made on formed and on cut cubic specimens with edges 10, 8, 6, 4 and 2 cm. They report that their studies did not confirm the assumption that the direct destructive strength increases as the samples are reduced in size. They point out, however, that in the low-strength concretes under study, the dispersion of strength is high, and increases as the

strength of the concrete decreases, which leads to a considerable dispersion of the values of the correction coefficient.

Weil and Daniel (1964) write (p. 269): "Two basic criteria of failure, size and normal tensile stress, are used in the Weibull theory. For a uniaxial stress field in a homogeneous isotropic material, governed by volumetric flaw distribution, the probability of fracture at a given stress σ is given by $S = 1 - \exp[-\int_V \{(\sigma - \sigma_u)/\sigma_0\}^m dV] = 1 - e^{-B}$, $\sigma \geq \sigma_u$; $S = 0$, $\sigma < \sigma_u$, where $B = \int_V [(\sigma - \sigma_u)^m/\sigma_0]dV$ is the risk of rupture and σ_u = zero probability strength (location parameter), m = flaw density exponent (shape parameter), σ_0 = scale parameter. The last three parameters are associated with the material and are independent of size." They derive the theoretical risk of rupture for material governed by volumetric flaw distribution and for material governed by surface flaw distribution, for various values of the loading factor k. For pure bending, $k = \infty$; for fourth-point loading, $k = 4$; for third-point loading, $k = 3$; and for center-point loading, $k = 2$. On p. 273, the authors write: "···for materials whose fracture is governed by a volumetric flaw distribution, the only specimen dimension entering the expression for the risk of rupture is the total volume V. For a fixed value of the parameter k, variations in length, width or depth of specimen which leave its volume unaffected will not affect the risk of rupture; the risk of rupture therefore is independent of (transversal) stress gradient. The dependence of the risk of rupture on specimen dimensions is more complicated for a material governed by a surface flaw distribution. For a given material, σ_0, σ_u and m are constants. If one is interested solely in the effect of depth [h] and width [b] on the risk of rupture, h and b should be regarded as being the only variables, whereas the length L and the extreme fiber stress σ_b should be kept constant. Then, the risk of rupture for a pure bending specimen···remains constant provided b and L [sic; h(?)] satisfy the relation $[h/(m+1)](1 - \sigma_u/\sigma_b) + b$ = constant."

Endo and Uede (1965) report the results of studies on the size effect of bending and twisting fatigue strength, which they summarize as follows (p. 314): "The relation between the twisting fatigue strength and the bending fatigue strength varies with temperatures and specimen sizes, as well as with presence of notches, flaws, and environments. In the present paper, the size effect of fatigue strength is discussed by calculating the probability of

flaws on the surface which affect the fatigue strength of materials by $1/(1 + 2\mu)$ under bending and by $1/(1 + \mu)$ under twisting [where μ = stress concentration]. And fatigue tests are carried out at elevated temperatures and at a low temperature for smooth specimens and also for notched specimens under bending and twisting moments. These results and the various past experimental results are applied to the calculation of the size effect discussed above, and the following explanation is given in a unified form: the relation of size effect under bending fatigue and under twisting fatigue, the size effect of smooth specimens and notched specimens, the effect of flaws, environments, and temperatures on the fatigue strength and notch factor, and the distribution of the fatigue strengths for various sizes of specimens."

Hughes and Bahramian (1965) note that it is known that the crushing strength of cube specimens does not represent the "true" uniaxial strength of concrete, but that the crushing strength of prisms or cylinders tends toward a constant value as the ratio of height to width of specimens is increased. These facts have led to a tendency to favor adoption of the cylinder as the standard specimen, even though the cube is more convenient. The authors propose a simple modification of the technique for the cube test which enables the uniaxial strength to be determined very readily, and recommend the continued use of the cube as the standard form of test specimen.

Izakson (1965) notes that three hypotheses have been advanced [see Davidenkov (1960)] to explain the scale effect in fatigue fracture: the statistical hypothesis, the energy hypothesis, and the technological hypothesis; also that some workers believe that the change of mechanical properties with changing dimensions of the specimen is due to the deviation from similarity in testing specimens of various sizes. He gives similarity criteria for the process of metal fatigue to show that the scale effect in fatigue is the result of the distortion of similarity, as is supported by the qualitative and quantitative conclusions of the theory. He compares the results of a calculation based on this thesis with the experimental data. He also claims to have shown that the "technological hypothesis" of the scale effect is a special case of the distortion of similarity during fatigue tests. He states, however, that the hypothesis of the distortion of similarity does not contradict the statistical and energy hypotheses of the scale effect, since these hypotheses show the physical significance and the internal mechanism of the

scale effect while similarity theory studies the external relations of the phenomenon.

Johnson, Mahtab and Williams (1965) summarize the results of experiments concerning geometric similarity in indentation as follows (p. 389): "Experimental results using cones and wedges to indent identical and similar metals showed that hardness can decrease with depth of indentation; the experimental circumstances were such that the principle of geometric similarity could apply. Continuous-loading and incremental-type-loading rigs were used and because they gave similar results, strain-rate effects were discounted as a cause of this phenomenon. It is attributed to a 'size-effect'. The authors believe their work may also have significance for experimental studies of machining and other processes." The size effect discussed in this paper is that on hardness, not on strength, but we have already seen [Auerbach (1891) and Tabor (1951)] that the two effects are closely related.

Karapetyan (1965) describes and gives the results of tests on the compressive strength of concrete prisms, 10 × 10 cm. in cross-section, of two different heights (40 cm. and 60 cm.), manufactured in vertical and horizontal positions. The increase in height of the prism from 40 to 60 cm. led to a decrease in the prism strength when testing the prisms perpendicular to the layers (prisms manufactured in a vertical position) by 15 percent, and when testing parallel to the layers (prisms manufactured in a horizontal position) by only 5 percent. The author attributes the difference to differences in the amount of water retained, but the reviewer (B. Kuzmanovic) believes that they account for only part of it, the remainder being due to Caquot's effect.

Kasai (1965) reports the results of a series of experiments to determine the effect of the size of the test cylinder on the compressive strength of concrete. It was found that if the size of the gravel used in making the concrete is 30 mm. or less and the specimens are cured in water, the average strength and the standard deviation of the strength are approximately the same for cylinders 10 cm. in diameter and 20 cm. long as for cylinders 15 cm. in diameter and 30 cm. long, but that both are significantly greater for cylinders 5 cm. in diameter and 10 cm. long.

Kermes and Bůžek (1965) state the conclusions reached as a result of a study of the size effect on the limit states of carbon steels under static and cyclic stressing as follows (pp. 352-353): "The results showed a considerable

influence of the size effect on the limit states of structural steels under static and cyclic stressing. The dependence of the limit tensions on the absolute dimensions of the test specimens may analytically be expressed as follows: for static stressing, $\lg \sigma_s = C_1 - C_2 \lg F$; for cyclic stressing, $\lg \sigma_c = C_3 - C_4 \lg F$. [The total fracture area F is defined as the size of the area at which failure results in the linear part of the usual working diagram.] The change of the notch coefficient in dependence on the absolute dimensions of the test rod is simultaneously evaluated. This paper affords no solution to the problem raised, yet, it is hoped that it may contribute to the verification of the effect of the studied factor on the limit states of the most widely used steels and add to the data for the calculation of parts of large dimensions and for further study of this problem."

Kogaev (1965) considers a statistical method for estimating the influence of stress concentration and scale factor on the parameters of the distribution of maximum ultimate tensile stresses. He concludes that available test data for steel, cast iron, and aluminum alloy show that it is possible to use the statistical strength theory of the weakest link to construct generalized equations $\sigma_{max} = f(P, d/\overline{G})$, giving the value P of the cumulative distribution function of the peak ultimate tensile stress σ_{max} for a given metal depending on the parameter $d/\overline{G}$ determined by the size and form of the part, where d is the diameter of the specimen in the critical section, $\overline{G} = 2/\rho + 2/d$ is the relative stress gradient at the base of the notch and ρ is the radius of curvature of the notch. These estimates give a good quantitative estimate of the influence of the stress concentration and the scale factor on the fatigue resistance of machine parts, including a dispersion factor, and can be used in strength estimates based on probabilistic considerations. The problem of choosing the proper distribution of peak ultimate tensile stresses requires further experimentation, but for the present the normal, lognormal, and Weibull distributions can be used in strength estimates.

Kozak (1965) gives theoretical values of the fatigue limits (at 2×10^6 cycles) of yield strength (200, 190, 180 and 170 kg/mm^2) and of breaking strength (250, 240, 230 and 220 kg/mm^2) for steel wires respectively 1.2, 1.5, 2.0 and 2.5 mm in diameter used in prestressed concrete. He does not indicate how these values were obtained, and his experimental values differ considerably from them, being 185, 233 and 186 kg/mm^2 and 205, 248 and 214 kg/mm^2 for

diameters of 1.2, 1.5 and 2.5 mm, respectively. No tests were performed on wires 2.0 mm in diameter.

Miyauchi, Yoshida and Fukui (1965) report the results of an investigation of the effects of size and shape of tensile sheet specimens on various mechanical properties for annealed aluminum, copper, 70/30 brass, 18-8 stainless steel, and mild steel. In particular, the effect of width and parallel gage length of specimens on mechanical properties, such as the strain hardening exponent, the ultimate uniform elongation, several measures of the local elongation and several strength characteristics, was investigated.

Nicodemi (1965) reports the results of an experimental study of the influence of specimen dimensions on the impact strength of three kinds of steel. For specimens 55 mm long and 10 mm wide, the impact strength showed a steady decrease as the thickness was increased from 2.5 mm to 10 mm in steps of 2.5 mm.

Ovsyannikov (1965) gives curves which show the variation of impact strength of various steels at different temperatures as a function of the notch bottom radius. His results make possible the comparison of the sensitivity of different metals to stress concentration and temperature, and hence a more effective evaluation of the tendency of metals to brittle fracture.

Pogoretskii (1965) reports the results of a study of the effect of test piece length on the fatigue strength of steel in air, which he summarizes as follows (p. 65 of translation): "1. It is established that the scale factor for short fatigue test pieces is a function of their diameter, length, and shoulder radius, the latter parameter having no effect on the fatigue properties of long test pieces. 2. Our results, demonstrating the dependence of the endurance limit (the principal fatigue strength characteristic) on the length and, consequently, the volume of the test piece, lend support to the statistical theory of the scale factor, the validity of which was doubtful before the existence of such a relationship was established. 3. The results of the present investigation show that the resistance of materials to cyclic stresses should be determined on test pieces of two distinct types: either on test pieces of 'zero' length [of which the gage portion constitutes a torus formed by a semicircle of radius r (the shoulder radius)] ··· which are least affected by the scale factor···, or on specimens long enough to exclude the effect of the shoulder radius."

Pogoretskii and Karpenko (1965a) report the results of a study of the effect of the size factor on the fatigue of steel in a corrosive medium, which they summarize as follows (p. 339 of translation): "1. The magnitude and direction of the effect of the size factor upon corrosion fatigue strength is determined by the relationship between a number of other factors, which may either increase or reduce the strength of the specimen surface layers. 2. The presence of a corrosive medium does not affect the nature of the relationship between the specimen length and the fatigue limit. 3. In the absence of technological effects (residual strains and stresses), it is advisable to carry out corrosion-fatigue tests on specimens with $\ell/d > 4$; this tends to reduce to a minimum the effect of the size factor."

Pogoretskii and Karpenko (1965b) report the results of a study of the effect of specimen length on the cyclic strength of steel, which they summarize as follows (p. 1874 of translation): "1. The cyclic bending strength of annealed medium-carbon steel is affected not only by the diameter of the specimen, but also by their [sic] length and the radius of transition from the working part to the end. In the case of long specimens the effect of this radius is eliminated. 2. In the region of relatively small specimen diameters (to 15 mm) the fatigue life is predominantly affected by the diameter. 3. In the region of large diameters the fatigue strength is affected mainly by the statistical factor. In this region the reduction of the fatigue strength is mainly due to the effect of irregularities in the metal whose number increases with increasing d and ℓ. 4. The fatigue testing by the application of pure bending with rotation is best performed on specimens of two shapes: with a zero length••• where the size effect can be a minimum, or on specimens with a length which eliminates the effect of the transition radii."

Robinson (1965) gives a summary of the Weibull theory, which is an attempt to account for the variability of measured strength in "brittle" material. He presents some new modifications, demonstrates the application of the theory to data analysis, and proposes a few design rules. In particular, he applies the Weibull distribution to the size effect on material strength. According to the Weibull theory, the probability S that a given specimen will survive an applied tensile stress σ of unspecified shape is given by $S = \exp[-\int (\sigma/\sigma_0)^m dV]$, where m and σ_0 are material constants (called the shape parameter and the scale

parameter, respectively) and V is the volume under tension. The strength in uniform tension of specimens differing in size from the test specimen may be predicted if m and σ_0 remain unchanged (normally a valid assumption, since they are constants of the material). In that case, the survival probability at the mean value is the same, so we get $\exp[-(\sigma_1/\sigma_0)^m V_1] = \exp[-(\sigma_2/\sigma_0)^m V_2]$, and therefore $\sigma_1/\sigma_2 = (V_2/V_1)^{1/m}$. The larger specimen is weaker and this size effect increases as m decreases.

Shashin (1965) gives a statistical analysis of bending and torsion fatigue tests on hardened and unhardened steels of diameters from 5 to 70 mm. The probable (median) value of the fatigue limit is plotted for spring steel under torsion, along with scattering zones and the coefficient of variation, as a function of the specimen diameter. The probable value decreases, but the coefficient of variation increases, as the diameter increases, rapidly for diameters up to about 20 mm, then more slowly. The slope index, plotted separately as a function of the specimen diameter, behaves similarly. Estimates of service life and cumulative damage are also given.

Stepanov (1965) writes (p. 1729 of translation): "There is a school of thought that the impact strength a_n increases with the increasing [sic; decreasing (?)] dimensions of the specimen if all other conditions remain the same. This is explained by the statistical theory of strength or by the energy theory; sometimes both theories are used, although the latter is normally preferred [Razov et al (1959)]. However, data has been published which shows a deviation from this apparently general behavior. For example, the data of Seita Sakui show that the impact strength increases when the thickness of the specimen is reduced to 6 mm, whereupon it begins to decrease. An examination of the experimental points obtained in our [earlier] tests shows that, at 0°C and above, a reduction of the specimen's thickness from 3.3 to 2 mm has no effect on a_n; however, at -40°C this reduction is clearly observed. It is interesting to note the decreasing effect of the specimen thickness in the range of brittle-tough fracture with increasing temperature; this corresponds to the general increase of plasticity of the metal. Two logical questions arise: Is it possible that at a certain temperature this effect is absent, and is an increase of impact strength with increasing specimen thickness possible? Even if so, this could be expected, apparently, only in the range of tough fracture." The

author describes new experiments on three steels and an aluminum alloy, whose results he summarizes as follows (p. 1731 of translation): "Our tests show that in the tough range an increase of specimen thickness leaves the a_n value unchanged or increases it; with increasing temperature the difference in impact strengths of specimens of various thicknesses decrease."

Swedlow (1965) reports the results of an extensive theoretical study of the thickness effect and plastic flow in cracked plates. In the introduction (Chapter I) he writes (p. 2): "Experimental work has•••shown that the thickness of the plate can exert a significant influence on the fracture stress. In certain alloys, for example, the fracture point in thin plates is over twice that in thicker plates. A few experiments demonstrating the thickness effect have been reported, but at present the results are not regarded as definitive nor is the effect itself fully understood." On the next few pages, the author summarizes typical experimental results and discusses stress and strain states and their relation to fracture. On page 7, he writes: "From this brief review of the thickness effect, certain features are evident: i) free surfaces normal to the plane of the crack have a strong influence on the local stress and strain fields; ii) complete description of these fields must ultimately involve both the elastic and plastic behavior of the material; iii) the stress-strain behavior may not be the same at all points of the plate; and iv) the fracture process itself can be a function of thickness." In Chapters II-IV the author gives a theoretical treatment of elastic considerations, inelastic behavior, and a plane stress case of elasto-plasticity, respectively. In his concluding remarks (Chapter V), he writes (p. 54): "In reviewing the physical situation associated with the extension of a crack in a metallic plate, it was observed that the thickness of the plate can exert a significant influence on the stress required for fracture. Certain inferences have been made as to the phenomena leading to fracture, and these have provided a basis for generating an hypothesis for the dominant features of the thickness effect. Qualitatively this hypothesis may be broken into three parts. The first involves the role of linear elasticity, the second deals with the elasto-plastic deformations in the plate prior to fracture, and the third is concerned with the interaction between such deformations and the fracture process."

Tkachenko and Pogoretskii (1965) point out that it is known that the results of fatigue tests on steel in corrosive media are markedly influenced by the size

factor, and state that the obvious explanation of this effect is that the similarity conditions for the action of the ambient medium upon the strength of geometrically similar specimens of a given material are not fulfilled. Using dimensional analysis and its principal tool, the π-theorem, they determined the similarity criteria for the case of the corrosive action of an ambient medium upon geometrically similar specimens of a given material subject to cyclic loads. They checked the validity of these conditions by experiment, determining the fatigue limit of steel 40 Kh specimens 5 and 20 mm in diameter under the following conditions: 1) in air; 2) in a 3% NaCl solution without meeting the requirements of the similarity criteria; 3) in a NaCl solution with the conditions adjusted to meet the requirements of the similarity criteria. As compared with the fatigue limit for the 5 mm specimens, that of the 20 mm specimens was slightly less for 1), much greater for 2), and practically the same (very slightly greater) for 3). The authors interpret their results to mean that in some cases field trials on large machines operating, for instance, in sea water, could be replaced by laboratory tests on much smaller specimens, provided that the parameters determining the action of the corrosive medium are accordingly adjusted.

Vagapov (1965) writes (p. 78, translation): "For bodies of dissimilar geometry, loaded in different ways, relationships between the probabilities of damage during initial fatigue macrofracture are established. For a border-line case, when macrofracture occurs on the surface, formulas are derived for the fatigue strength distribution function of the body depending on its shape, dimensions, and method of loading. For brevity and convenience in analyzing sample distributions, transformation coefficients, the characteristic relationship between damage probabilities for bodies of dissimilar geometry and differently loaded, are introduced. This has been demonstrated when these coefficients depend on the level of stress amplitude at the critical point of the body. The theoretical results are compared with experimental data on steel samples of various shapes and dimensions. From this comparison conclusions on the limits of adaptability of the theory are drawn. Theoretical data have been obtained on the basis of statistical presentations on the brittle fracture of macroscopically heterogeneous bodies formulated in 1933 by Aleksandrov and Zhurkov (the distribution of the non-uniformities by volume of a fragilely destructive medium is uniform, damage of the individual elements of the body is independent,

the moment of destruction of the weakest element of the body and its fracture coincide). These presentations were used by Weibull [(1939a)] and then Kontorova and Frenkel [(1941)] for predicting the scale effect by the mean strength value. These results are generally formally transformed to strength form. In the article special attention has been paid to suitable limitations."

Bartenev and Sidorov (1966) propose a statistical theory for strength of glass fibers which takes account of different types of surface defects and their distribution along the length of the fiber. They assume that the probability of not encountering a defect in the length ℓ of a glass fiber is given by the function $\omega(\ell) = \exp[-(\lambda\ell)^n]$, in which λ and n are constants. The exponent n ($n \geq 1$) takes into account the character of the distribution of the defects along the length of the glass fiber. For an exponential distribution of the defects along the length, n = 1; for any other distribution, n > 1; for defects occurring at a fixed interval $1/\lambda$ along the length, $n = \infty$. Defects of two kinds are encountered in glass fibers--microcracks and microbreaks of the surface layer--therefore, they exhibit three standard strengths. A fiber has strength σ_A if neither kind of defect is present, σ_B if a defect of the first type (surface microcrack) is encountered in it but not one of the second type (microbreak of the surface layer), and σ_C if a defect of the second type is encountered. The authors show that the mean strength of a fiber of length ℓ is given by $\bar{\sigma}_\ell = (\sigma_A - \sigma_B)\exp[-(\lambda_1\ell)^{n_1} - (\lambda_2\ell)^{n_2}] + (\sigma_B - \sigma_C)\exp[-(\lambda_2\ell)^{n_2}] + \sigma_C$, which can easily be generalized to the case of m different strengths (m - 1 kinds of defects). Even defects of one kind may not be identical, for instance, microcracks of different depths; consequently, the strength of glass fibers exhibiting one kind of defect is not a constant, but a variable distributed around some value typical of the given defect. This can be taken into account if instead of the strength of each individual fiber we take the mean value of the strength characteristic of defects of the given type. A comparison with experimental data shows that the theory gives a good description of the strength distribution curves and the relation between the mean strength of fibers and their length. The authors conclude that the distribution of defects along the length of fibers obtained by the continuous draw-plate method is not purely random, probably because of the nature of the manufacturing process.

Endo, Yano and Okuda (1966) note in their introduction that fiberglass reinforced plastics (FRP) show different characteristics of mechanical strengths

depending upon the kinds of resin and fiberglass and the methods of treating and processing them. In their conclusion (p. 108), they write: "In the present paper, the fatigue strength of FRP is discussed considering that the failure occurs by the stress concentration of the flaws at the interface of the glass and resin though the load is supported by both materials. The effects of the content of glass, the fiber-glass orientation and the size of the test specimen on fatigue strength are deduced from the discussion. The plane bending fatigue tests are carried out with roving FRP and satin woven cloth FRP of various contents of glass at various temperatures. The test results are compared with the above criteria and good coincidences are found." For roving FRP, the ratio α of the fiberglasses that support the load to all fiberglasses passing through the minimum cross section (and hence also the strength) depends on the size of the test specimen. For satin woven cloth FRP, the strength is affected by the size of the test specimen except when θ, the angle between the warp orientation and the load direction, is either 0° or 90°.

Forman (1966) reports the results of tests to investigate the dimensional similitude requirements for testing plane stress fracture toughness, which were conducted on three kinds of aluminum sheet (.063 gage 7075-T6, .060 gage 2024-T3 and .060 gage 2024-T81), .020 gage AM 350 CRT steel sheet and .020 gage AM 355 CRT steel coil. The purpose of the test program was to determine the change in fracture toughness due to 1) stress level and crack length, 2) panel length to half crack length ratio (L/a) and 3) crack length to panel width ratio (2a/2b). In his summary, the author states that the test program has shown that the plane stress fracture toughness of centrally notched Griffith panels will remain relatively constant with changes in panel width and panel length if the crack length remains constant, but is considerably affected by the stress level and the crack length. More specifically, the results indicate that for a given crack length, the fracture toughness will remain constant for $L/a > 3$, but is moderately affected by variation in panel width.

Grover (1966), in a section on statistical approaches to the nature of fatigue (pp. 21-22), writes: "Afanasev [(1940)] advanced one of the early 'statistical theories' of fatigue failure. Freudenthal [and Gumbel (1956)--see Bibliography] and many others have made notable contributions.•••The statistical approach•••suggests, as has been pointed out by Freudenthal,

implications concerning 'size effect' and 'cumulative damage'. Nevertheless, the statistical approach cannot provide a wholly satisfactory theory of fatigue. •••Some of the 'conclusions' from the statistical approach are•••less informative than they appear at first. For example, probability considerations of any strength property imply a lower macroscopic strength for larger specimens, but they do not directly imply the magnitude of this 'size effect' or the factors which govern the magnitude. Fatigue strength might be low at a thickness of a few hundred atomic distances, with no further decrease of engineering significance anticipated as thickness increases over such practical ranges as ¼-inch to 1 inch. This information•••is not likely to come, in any unique sense, from the statistical approach." Again, in a section on material factors in the fatigue of components, Grover writes (p. 156): "There have•••been observations of an apparent size effect in fatigue. Large monolithic structures show significantly lower fatigue strength (especially in bending) than do small test coupons. Some of this may be explainable statistically. Some results may reflect the influence of differences in surface finish or in residual stress. In any event, this means an added precaution in the use of results of tests on small specimens to predict behavior of large components."

Jancelewicz (1966) writes (English summary, p. 144): "The investigation described concerns the fatigue life properties of 1 mm and 2 mm PA-7T specimens under constant amplitude loads. An adequately calibrated ϕ 3 mm central hole was provided to act as a stress concentrator. Higher increase of the fatigue life was observed for narrow specimens and low stress levels. For wider specimens and higher stress levels this increase was distinctly lower. In the range investigated the increase of the fatigue life in function of plastic deformation of the diameter of the hole was found to be linear, or quasi-linear."

Jaworski and Rudowski (1966) describe the results of experimental studies of the effect of a transverse hole on the fatigue limit of a cylindrical element subjected to rotating bending. They define the concept of effective stress concentration factor $\beta_{k,z}$, and describe the method of experimentation. As a result of the studies, they give the relation between $\beta_{k,z}$ and the ratio δ of the diameter of the hole to that of the shaft. In the case of mild steel, they conclude that one can apply the universal factor $\beta_{k,z} = 1.2$ regardless of the value of δ.

Karpenko, Pogoretskii and Tkachenko (1966) report the results of a study of the effect of specimen length on the corrosion-fatigue resistance of steel in alternating tension. Tests were carried out on steel 40 Kh specimens, 4 mm diameter, 20 and 72 mm gage length, at two stress levels, corresponding to dynamic load amplitudes of 100 and 75 kg, with a 3% NaCl solution in tap water being used as the corrosive medium. Increasing the specimen length from 20 to 72 mm produced a reduction in endurance, the effect being more marked at the lower stress level. The authors state that this is evidently because with increasing test duration the probability of formation and growth of a fatigue crack leading eventually to fracture is larger in the case of a longer specimen, and that, consequently, the results obtained should be closely associated with the statistical nature of the size effect. The authors postulate that, since the weakening of the metal is more intensive for specimens of smaller diameter, the effect of the NaCl solution on the fatigue strength in the case of tests in alternating tension should also decrease with increasing specimen diameter, which should produce a reduction in the effect of specimen length on the test results. To check this postulate, they carried out a series of tests on 10 mm diameter specimens of the same steel 40 Kh with gage length of 90 and 150 mm. The fatigue curves obtained show that the fatigue and corrosion-fatigue strength of 10 mm diameter specimens is practically independent of the specimen length; the difference in the fatigue limits does not exceed 1 kg/mm^2, which corresponds to a difference of 1.7% for tests in air and 2.1% for tests in NaCl solution. The authors regard this as sufficient evidence to assert that, in the case of the steel studied, the statistical factor (heterogeneity of the metal) has no significant effect on the fatigue and corrosion-fatigue resistance of steel in alternating tension when the specimen diameter is as small as 10 mm. They point out, however, that this fact does not contradict the statistical theory, but proves its basic tenet according to which the influence of the heterogeneity of a material attenuates and vanishes when the body of the stressed specimen becomes saturated with a limiting number of harmful defects; when this stage is reached, the absolute dimensions of a specimen no longer affect its fatigue resistance.

Litle and Paparoni (1966) describe an experimental study of the influence of geometric scale on the ultimate strength of reinforced mortar beams failing in a flexural mode. Included in the test were 132 beams, with different

geometric scale ratios and two reinforcement ratios. The authors state the following conclusions (p. 1203): "1. A significant size effect was observed in the reduced scale model testing of under reinforced concrete and mortar beams. Experimentally determined ultimate moment capacities deviated from those predicted by commonly accepted theory by up to 43 percent. 2. These discrepancies must be caused by either one of or a combination of two factors, namely, (1) a relative increase in mortar compressive strength as size is decreased, perhaps due to different stress gradients and (2) the development of an increasing hyperstrength in the embedded reinforcement wires as their size is decreased. Additional studies are needed."

Mann (1966) gives a review of the literature and the then-current state of the art on the fatigue of metals. He mentions the size of specimens as one of the factors which influence fatigue behavior. In connection with the standardization of fatigue testing, he notes that one of the difficulties is that there are hundreds of different configurations of fatigue testing machines taking different sizes and shapes of specimen as well as differing in other important respects. He also discusses the effect of notches, and warns of the danger in accepting fatigue data on unnotched specimens as the criteria for the selection of a material or process under cyclic loading conditions.

Matting and Neitzel (1966) investigate the relationship between size of welding defects and effective strength reduction. They report results for ten specimens with four different welding defects and for eight unwelded specimens with geometric stress raisers. They plot data given by Phillips and Heywood (1951) for direct stress tests. One figure shows the alternating fatigue limit as a function of the diameter of the test section, constant for plain specimens, decreasing for transversely bored specimens with the diameter of the hole 1/6 that of the test section. Another shows the strength reduction factor K_f as a function of the diameter of the test section for 2½% NiCr steel and for mild steel, again with the diameter of the hole 1/6 that of the test section. In both cases, K_f increases rapidly for test section diameters up to about 1 inch, then more slowly; for 2½% NiCr steel, it approaches asymptotically the theoretical value $K_f = 2.8$, while the asymptote is somewhat lower ($K_f \doteq 2.25$) for mild steel.

Mowbray, Brothers and Yukawa (1966) report the results of single edge notch and notch bend fracture tests on three steels for fatigue cracked specimens

with ranges of thickness from 0.02 to 6 in. and temperature from -320 to 190°F. The tests were conducted to determine the capability of various size specimens for providing valid plane-strain fracture toughness (K_{Ic}) values at various temperatures. At very low temperatures all specimens gave similar K_{Ic} values. With increasing temperature, K_{Ic} values obtained from the larger specimens remained relatively constant and then increased rapidly. At higher temperatures, valid K_{Ic} values could not be measured directly from small specimens, but were estimated from relations between fracture toughness on the one hand and either shear-lip thickness or bend angle and crack-opening-displacement on the other.

Neville (1966) shows that the strengths of concrete test specimens (cylinders, cubes, and prisms) can be related to one another by simple expressions. He presents substantiating test results, and discusses the secondary influence of the fineness modulus of aggregate on this relation. He writes (Conclusions, p. 1107): "Although the level of strength of concrete and the fineness modulus of aggregate influence the relation between the strengths of concrete specimens of different shapes and sizes, an over-all relation valid for different strength levels, aggregate gradings, curing conditions, and ages at test can be derived. This empirical relation describes the strength ratio P/P_6 [where P is the strength of the specimen and P_6 is that of a 6-inch cube] in terms of volume of the specimen V, its height h, and maximum lateral dimension d. In the general form the independent variable is expressed as V/(hd) + h/d, and in the simplified form as V/(hd). Tests of these relations made on published data show that they make reasonable prediction of strength of specimens of the usual shape possible from the strength of other specimens. An increase in the fineness modulus of aggregate probably enhances the difference in strengths of specimens of different shapes."

Sigwaldason (1966) discusses the influence of testing machine characteristics on the cube and cylinder strength of concrete. He writes (Conclusions, p. 205): "Cube and cylinder strengths are influenced differently by the method of end loading. With carefully centred specimens, differences of 7% occur between loading cubes with both ends pinned and both ends fixed, while the corresponding difference in cylinder strengths is only 1%. Cylinder/cube strength ratios, as a result, are very dependent upon the exact method of end loading. The cylinder/cube strength ratio is also very dependent upon the degree of uniformity of the concrete, because of the different directions of

testing, in relation to the direction of casting. Whereas the results of cube specimens, which are tested at right-angles to the direction of casting, represent the average strength of the concrete, cylinder strengths indicate the strength of the weakest portion of the material. As a result, cylinder/cube strength ratios are considerably smaller with the naturally segregating concretes than with uniform concretes."

Walker (1966) summarizes a program, consisting of an analytical study and a supporting experimental study, whose objectives were to define, verify, and present in useful form a synthesis of strength-limiting parameters for fatigue-cracked panels which would be applicable to the wide range of conditions of interest in the engineering problem of strength analysis. The experimental program provided information on the behavior of fatigue cracks for bare 2024-T3 aluminum with crack lengths ranging from .5 to over 10 inches, panel widths 30, 20, 12 and 9 inches, and nominal panel thicknesses .080, .063 and .032 inch. Limited test data were also obtained for duplex annealed titanium 8Al-1Mo-1V with panel widths 12 and 9 inches and thicknesses .045 and .020 inch. Buckling restraints were used for approximately half of the panels tested. The author concludes that, in narrow panels, width has a significant influence on crack extension that cannot be adequately accounted for by current fracture mechanics theory, as well as an influence on the reduction in strength due to buckling. While he admits that the thickness of a panel can have a significant influence, he states that in this study differences in thickness have for the most part been ignored.

Weiss, Schaeffer and Fehling (1966) write (Abstract, p. 675): "The effect of section size on the room-temperature notch strength of H-11 steel, 2014-T6 and 7075-T6 aluminum alloys, and Plexiglas was investigated in three different series of notched cylindrical tensile specimens and notched rectangular bend specimens. The three test series permit the separation of the influences of the geometrical parameters, which determine the stress field near the base of the notch, on the notch strength as the size of the specimen increases. The notch strength decreases with increasing size for all series investigated. The decrease in strength is most pronounced for the sharp machine-notched and fatigue-cracked specimens but it is also observed, to a much smaller degree, in the other series of notched specimens for all materials. An analysis of the results of this study indicates that the loss in strength with increasing

section size is due to at least two factors: An increase in stress concentration factor and an increase in critically stressed volume with increasing size. With a superposition of stress concentration and volumetric effects, a loss in strength with increasing size greater than that predicted by existing fracture concepts for a brittle material may be anticipated and was actually observed for sharply notched H-11 steel specimens. Insight into the size effect and experimental scatter observed in notched specimens of real materials is gained from a consideration of the behavior of a model of an inhomogeneous material in the presence of external notches. The constant fracture stress concept applied to this model yields a size-effect prediction on notch strength and an expression for experimental scatter in terms of the interflaw spacing and the notch root radius."

Cooper and Kelly (1967) describe experiments designed to show how a material reinforced with aligned fibers fails at the root of a notch. To investigate the effect of fiber diameter on fracture toughness, measurements were made on composites of vacuum-cast copper reinforced with brittle tungsten wires of varying diameter. All composites had a constant-volume fraction of 53 per cent fibers. An important result is that the work of fracture varies linearly with the fiber diameter and in a copper matrix at room temperature is less than 10^7 ergs/cm^2 for a fiber diameter of 20μ.

Endersbee (1967) describes recent researches into fracture in concrete and in rocks. He outlines the general theory of the strength of brittle materials, based on the concept that brittle bodies contain a multitude of crack-like flaws, and uses it to describe the observed fracture behavior of concrete and rocks. He draws attention to the relation between size of structure and probable stress at fracture. He points out that the size effect was noted by Leonardo da Vinci, who observed that shorter wires are stronger than longer wires from the same stock, a statistical consequence of imperfections of material still not generally recognized over 460 years later. He also reviews the work of Griffith (1920) on the strength of brittle materials. He shows that an increase in scale by 100 times (as compared with a 6 in. × 12 in. test cylinder) reduces the strength of concrete by about one-half. He concludes that, in the case of brittle materials, the decrease in strength with increasing size of structure must be an important consideration in the design of large, highly-stressed structures, such as dams.

Ford, Radon and Turner (1967) present the results of a series of slow-bend and Charpy V-notch impact tests on medium-strength manganese molybdenum UXW steel over a range of temperatures to show the effect of specimen size, notch intensity, and strain rate on the fracture behavior of this material. Conclusions relative to the size effect include the following (p. 859): "1. The concepts of linear fracture mechanics can be applied to laboratory-size specimens of medium-strength steel under certain limited conditions. For the low strain rates normally encountered in slow-bend tests, the principal restriction is the temperature at which at- or below-yield fracture takes place. These tests indicate that temperatures much below those normally encountered in service are necessary before the concepts can be employed. 2. The experiments on full plate thickness specimens (1 in) confirmed that below-yield fractures could not be induced at normal service temperatures by fatigue-cracking and side-notching. Full plate thickness specimens, precracked and side-notched, exhibited plane strain below-yield fractures in slow bend tests at -85°C and lower, plane strain above yield fractures at temperatures from -85°C to -30°C, and plane stress fractures above -30°C. 3. The small specimens fractured at stress much above those of full plate thickness. Below-yield fractures on small specimens were not encountered at temperatures above -160°C, while the large full plate thickness specimens exhibited below-yield fractures at temperatures of -85°C and lower. The size of the specimen in slow bend tests is, thus, most important in promoting the below-yield nature of the fracture, but the stress intensity factor K_{IC} in slow-bend testing appears to be independent of size, provided fracture takes place at stress below gross yield."

Glucklich and Cohen (1967) write (Abstract, p. 278): "Attention is called to a new [see, however, Davidenkov (1960)] phenomenon: in certain materials, specimen size (or, its energy-storage capacity) influences its brittle-ductile transition and strength. This effect is not the recognized statistical one (which only concerns the nucleation of fracture), but derives from the strain-energy in the system and concerns the stability of slow-growing cracks after nucleation. Recent experimental observations, in which this effect was noted, are cited and a theory proposed to account for them. This theory is based on the fact that a highly unstable equilibrium exists between the respective rates of strain-energy release and energy demand. When the system is over-stocked with strain-energy, any sudden drop in energy demand creates an excess of

energy released. This then takes the form of kinetic energy capable of doing work against the remaining resistance, in turn, resulting in a lower fracture load and reduced ductility. Specimen size enters these considerations only insofar as it governs the amount of stored energy; consequently the effect is most pronounced in flexure. Although this effect was so far observed in materials with moderate ductility, it is speculated that fine techniques might reveal its presence [in] both 'ideally' brittle and 'ideally' ductile materials."

Juvinall (1967) discusses the effect of size in dynamic loading. He writes (p. 231): "With bending and torsional loads, endurance strength tends to decrease as size increases. Small wire usually exhibits a higher endurance strength than a standard 0.3-in.-diam specimen. As the diameter is increased above 0.3 in., the endurance strength quickly drops, usually by 5 to 10 percent, and then tends to remain constant up to at least 2 in. diameter. To deal with this effect, we introduce a size factor, or diameter factor C_D, defined as the ratio of the endurance limit [at 10^6 cycles] for samples of arbitrary size to the endurance limit of otherwise identical samples of 0.3-in. diameter." The author summarizes graphically the results of tests by various investigators for steels with ultimate strengths ranging from 50 to 165 ksi and diameters up to 2 in. He states that on the basis of such tests, it may be conservatively assumed that the size factor is 0.9 for parts larger than about 0.4 in. diameter and 1.0 for smaller parts. He attributes the size effect to stress gradients. This would indicate that there should be no size effect for axially loaded parts, which is confirmed by experimental results for axially loaded parts up to the 2-in.-diam range, and that the effect should disappear for parts loaded in bending or torsion if they are increased in size above 0.5 in. diameter, since the steep stress gradient is lost. According to this reasoning, we would expect size factors to remain unchanged for diameters above two inches, but test results appear to give evidence to the contrary, since many tests of parts in the 4- to 12-in.-diam range indicate apparent size factors of 0.75 or less for all types of loading. The author attributes this to metallurgical factors inherently related to specimen size, and not to a true size effect associated with stress gradient, but advises engineers to assume that in effect the size factor for large parts subjected to any loading type will be from 0.6 to 0.75 unless specific test results support the use of a higher value. He states that the effect of size on static strength and of

10^3-cycle strength is much less pronounced and is commonly neglected except for very large sizes.

Kadlaček and Špetla (1967) give an experimental verification of a relationship between the slenderness ratio of test specimens--cylinders and prisms (compacted both in vertical and in horizontal positions) and strength of concrete in direct tension. They report that strength decreases as the slenderness ratio increases up to about H/D = 2 for cylinders or H/a = 3 for prisms (H = height, D = diameter, a = side of base), then remains almost constant. There is no substantial difference in the relation between slenderness ratio and strength for concrete made with gravel aggregate and with crushed stone. From the general curve of growth of apparent strength, expressed in percentages of the initial value for lower slenderness ratios than the basic one, it is possible to find the actual strength of concrete in direct tension for cylinders or prisms with any desired slenderness ratio.

Kaufman (1967) writes (Abstract, p. 189): "Tests of 1 3/8-in.-thick 2024-T851 aluminum alloy plate were made with notched specimens of different types and sizes with different notch radii and associated stress-concentration factors. Notch-tensile strengths varied widely, dependent upon notch geometry (shape, size, notch depth and sharpness, and location of notch), even for those specimens having identical theoretical stress-concentration factors. The data demonstrate that it is not sound practice to associate required levels of toughness as measured by notch-strength ratio or notch-yield ratio with specific values of stress-concentration factor without also specifying the exact specimen and notch geometries. The primary usefulness of notch-tensile data is to establish relative indices of notch toughness, based upon tests of notched specimens of a single design."

Mann (1967) writes (p. 126): "Much of the information relating to the fatigue behaviour of materials has been obtained from investigations using small and relatively simple laboratory specimens.•••However, large specimens or full-size components exhibit lower values of fatigue strength than geometrically similar small specimens or small models. This phenomenon has been referred to as the <u>size effect</u>. It is particularly evident when a stress gradient occurs in the section, e.g., under bending or torsional loading, or in cases of non-uniform stress distribution, e.g., when a stress concentrator is present. The unnotched fatigue limit under axial loading appears to be

nearly independent of specimen size. It is difficult to assess correctly the results of size effect investigations because large specimens can never be regarded simply as scaled-up versions of small specimens, e.g., differences in stress gradient, tensile strength, metallurgical structure, grain size, depth of deformed surface layers, internal stresses, surface finish and condition are inevitable. The actual test conditions, also, which include frequency of loading, accuracy of load application and measurement and specimen temperatures are usually different for the various sizes considered. The variations in fatigue behaviour between large and small specimens very likely indicate to what extent these technological factors are influencing fatigue behaviour, and they probably completely mask the existence, or otherwise, of a true geometric size effect." The author shows on a graph the reduction in fatigue strength with increasing size of shaft (up to 2 in.-diameter), loaded in bending or torsion, and states that a correction factor derived from the graph should enable some preliminary allowance to be made for size effect in design calculations.

Milashauskas and Slavenas (1967) note that the lifetime in the case of nonsteady loading depends on a number of factors, of which the effect of the scale factor has been least studied. With increase of the diameter of the specimen from 4-6 to 40 mm., the sum of the relative life decreases by a factor of two or three. For sufficiently large statistical tests, sums of the relative life are obtained in the interval 0.5 - 2.0. It has been shown experimentally that the sum of the relative life for short specimens is considerably greater than for long specimens, which is true because of the larger number of flaws in the long specimens. [The compiler has been unable to obtain a copy of the original of this paper, so the above information is based on the review, which is a translation (apparently rather imprecise) of a review by L. M. Shkolnik in a Russian journal, Referativnyi Zhurnal, Mekhanika, USSR.]

Nagaraja (1967) discusses the use of structural concrete models in engineering. He writes (Summary, p. 511): "With the increasing magnitude of structural activity as we have today, the variety of sizes and shapes which result from the imaginative intellect of the architect and the designer often defy an easy and accurate mathematical treatment. The model study of such complex shapes takes into consideration all the influences--primary and secondary--and the behaviour of the structure, under load as a whole, is placed before the

designer for a safe, economical, and aesthetical proportioning of the structure." He points to size and shape effect and scale effect as possible sources of errors in models. He writes (p. 517): "The scale effects are unknown influences which are due to change in size of the components of the structure. One of the methods of estimating the scale effect is to conduct [tests] on a number of geometrically similar models constructed to different linear scales and then to compare the results."

Nakamura and Hatsuno (1967) note that various fatigue tests have been carried out on the relationship between working stress and fatigue intensity, but that most of the voluminous data for rotating bending tests is for small size test pieces, while data for rotating bending tests on large pieces is virtually nil. Practical application of the existing data obtained from small test pieces would be convenient, they say, but such simplicity seems unacceptable because of the size effect. They describe new equipment designed for rotating bending tests on large pieces and some of the tests performed on SS41 steel by use of this equipment. In particular, they study the relation between the stress concentration factor α and the stresses σ_{w_0}, σ_{w_1} and σ_{w_2}, where σ_{w_0} is the fatigue limit (at 10^7 cycles) of the unnotched test piece, σ_{w_1} is the maximum value of the stress range for which the bottom of the notch will remain uncracked regardless of the number of stress cycles, and σ_{w_2} is the maximum value of the stress range for which the test piece remains unbroken regardless of the number of stress cycles. They conclude that σ_{w_1} is roughly equal to σ_{w_0}/α, that the σ_{w_1} and σ_{w_2} curves diverge at about $\alpha = 3.5$, and that when $\alpha > 3.5$, σ_{w_2} is about 6 kg/mm^2.

Nishimatsu (1967) first notes that Fisher and Hollomon (1947) developed their statistical theory of fracture, application of which is limited to the case of the tensile stress state, on the basis of a statistical distribution of the major axis of Griffith cracks. Under the assumption that fracture is caused by the stress concentration around Griffith cracks, he then gives a statistical theory of brittle fracture applicable to an arbitrary stress state. He assumes that the sizes of the cracks obey a Weibull distribution. He examines the model for uniaxial compression and uniaxial tension, but it can be extended to loading under a confining pressure and an axial load. The

analysis is based on the number of cracks (size effect). When the number of flaws is increased from 10^3 to 10^6 (corresponding to a tenfold increase in each linear dimension), the strength is reduced by a factor of 4. The ratio of the compressive strength to the tensile strength is rather insensitive to size (8.2 for 10^3 cracks and 8.0 for 10^6 cracks, as compared with 8 for the original Griffith theory). The author discusses the values of the Weibull shape parameter for granite obtained by several investigators.

Oh and Finnie (1967) note that if one assumes that the tensile strength of glass is constant, the load P on a spherical indenter of radius R required to produce ring cracking should be proportional to R^2. They point out, however, that experiments show that the mean fracture load is proportional to R^2 only for very large indenters; for small indenters, $\overline{P}$ is more nearly proportional to R, as Auerbach (1891) and later investigators found. This result, often referred to as Auerbach's Law, shows that the maximum tensile strength corresponding to the mean fracture load is inversely proportional to the cube root of the indenter radius, a striking size effect. Weibull (1939a, b) gave a general explanation of the effect of specimen size on the strength of a brittle material. Weibull (1939a) indicated that Auerbach's Law might be a consequence of the statistical nature of brittle strength, but reported no experimental data in support of this observation. The authors use the Weibull theory to compute the mean and the standard deviation of fracture load, and an extension of that theory to compute the mean crack location. They report good agreement with experimental results, over a wide range of indenter sizes, of predictions of mean load, standard deviation of load and mean crack location, all as functions of indenter radius, from material properties determined in bending tests.

Regester, Gorusch and Girifalco (1967) report the results of a study of the mechanical properties of aluminum oxide whiskers in relationship to their size and geometry. Finding that tensile strength varies inversely with surface area, they hypothesize that fracture is initiated by surface defects.

Serensen (1967) reviews the Russian literature (156 publications) on the fatigue testing of materials. Concerning the size effect, he writes (pp. 1494-1495 of translation): "The conditions of similitude for fatigue failures and the simulation methods for endurance tests is being investigated on specimens of various sizes and with uniform and nonuniform stress conditions.···On the basis of statistical representations for the failure conditions the similitude

loading conditions of the surface layer are formulated for modeling purposes. Modeling involves the similitude of loading conditions at the surface and the comparison at a lower limit of dispersion of small-sized models and members with due regard for the dependence of dispersion on the stressed volumes; at the crack development stage, this is based on a loose relationship between the relative duration of the crack development and the absolute dimensions as well as the stress level.•••Analysis of similitude conditions in the crack development stage and during the occurrence of the final failure has led to an examination of the combined influence of inhomogeneity in the stress condition and of the absolute dimensions upon these conditions.•••The dispersion of the fatigue characteristics, which is closely associated with the localized character of the appearance and development of the failure, has necessitated the broad use of statistical methods both for the analysis of results and for planning fatigue tests. In order to describe the effect of stress concentrations and the scaling factor on fatigue strength, statistical theories for the strength of materials are employed [Afanas'ev (1953), Volkov (1960), Bolotin (1961)]. The probability treatment of fatigue failure conditions based on a distribution function for the random stress values on individual grains in a polycrystal has made it possible to characterize the conditions for the onset of crack formation as a function of non-uniformity in the stress distribution over the cross section and of the absolute dimension [Afanas'ev (1953)]. This treatment has explained the effects of inhomogeneity in the stress distribution and of the absolute dimensions on the average values of the fatigue strength. The subsequent development of the statistical description for fatigue failure similitude conditions has been related to the experimental evaluation of overall probability diagrams of fatigue for specimens having various dimensions and stress concentration levels and with the inclusion of statistical theory for the strength of 'weakest grain' in the quantitative description of the similitude rules for fatigue failure [Bolotin (1961), Kogaev (1965)]. The application of the theory mentioned has made it possible to advance a criterion of similitude for fatigue failure that is expressed quantitatively by the ratio of some linear dimension of the stressed area to the relative gradient of the first principal stress in the concentration area. This aspect has permitted the precalculated and experimental data on the fatigue strength and dispersion of endurance characteristics to be closely

correlated for specimens of different dimensions and stress concentration levels [Kogaev (1965), Stepanov (1965)]."

Tetelman and McEvily (1967) discuss the size effect on the fracture of structural materials under various conditions. On pages 16 and 17, they consider the effect of size and shape of a discontinuity and of the specimen itself on the elastic stress concentration factor around the discontinuity. On page 26, they deal with the size effect in the case of local yield around a notch; on page 90, with the relation of flaw size to specimen size; and on page 102, with the relation of the relative amount of radial fracture to the specimen size. On pages 325-329, they discuss the effect of variations in notch and specimen geometry, including that of specimen thickness and plate width. On pages 382-385, they consider the size effects in fracture under cyclic loading (fatigue). They write (p. 382): "Three types of size effect influence fatigue behavior. These are (a) a statistical size effect related to the probability of finding a critical flaw within the most highly stressed region, (b) a metallurgical size effect that is associated with a change in metallurgical structure and properties as a function of absolute size, and (c) a notch size effect related to the steepness of the stress-gradient at the root of a notch as a function of the notch radius." On page 415-417, they discuss the effect of specimen size on creep rupture behavior. On pages 590-594 they deal with some observations on glass strength. They point out that what Griffith thought to be a true size effect for glass fibers is really a surface effect, since glasses fail below their theoretical strength because of the presence of surface flaws introduced in processing or handling rather than a statistical distribution of flaws throughout the bulk.

Freudenthal (1968) writes (Summary, pp. 617-618): "The statistical approach to brittle fracture is concerned with two problems: the distribution function of brittle fracture strength of nominally identical specimens under nominally identical conditions, and the effect on brittle structure of specimen size, stress distribution, and state of stress. These problems are interrelated, and their solution requires the construction of plausible physical-statistical models of the fracture process. Three models can be formulated, each of which leads to a different distribution function: 1. The uniform defect model produces a gamma distribution. 2. The weakest link model produces the third asymptotic distribution of smallest values [Weibull distribution]. 3. The

classical bundle model produces a Gaussian (normal) distribution. While the analytical expressions of these three distributions are quite different, they are not always easily distinguishable when used for the representation of test results of a moderate number of replications. Thus, the relevance of the individual distribution function for extrapolation beyond the range of experiments usually cannot be deduced from considerations of statistical inference based on experimental results, but must be assessed, rather, on the basis of the assumed physical relevance of the fracture model underlying the particular distribution function. In most cases of fracture of truly brittle materials (ceramics, glass), the third asymptotic distribution of smallest values will, therefore, provide the best representation of the test results and the most reliable basis for extrapolation." Concerning the relation of the size effect to the statistical approach, he writes (p. 593): "The brittle fracture strength of a material under a homogeneous state of stress is related to its defect structure, that is, to the concentration and severity of defects in the specimen. Statistical considerations, therefore, lead to an effect of size on fracture strength: if either a certain number or a certain severity of flaws is required to cause fracture, the occurrence of fracture is dependent on the statistical expectation of encountering the critical condition(s) in the specimen. This expectation is directly related to size; the smaller the specimen, the less likely the critical condition. While the existence of a size effect in brittle fracture is thus an integral part of the physical phenomenon, its character changes with the various assumptions that can be made concerning the nature of the fracture process. Since the size effect reflects the operation of a statistical process, its theory emerges as part of the general statistical approach to brittle fracture." He shows that, under the weakest link concept, the strength is inversely proportional to the (1/m) power of the volume, where the Weibull shape parameter m is a constant of the material.

Glucklich and Cohen (1968) write (Abstract, p. 22): "A set of simple static experiments was conducted with gypsum to demonstrate a hitherto unreported size- and strain-energy effect on strength. The experiments were tensile, compressive, and flexural. A size effect was shown for flexure only, while for tension and compression a strain energy effect was shown by adding springs in series with the specimens. The spring reduced the strength in tension and compression by approximately 30 percent; the length effect in

flexure was approximately 18 percent for a halved span, and 30 percent for a quartered span. The spring effect in flexure was of the order of 12 percent. These effects were over and above the familiar statistical effect, since the fracture cross-section was predetermined by means of a notch. Besides the effect on strength, the strain energy and size show an appreciable effect on ductility and on the scatter of strength results. Ductility decreases and scatter increases with increasing strain-energy content. Size has a similar effect on ductility, while its effect on scatter is still not clear. The authors outline a possible theory, based on the principles of fracture mechanics, to account for the observations. Recommendations are made for future work in this new field."

Hasofer (1968) writes (Abstract, p. 439): "A statistical model [of the 'bundle of fibers' type introduced by Daniels (1945)] for the brittle fracture of steel is formulated and analyzed. Fracture is assumed to occur by coalescence of a number of arrested cracks initiated at different points. The upper tail of the distribution of maximum stress is shown to be given by the formula $1 - (x/\sigma_0)^{\theta}$, where σ_0 and θ are functions of the microstructural constants of the material. It is further shown that the size effect in this model is much less marked than in the case of weakest-link models. Finally, some experimental results from a pilot program are presented and suggestions for further experimental work given." The reviewer (R. G. Forman) states that he does not believe that the proposed fracture mechanism is in agreement with current fracture mechanics theory, particularly with respect to crack arrest behavior, so that the new statistical model needs basic experimental verification.

Hatano (1968) gives a linear relation between the equivalent strain and the first strain invariant in the fracture of brittle materials such as concrete. His fracture criteria can account for the time, temperature, and repeated load effects, and he extends them, by use of the Weibull distribution, so as to estimate the probability of failure in concrete structures. He estimates the effect of the strain distribution and the size effects as well as the effect on fracture of the probable distribution of defects in space. He shows, as other authors have done previously, that the compressive strength is inversely proportional to the volume raised to a power which is the reciprocal of the Weibull shape parameter, a material constant. He speculates that his fracture theory, formulated for concrete, can be extended for use in rock mechanics.

Johnston and Sidwell (1968) report that they found, by the analysis of the results from compression tests on 147 pairs of 4 in. square specimens of concrete, that the specimens with a height/width ratio of 3 were insignificantly weaker (by 0.28%) than the 2:1 specimens. Nevertheless, they recommend a height/width ratio of at least 2.5 for tests involving strain measurement on an adequate gauge length subjected to uniaxial compression.

Kadlaček and Špetla (1968) write (English summary, p. 426): "The Authors have proved by experiments a relationship existing between the size and shape of test specimens--cubes and prisms--and the strength of concrete in direct tension •••. Although two different kinds of concrete were tested (natural gravel concrete and crushed aggregate concrete), there is almost no difference •••between the growth of concrete strength in direct tension depending on the size (volume) of the testing specimens. The shape of the testing specimens--cylinders and prisms--referred to a common reference volume $V = 5.3\ dm^3$ [the volume of a cylinder 15 cm. in diameter and 30 cm. high] does not show any practical difference in percentual [sic] growth of concrete strength in direct tension •••. The general curve representing the relationship between the percentual growth of concrete strength and the decreasing volume of test specimens•••is used to determine the strength of specimens of fundamental size, provided the maximum grain size of the aggregate is smaller than 1/3 of the least dimension of the studied specimen and its slenderness ratio corresponds to $\lambda = H/D = 2$ for cylinders or $\lambda = H/a = 3$ for prisms, or is greater than these values. The effects of size and shape of specimens (expressed in terms of their volume) on the strength of concrete in direct tension may be neglected in the region of currently used size of test specimens, unless this strength is influenced by their slenderness ratio (i.e., if $\lambda = H/D < 2$ for cylinders and $\lambda = H/a < 3$ for prisms •••."

Kálna (1968) writes (Abstract, p. 481): "The size of a body, the shape and length of the notch affect the strength characteristics and the temperature ranges of the transition from plastic to brittle state, during static tension loading. The effect of the cross-section size in the range from 1 to 2400 sq. cm. on failure strength, on the yield point, and on the transition temperatures has been studied on bodies without a notch, with a circular, and with a sharp notch. The size effect is definitely displayed by bodies having a sharp notch, although this effect is not caused by the length of the notch, but by the size

of the cross-section. This regularity was verified for the brittle and plastic failure of steel bodies and for an aluminum alloy. The crack-arrest temperature depends on the stress and the thickness of the body. It will increase with an increased thickness of the body, till the limit value is reached. In the strength diagram, it is necessary to determine the CAT--nominal stress dependence of bodies which have an actual thickness, and are fabricated in accordance with an actual technology process and, eventually, with actual weld seams."

Kokotailo, Kuslitskii, Starovoitov and Karpenko (1968) point out that vacuum or slag remelting (or other refining method) not only improves the quality of steels and their physicomechanical properties, but also reduces the anisotropy of their mechanical properties. They reason that these refining methods can therefore be expected to affect the size effect in cyclic loading, and report the results of tests on vacuum-poured steels to verify this. They write (p. 115 of translation): "With increasing specimen size the fatigue strength decreases, but for a vacuum steel this decrease is smaller than for ordinary steel. Consequently, the size effect is smaller in the case of a vacuum steel; this is seen very clearly in testing transverse specimens. As the specimen diameter increases the difference between the fatigue strengths of the vacuum and ordinary steels increases (the difference is greater in the case of transverse specimens and smaller in the case of longitudinal specimens). This suggests that the fatigue tests should be carried out on specimens which imitate the actual structures in size as closely as possible; this is especially important when comparing various steels or investigating the effect of a refining method. The size effect is more apparent in testing specimens with stress concentrations •••. For example, a change in the working part diameter of the specimen from 5 to 25 mm reduces the fatigue strength of smooth transverse specimens of ordinary steel by only 5% whereas in the case of a notch the reduction is about 30%. Here, too, the vacuum steel was less sensitive to the size effect although to a lesser degree than was observed in testing smooth specimens. It should also be noted that the testing of smooth specimens with a diameter of 5-25 mm showed a tendency to reduce the size effect; for notched specimens this tendency was much smaller than for smooth specimens."

Markov, Mikhailov and Rebinder (1968) discuss the effect of loading sequence and scale factor on the strength of brittle bodies (specifically concretes) during complex loading by compression and tension. They point out that during

initial loading of the material with tension stresses, the stresses do not depend on the static friction of the specimen ends, but when the initial loading is in compression, the static friction of the specimen hinders the uniform distribution of the tension stresses along a section of the specimen parallel to the compression stresses. Therefore allowance must be made for the scale factor, since the static friction depends on the size of the specimens, primarily on the height-width ratio h/d. The authors cite experimental data which show that the strength of concrete in axial compression is approximately 1.5 times as great for specimens with $h/d = 1$ as for those with $h/d = 7.5$, with little or no further decrease for $h/d > 7.5$. The corresponding values of the strength in tension are only about 1.2 times as great for $h/d = 1$ as for $h/d = 7.5$.

Nisitani (1968) writes (Introduction, p. 947): "The size effects in rotary bending fatigue tests of metallurgically uniform materials may be considered to be controlled mainly by the following two factors: (1) The stress gradient; (2) The extent of area exposed to the risk of fracture. The factor (1) is concerned with the mechanical cause, and the factor (2) is concerned with the statistical cause. In some studies, the above two factors are treated without any distinction as the one which concerns the volume exposed to the risk of fracture. According to the author's opinion, however, they must be distinguished. In this paper, paying attention to the factor (1), the effects of size on the fatigue limit and on the branch point (i.e., the point at which the lines showing the fatigue limit based on crack initiation and the crack strength intersect) are investigated." He states the following conclusions (p. 957): "(1) In the case of rotary bending tests, the root radius of a notch ρ_0 at the branch point is a constant to be determined by the material, and has no relation to the diameter d, of the minimum section and to the notch depth t, or to the existence of work hardened layers. This is due to the fact that the stress distribution near the root of a notch is almost completely determined by the root radius of the notch ρ alone, except for the special case, irrespective of d or t. (2) The cause of size effect of the fatigue limit σ_{w1} is different from that of the crack strength σ_{w2}. a) The size effect of σ_{w1} (the fatigue limit based on crack initiation) appears from the cause that the maximum stress at the root of a notch at the crack initiation limit becomes constant, irrespective of d, t or α [the stress concentration factor]

if the stress gradient is constant. b) The size effect of σ_{w2} (the fracture limit within the range of the non-propagating crack existing) appears from the causes that the root radius of a notch becomes constant at the branch point and the crack strength has no relations with ρ."

Paul (1968) writes (Introduction, p. 315): "The main purpose of this chapter is to define those states of stress which produce yielding of ductile materials and fracture of brittle materials. It is not immediately obvious that the state of stress, by itself, determines the onset of yielding or fracture. It would seem reasonable to assume that other factors would bear on the problem, e.g., the effects of time, temperature, stress gradient, microstructural features, size effects, etc. However, as will be seen, there is an important range of materials and circumstances, such that the latter effects are of secondary importance." He reiterates (Summary and Conclusions, p. 453): "Despite the complexities introduced by size, time, and temperature effects, there is an important range of circumstances where the limit of elastic behavior is determined primarily by the instantaneous state of stress."

Riley and Reddaway (1968) derive a theory for predicting the tensile strengths and failure mechanisms of fiber composites where the fibers are aligned in the direction of tensile loads and are flawed to some extent. The theory agrees reasonably well with experimental results, and the authors suggest that it may be qualitatively applicable to composites containing randomly aligned fibers. Several modes of failure can occur, depending on the initial length of the fibers and on the degree to which the fibers are flawed. Concerning the effect of flaws on the strength of fibers, the authors write (pp. 41-42): "Any defects, either in the fibres or on the surface, affect the strength properties. The effects vary with the length and diameter of the fibres. The mean strength of a number of fibres in a bundle decreases with increasing length of the fibres.•••Defects also result in a variation in strength between individual fibres, so that when a bundle of fibres is tested, some fibres will break before others. This increases the load on the remaining fibres, and the strength of the bundle is less than the mean strength of the fibres in the bundle.•••The bundle strength has been related to the mean strength, and to the coefficient of variation of the individual fibre strengths about the mean, by Coleman [(1958)]. Finally, it is known that fibres become stronger with decreasing diameter. This effect has generally been attributed

to the fact that, as fibres become smaller in diameter, there are fewer surface flaws on brittle fibres, and fewer dislocations in ductile fibres ...."

Serensen and Kogaev (1968) discuss the statistical formulation of the strength requirements for machine and structural elements subjected to steady cyclic loading with random variations in steady stress and in fatigue strength. The distribution function of fatigue limit, which can be found from experimental data, may be approximated by a normal distribution function, or more closely by a lognormal or a Weibull distribution function. The authors discuss the relationship between specimen size (or the size of the machine element concerned) and the probability of failure. They write (p. 13 of translation): "In evaluating the probability of failure of machine elements, it is essential to determine the relationship between the parameters of the distribution function of σ_{max} [the maximum stress in the stress-concentration zone] and the size of the component concerned and the degree of heterogeneity in the stress distribution. It is virtually impossible to obtain this relationship experimentally owing to the great variety of possible shapes and sizes of components. For this reason, an aspect of great importance in the investigation of the similarity relationships (the effect of size differences and stress concentration on fatigue strength), is the use of the statistical theory of strength, and particularly the theory of the 'weakest link'....This theory [see Kogaev (1965)] produces a relationship correlating ... σ_{max}, the ratio $d/\overline{G}$ between the specimen diameter and the relative gradient of the first principal stress, and the probability of failure P....For a given value of probability P, σ_{max} depends only on ratio $d/\overline{G}$, which gives the combined effect of the size factor and the stress concentration on the fatigue life. Consequently, if the size and shape of testpieces with the same $d/\overline{G}$ ratio vary, ... they will have a common distribution function of σ_{max}, a fact which greatly simplifies the strength calculations."

Spiers and Cullimore (1968) write (Abstract, p. 253): "A study has been made of the geometrical factors affecting the fatigue strength of shear-type double cover plate joints, in which the load is transferred wholly by friction between the connected parts. The basic variations of joint geometry which have been examined are: - plate thickness, total joint thickness, plate width, edge distance and disposition of bolts. Fatigue tests, mainly with alternating load, have been carried out to establish the effects on the S-N curve of

variations of each of the parameters and the results obtained have enabled some conclusions to be drawn regarding the choice of suitable joint proportions for design against fatigue."

Sullivan and Pierce (1968) write (Concluding Remarks, pp. 17-18): "Test results of [burst tests at cryogenic temperatures on through-cracked] cylinders fabricated from 0.020-inch (0.051-cm) thick sheet of the titanium alloy Ti-5Al-2.5Sn ELI showed that when the cylinder diameter was increased from 6.00 to 18.50 inches (15.2 to 47.0 cm) the bulge coefficient remained essentially constant. It is reasonable to believe that tests of larger diameter cylinders would result in no change in the value of the bulge coefficient obtained. It is concluded that the bulge coefficient is independent of cylinder diameter. Therefore, tests of scale-model cylinders can be used to accurately predict the fracture strength of full-size, through-cracked cylindrical pressure vessels of the same wall thickness. Tests results of 5.62- and 18.26-inch (14.3- and 46.4-cm) diameter, 0.060-inch (0.152-cm) wall cylinders machined from 2014-T6 extruded aluminum tubing showed a variation in the bulge coefficient. This variation was probably due to the fact that the two different diameter cylinders came from two heats of material. However, use of the bulge coefficient obtained from the smaller diameter cylinders resulted in predicted hoop fracture stresses for the larger diameter cylinders only slightly higher than those obtained experimentally. The experimental results were compared with an expression for predicted hoop fracture stress of through-cracked, cylindrical pressure vessels proposed by Eiber. The aluminum alloy data fit the predictions of this expression very well while the titanium alloy data did not. The Eiber expression may not be applicable to cylinders with walls so thin that the bulge effect is very pronounced."

Tychowski (1968) reviews the methods proposed by several authors to determine the true compressive strength of metals, without the influence of the friction between the compressed plates, by means of a graphical extrapolation of the curves giving the value of the compressive stress (σ) or strain (ε) in terms of the ratio of the diameter (d) to the height (h) of the specimens. He proposes fitting the curves by the method of least squares in order to avoid the inaccuracies resulting from graphical extrapolation. The difficulties in this approach in the case of curvilinear representations are overcome in the case of linear functions $\sigma = f(d/h)$, where the ratio d/h varies during the

deformation. These functions, being plotted for fixed strains, remain straight lines when the logarithmic deformation is considered instead of the unit compression strain itself. The results of applying this method to two kinds of mild steel, for not too large strains, compare favorably with those provided by the same method, when the linear deformation depending on the ratio d/h is taken into account.

Wadsworth and Spilling (1968) present the results of a theoretical and experimental study of the transfer of loads from broken fibers in composite materials (high-modulus carbon fibers placed in a resin matrix on an aluminum backing. They write (Conclusion, p. 1058): "If the interface between the fibres and the resin is weak, then when a fibre breaks the interface fails and the fibre ends contract in the hole in the resin they previously occupied. The load in the fibre is then transferred to the other fibres (or to the base plate) over a long frictional transfer length, determined by the (low) frictional force in the slipping interface. This length is much longer than the elastic transfer length over which loads small enough for the interface not to fail can be transferred from one fibre to another, so the nearby fibres can shed almost all their increase in load to others further away or to the base plate. A break in one fibre causes little increase in the load in others, and the crack does not spread. The cracks in neighbouring fibres are uncorrelated. If the interface is strong the load is transferred to nearby fibres over the same elastic transfer length as they need to shed it. Thus the stress concentration is larger and the crack is likely to run into neighbouring fibres. This can cause a change in the mode of failure of dense composites." The authors note that the behavior of a single fiber, assumed to be uniformly strong along its length, can be simply predicted. They write (p. 1052): "When the strain in the fibre reaches the breaking strain of the weakest part the fibre breaks there, the ends slip back in the resin, and a frictional shear force acts on the fibre over the slipped length. This force must transfer the load carried by the fibre. Thus after the first break there is a distance, ℓ_0 say, on each side of the break over which the stress in the fibre changes from zero at the end to the value corresponding to the applied strain. As the applied strain is increased slightly the next break will occur at the next weakest point in the fully stressed part of the fibre. When the average spacing between the breaks is still much greater than ℓ_0, the 'frictional transfer

length', the breaks will be distributed randomly and the lengths of the fibre pieces will be distributed in a Poisson distribution, with the proviso that lengths less than ℓ_0 cannot occur. The number of pieces of lengths between ℓ and $\ell + d\ell$ will be proportional to $\exp(-\ell/\lambda)d\ell$, where $1/\lambda$ is the expectancy of breaks [per unit length], and the fraction of pieces which have a length less than ℓ will be given by $f = 1 - \exp\{-(\ell - \ell_0)/\lambda\}$." They also consider the behavior of bundles of fibers. They point out that, at fairly small strains, the probability of failure of a small, close group of n fibers is n times that of one fiber if a crack in one fiber spreads at once to the others. Thus, if the cumulative distribution of length for a single fiber is given by $f = 1 - \exp\{-(\ell - \ell_0)/\lambda\}$, then the cumulative distribution for the group should be given by $f = 1 - \exp\{-(\ell - \ell_0)n/\lambda\}$. This fits the observed distribution for a strain of 1.0% in the case of a bundle of 4 fibers very well with $\ell_0 = 0.3$ mm, $\lambda = 1.75$ mm and $n = 3.2$. Presumably $n < 4$ because the 4 fibers were not all very close and some cracks did not spread to the others.

Wallace, Vishnevsky and Briggs (1968) report the results of research whose purpose was to extend the knowledge of fatigue properties of cast steel by presenting a study of the effect of notches on fatigue properties and the effect of specimen design and the method of loading on the fatigue properties of cast steel. They conclude that a severe notch (0.015-in. radius) in fatigue testing of cast steel results in a 36 percent reduction in endurance limit based on unnotched specimens, and that an extremely severe notch (0.001-in. radius) results in a 42 to 53 percent reduction. With regard to the effect of specimen design, they point out that it is known that the size of the test specimen and its shape affect fatigue results, a lower fatigue strength being observed for larger specimens, and that the size effect is observed in cyclic torsion tests as well as in rotating beam tests. They report the results of tests not only on the standard R. R. Moore specimens, but also on specimens of other shapes and sizes.

Zweben (1968) writes (Abstract, p. 2325): "In this paper two modes of composite tensile failure are investigated. The failure loads predicted by these analyses are significantly closer to experimental data than predictions of other theories. The first study considers composites containing a planar array of parallel fibers which exhibit a large number of isolated fiber breaks before failure. A statistical analysis that includes the effects of stress

concentrations is employed to describe the mechanics of failure. The second study, which is also statistical in nature, considers monolayer and multilayer unidirectional composites that do not display many isolated breaks before failure. Failure criteria are established for each mode, and the implications for nondestructive testing are discussed." Both statistical studies are based on the weakest-link theory, and make use of the Weibull distribution. Concerning the size effect, the author writes (p. 2331): "It is interesting to note that both the cumulative and non-cumulative failure modes predict that composite failure strength decreases with the size of the body. This is in contradiction with the theory of [Rosen (1964)] which predicts a failure strength that is essentially independent of size.•••It is well known that brittle materials like ceramics exhibit a significant size effect that is generally attributed to flaw sensitivity. This dependence of strength on size has also been well documented for the high modulus fibers such as glass, boron, and graphite, which are currently in use. However, little work has been done to investigate this effect in composites. Kies reported a significant decrease in strength with size increase for filament-wound pressure vessels. But, as he noted, the nonuniformity of stress in such structures prevents the formation of any concrete conclusions. The resolution of this dilemma requires a great deal of careful testing."

Bartenev (1969a) presents experimental results to prove the existence of a thin surface layer in glass fibers of the alumina silicate group and discusses the reasons for the increased strength of the surface layer as compared with the strength of the core of glass fibers and massive glasses. He also discusses five different levels of strength of glass, connected with different surface flaws, and the effect of chemical composition of a glass fiber on its strength, which is very marked in more perfect materials. In a related paper, Bartenev (1969b) again describes five characteristic ranges of glass strength with respect to the sizes of the surface flaws. He reports experimental results which lead to the conclusion that glass fibers have structural anisotropy, but that massive glasses are essentially isotropic. He writes (p. 180): "Because structural anisotropy in silicate glass fibers requires exceptionally careful techniques to detect it, it is likely that the structural anisotropy is weak. It therefore cannot be the main reason for the high strength of glass fibers. This conclusion agrees with the observed weak dependence of

the strength upon the diameter ···. This weak dependence can be explained by the slight change of the structural anisotropy associated with changes in the degree of drawing during molding." He also reports that in flawless glass fibers a surface layer one order of magnitude thinner than normally detected is present, probably as a consequence of the drawing process. Basic strength of flawless glass fibers in vacuum can reach one million pounds per square inch of cross section and is independent of length.

Crosley and Ripling (1969) report the results of an evaluation of the fracture toughness of A533 steel at room temperature, 0° and -70°F., with 1- and 2-in-thick contoured double cantilever beam specimens. To establish the effect of rate on thickness requirements, they measured both initiation and arrest toughness in tests covering a wide range of loading rates. They write (Discussion, pp. 531-532): "The results indicate that the direct influence of strain rate on toughness is modest, but there is an indirect effect traceable to the strain rate dependence on the yield strength. The latter effect can be viewed in the larger context of the influence of specimen size.···the only evidence of a thickness effect is in the room temperature data on 1-in-thick specimens tested at low loading rates, and this effect persists to lower rates for arrest than it does for initiation."

Forrey (1969) gives a method of predicting the minimum tensile strength of the wire command link in a small anti-tank missile which is based on extreme-value theory [Gumbel (1954), Kase (1953)]. He writes (pp. 233-234): "Briefly the assumption is that the strength of a length of ··· wire is determined by its weakest segment and is similar to the concept that a chain is no stronger than its weakest link. The reason for the weakest segment is that the wire contains randomly distributed flaws and their magnitude is also randomly distributed. Therefore, the wire will break (under a straight tensile load) where the largest flaws exists. It is also assumed that the breaking strength S is a linear function of the proportion of the cross-sectional area of the largest flaw to the cross-sectional area of the wire, and since the cross-sectional area of the wire is essentially constant, the above may be represented by $S = S_0(1 - cZ)$ where S_0 = the theoretical tensile strength of the wire, c = a constant and is equal to the reciprocal of cross-sectional area of the wire, Z = the area of a flaw. It has been found ··· that the size distribution of flaws, f(Z), fits the exponential density function $f(Z) = \lambda e^{-\lambda Z}$, $Z > 0$, where

$f(Z)$ = probability density function, λ = the parameter of the distribution, $e = 2.718 \cdots$. If the underlying distribution of all breaking strengths is considered, then the smallest value in repeated large samples from this distribution will have a distribution of its own which, as the sample size becomes larger and larger, approaches closer to a limiting distribution. This limiting or asymptotic distribution is ••• of the form $\phi(S) = \alpha \exp[\alpha(S - \beta) - \exp(\alpha(S - \beta))]$ where S = the minimum breaking strength of a length of wire, β = the modal value of the breaking strength of wire, α = the reciprocal of a measure of dispersion." The author points out that the theory can be applied to the data at hand (minimum breaking strength S of 20 one-hundred-foot lengths of wire from each of 131 spools) by plotting S on extreme-value probability paper (available commercially) or by plotting the reduced variate $Y = -\alpha(S - \beta)$ on ordinary rectangular coordinate paper. The plotting position $n/(N + 1)$ is used, where n is the cumulative frequency, counting from the largest observation down, and N is the total frequency.

On the size effect in fatigue of cast steel, de Kazinczy (1969) writes (Abstract, p. 40): "Fatigue fracture in cast steel starts at small internal defects. Their size distribution can be reasonably well represented by a Weibull function. The probability of finding a larger defect in a specimen increases with increasing specimen size. Since the endurance limit is lowered by such defects, a stronger size effect will be observed than in defect free materials. The ratio between the endurance limit of two specimens with different stressed volumes (w) could be expressed by: $\sigma_1/\sigma_2 = (w_2/w_1)^{1/m+n/p}$ where 1/m is the exponent in the absence of defects, n is obtained from the defect size distribution, and p is related to their notch effect."

Knowles and Park (1969) give the results of an investigation into axially and eccentrically loaded tubular columns covering a wide range of slenderness ratios. Both hollow and concrete filled steel tubes were tested. Columns of lengths 1 ft. to 5 ft. were tested. Two sizes of circular tube (3.50-in. and 3.25-in., with wall thicknesses 0.230 and 0.055 in., respectively) and one size of square tube (3.0 in., with wall thickness 0.131 to 0.133 in.) were used. A study was made of the effect of the slenderness ratio on the lateral pressure exerted by the tube on the concrete, as well as on the strength of the column. The authors show that the tangent modulus approach to ultimate strength determination of axially loaded columns agrees with test results.

They also compare the results of the eccentrically loaded columns with a straight line interaction formula.

Konstantinov and Strelyaev (1969) consider the mechanical characteristics of oriented glass-reinforced plastics (GRP) stressed in shear. The interlaminar shear strength of GRP was investigated by conducting bending tests. The span-to-depth ratio ℓ/h varied between 5 and 50, with 20 tests per point. Depending on the ratio ℓ/h the specimens failed as a result of either the normal stresses or the shear stresses, the boundary between the two modes of failure being indistinct because of the considerable spread of shear strengths. An analysis of the experimental bending data showed that shear failure occurs for $\ell/h < 10$, while failure for $\ell/h > 15$ results from the action of the normal tensile stresses, with the transition from normal to shear fracture taking place for $\ell/h = 10 - 15$ depending on the variance of the strengths σ_u and τ_u and the type of GRP. The rupture shear stress remains almost constant on the ℓ/h range investigated.

McKee and Sines (1969) write (Abstract, p. 185): "A statistical model for the tensile fracture of parallel fiber composites is based on a stress criterion for crack propagation. The stresses in fiber[s] which surround a crack nucleus are evaluated and the number and size of such nuclei are evaluated statistically. Failure is predicted when, statistically, a new break is expected at any crack nucleus. The predictions of this model are in good agreement with measured tensile strength, the variation of strength with size, the variability of tensile fracture strength, the very small creep strain, and the observed mechanism of fracture of fiberglass composites." They state the following conclusions (pp. 192-193): "The most meaningful predictions of the proposed theory have been mentioned, that reinforced fiber structures will exhibit a regular decrease in fracture stress as volume is increased and will exhibit little more 'creep' than their constituent fibers. The variability of tensile strength of large specimens is predicted to be only slightly less than that of the individual fibers, in agreement with experimental experience."

Nakamura, Tanaka, Hatsuno, Yaguchi and Mōri (1969) report the following results of various fatigue tests carried out on a full size wheel-axle testing machine: (1) The fatigue strength of the actual car-axle is lower than that of the large size press-fitted specimen; (2) When the (S/2.3 - N) curve is used to calculate the fatigue life of an induction-hardened car-axle, the estimated fatigue life agrees well with the actual life.

Ohuchida and Ando (1969) summarize the results of low cycle fatigue tests on unnotched, notched and cracked specimens and tensile tests on cracked materials of various dimensions on two types of low alloy steels, SNCM-2 and SNCM-9, as follows (p. 26, translation): "(1) The fatigue strength in unnotched specimens hardly decreased up to 10^4 cycles when compared with σ_B [tensile strength]. Although the static strength of notched specimens is higher than σ_B, the fatigue strength decreased linearly with the number of cycles. (2) When the outside diameter of a cracked specimen is greater than 25 mm., K_{IC} [the stress intensity factor obtained from the fracture load in tensile tests and the crack tip diameter] becomes constant and the fracture mechanics theory becomes applicable. Accordingly, the effect of dimensions on static strength of specimens can be estimated by this theory. (3) As long as the diameter of the test specimens is in the range greater than 25 mm., where fracture mechanical considerations are valid, the fracture conditions in low cycle fatigue coincide with those of fracture mechanics and $K'_{IC}/K_{IC} = 1$. [K'_{IC} is the stress intensity factor obtained from the crack tip diameter and the test load at final fracture.] On the other hand, when the diameter is smaller than this range, K'_{IC}/K_{IC} becomes significantly greater than 1 and fracture mechanics is no longer applicable."

Olubode and Crossland (1969) give a brief review of theoretical and experimental work on the yield of steels under static contact loads, and conclude that the experimental evidence is consistent with the well known effect of size or stress gradient on the yield of mild steel. Experimental work was carried out on a 0.19 per cent carbon steel, given various heat treatments to vary the grain size, and on Vibrac V30 or EN25 steel. The shear yield stress in ball loading tests is dependent on ball size, and it is significantly higher than the shear yield determined from torsion tests for 0.19 per cent carbon steel, the discrepancy increasing with grain size. For Vibrac, the effect of ball size appears to be negligible for 0.75 in. diameter and above, but it is appreciable for 1/2 in. diameter balls and smaller. The effect of size in ball loading tests can be accounted for on the assumption that the yield will not occur until the shear stress exceeds the "true" yield stress in a critical volume of material or in a critical number of crystals. For the 0.19 per cent carbon steel stress relieved at 600°C it appears that the "true" yield stress is 12.3-12.4 tonf/in^2 and that the critical number of crystals is 10-17 except for small balls, where it is less.

Peras (1969) writes (English summary, p. 139): "An equation ··· which establishes the relation between the average strength of a brittle porous material [in pure bending] and the sizes [sic] of a sample, taking into account the parameters of the material and its porosity, is derived. It is established that in the most cases the influence of the length of the sample on the average strength of material is expressed by the well-known Weibull's theory relations. It is also proved that the dependence of the strength on the area of the cross-section of the sample has a non-monotonic character. At low cross-section areas the average strength increases with the increase of the cross-section, reaches the maximum, and than [sic; then] decreases at the further increasing of the cross-section of the sample."

Phillips and Armstrong (1969) write (Summary, p. 265): "It is shown, by comparison of fatigue and yield strength results previously reported for several materials, that a rational explanation can be given of the seemingly complicated manner in which the fatigue properties of annealed low-carbon steel depend on the number of grains in a specimen cross-section, the polycrystal grain size and the yield point behaviour. This dependence varies according to whether the fatigue stress is greater than, equal to, or less than the yield stress of the same material."

Piontkovskii and Morozov (1969) write (p. 595): "Specimens having large cross sections are employed for testing ··· because it is well known that as the section is increased, the static characteristics are reduced considerably less than the fatigue performance. For example, with specimens 150 mm in diameter made from 35L and 22K types of steel, the fatigue limit is reduced by 30 to 50% compared to specimens 10 mm in diameter but the yield point is only reduced by 15% [Kudryavtsev and Naumchenkov (1959)]. Thus the use of large specimens makes it possible to carry out a test with pulsed loads that are considerably higher than the endurance limit but not higher than the yield point, which thus ensures uniformity of the loading cycle with time."

Popovics (1969), in a review paper on the fracture mechanism in concrete, discusses the criterion for crack propagation due to Griffith (1920) and two statistical hypotheses, the "weakest link" theory [Peirce (1926), Weibull (1939a)] applicable in series loading and the theory of Daniels (1945) for bundles of threads loaded in parallel. He states the following conclusions (p. 541): "Of the available failure hypotheses, the Griffith criterion seems

to be the most suitable for concrete because it provides at least a qualitatively correct picture of the crack propagation in concrete. The most serious difficulty with the numerical application of the Griffith criterion [$\sigma_u = (2ET/\pi c)^{1/2}$] is the elusiveness of the E, T, etc. values [σ_u = applied tensile stress far from the tip of the flaw, which is uniform and normal to the plane of the flaw; E = modulus of elasticity; T = surface energy of the material per unit area]. Weibull's calculations, as well as the other statistical methods, predict properly that large specimens will be weaker and the scatter of their measured strength will be less. However, quantitatively, there are discrepancies between the calculated and experimental values. Therefore, these statistical flaw theories, as well as the Griffith criterion, can be considered to be only marginally successful for concrete." He gives 45 references and a bibliography of 16 items on the mechanics of fracture in general and the failure of concrete in particular; many of these are relevant to the size effect.

Yen (1969), in Section 5.1 (pp. 141-144), discusses how the theories of probability and statistics may be used to explain the size effect in static failures and to explain the size effect, notch effect, and the slope of S - N curves in fatigue failures. For static failures, he expounds the statistical failure theory of Weibull (1939a), based on the weakest link theory and the Weibull distribution, which states that the probability of fracture S at stress σ is given by $S = 1 - \exp[-V\{(\sigma - \sigma_\mu)/\sigma_0\}^m]$, where V = the volume of the component subjected to tensile stress, σ_μ = the lowest limiting stress below which the fracture cannot occur (Weibull location parameter) ···, σ_0 = the strength of a 'flawless' specimen (scale parameter), m = flaw density per unit volume in the body (shape parameter). If one assumes, as Weibull originally did, that $\sigma_\mu = 0$, it follows that $\sigma_1/\sigma_2 = (V_2/V_1)^m$, i.e., that the strength is inversely proportional to the m^{th} power of the stressed volume. The author writes (p. 142): "This theory appears to be valid for some materials but not for others. Sometimes, instead of the volume, the surface area is more important. With glass and similar materials not even the assumption of a proportionality between the number of flaws and the surface area is likely to be correct. Experiments have shown that for glass and silica tubes (and perhaps also for fibers and rods) the number of flaws is probably proportional to the length of the tube rather than to its surface area." The treatment of fatigue failures is based on the work of Freudenthal (1946). Yen writes (p. 143): "If a nonuniform

stress distribution due to a notch or to flexural action in a beam is approximated by a discontinuous step function with two intensities, it can be assumed that a percentage i of the total Q bonds is subjected to the high stress intensity in the vicinity of the notch, whereas the majority of bonds are in the field of comparatively low homogeneous stress. If p is the probability of destruction of a bond in the field of low stress intensity and p_1 is this probability in the immediate vicinity of the stress concentration, it is shown that the probability P_2 of rupture of all Q bonds under nonuniform stress distribution for N load repetitions is $P_2 = 1 - (1 - P)e^{-iQN(p_1-p)}$, in which e is the base of natural logarithms and P is the probability of rupture of all bonds under the homogeneous low stress, without stress concentration. According to this equation, the probability P_2 increases rapidly with increasing [absolute] value of the exponent of e; that is (a) with increasing number of bonds iQ in the volume affected by the stress concentration, (b) with increasing load repetitions N, and (c) with intensity of stress concentration $(p_1 - p)$. These three influences are of the same order of importance; a similar increase in the probability of rupture will be obtained either by increasing the volume affected by stress concentration or by increasing the number of load repetitions. Therefore the effective strength reduction due to notch should increase considerably at the lower stress levels (for which the number of load repetitions is large). The notch effect $(p_1 - p)$ is multiplied by the factor N simply because the load is repeated N times." In subsection 5.3.3 (pp. 153-157), the author discusses extreme value theory and the Weibull distribution (also called the third asymptotic distribution of smallest values) and their application not only to static strength and fatigue life, but also to other natural phenomena.

ASTM Committee E-24 on Fracture Testing of Materials (1970) outlines a method (ASTM Method E399-70T) of determining the plane strain fracture toughness of metallic materials by a bend or a compact tension test of a notched and fatigue-cracked specimen having a thickness of 0.25 in.(6.4 mm.) or greater. Because of the size effect, the specimen size required for valid test results increases as the square of the ratio of toughness to yield strength of material; therefore, a range of proportional specimens is provided.

Barrois (1970) discusses, in Chapter IV, the size effect on the static strength of notched or cracked components. In particular, he reviews the

experimental work of Weiss et al (1960, 1966) on the size effect for V-type circumferential notches and on the section size effect in tension and bending, and the weakest link theory of Weibull (1939a, b), based on the statistical theory of extreme values, relating the probability of failure to the volume subject to stress. Concerning the latter, he writes (p. 137): "The fundamental assumption ··· implies the existence of such volume V_0 that the probability function $S_0(\sigma)$ for a localized fracture to cause complete failure is the same for all the domains. If stress σ is variable, the domain V_0 must be small enough for σ to remain substantially constant in this domain; on the other hand, V_0 must be large enough--compared to the flaws which represent potential fracture initiations--for $S_0(\sigma)$ to remain stationary in the solid. These requirements are rather contradictory. If, for example, the flaws are very small compared to the volume V_0, it is unlikely that a localized fracture will extend through the entire volume V_0 unless the flaws in the neighbourhood are substantially of the same severity, i.e., if the scatter due to the flaws is low and if the decrease in stress on the neighbouring flaws caused by plastic adjustment near the primary fracture is less than the increase in stress caused by elastic redistribution." The author notes that Weiss et al (1966) have presented a simple model to explain the effect of inhomogeneities on the strength of sharply notched H-11 steel specimens, and states his belief that this model is more appropriate than Weibull's theory to account for the effect of inhomogeneities on the strength of notched specimens made from engineering metals which are always ductile to some extent. In Chapter V he considers physical changes and damage during fatigue. In particular, on page 236 he discusses the size effect on crack propagation and on pages 268-270 he writes: "Furthermore, there is a size effect related to the stress gradient during the crack initiation period and to the stress intensity factor during the crack propagation period. In geometrically similar specimens the stress gradient decreases with increasing absolute size.···Like the static strength of notched specimens ···, <u>the fatigue strength decreases with increasing absolute component size</u>. This has already been stated with regard to the residual static strength of cracked components ··· and with respect to the fatigue crack propagation rate ···, these two phenomena being governed by the stress intensity factor in which the square root of the absolute crack size is an important factor. In all cases the size effect is mainly dependent on the stress gradient.

The second parameter that determines the size effect is the absolute size of a surface element with physical properties differing from those of elements in depth. It may be, for example, the average size of grains since surface grains, being less supported by their neighbours, are more subject to deformation and yield under load without breaking. It may also be the thickness of a work-hardened surface layer containing compressive residual stresses which delay failure and crack initiation. It appears that great caution is needed when using the results of small-specimen fatigue tests for prediction of the strength of large-size components."

Brown and Srawley (1970) offer comments on present practice in plane strain fracture toughness testing, especially on the specimen size requirements of ASTM Method E399-70T, reduction of which has been proposed by the authors of other papers in the same volume [Rolfe and Novak (1970) and Steigerwald (1970)]. They write (pp. 219-220): "The concept of K_{Ic} [plain strain fracture toughness] ··· entails two independent size effects associated with the two essential conditions of (1) linear elastic behavior of the material over a field which is large compared with the plastic enclave that surrounds the crack front and (2) tritensile plane strain constraint within this enclave and somewhat beyond it. To a practical degree of approximation the specimen thickness must be sufficient to maintain this constraint up to the test load that corresponds to K_{Ic}, and the other dimensions must be sufficient to maintain boundary conditions around the plastic enclave which match the crack tip stress field components of a linear elastic model. Any useful K_{Ic} test method must provide for these basic limitations by restriction of the valid range of a test to that for which the plastic enclave size factor $(K_I/\sigma_{YS})^2$ is within some specified fraction of the most critical dimension of the specimen. In ASTM Method E399-70T, for example, this factor is restricted to the smaller of 0.4 a_0 or 0.4 B, where a_0 is the initial crack length and B is the specimen thickness, and if the material property $(K_{Ic}/\sigma_{YS})^2$ is beyond this limit the specimen is not large enough in at least one respect to determine K_{Ic} for the material concerned. The crack length limit in ASTM Method E399-70T also controls other specimen dimensions such as the width W (depth of a bend specimen), which are held proportional to a_0 within narrow limits." On page 226 they write: "To summarize our views on the issue of specimen size, we believe that the available evidence suggests that the provisions of the 1970 ASTM E399 Tentative Method may not be sufficiently

restrictive to ensure that spurious results could not be obtained. Almost certainly they are not unduly restrictive in any respect." In the course of a discussion of "screening" tests with subsized specimens, they write (p. 244): "Most of these tests have considerable engineering value when properly used, but they can be seriously misleading if their limitations are not understood clearly. In particular, it must not be taken for granted that the order of ranking of materials by a given subsized specimen test will be the same as the order of ranking for K_{Ic}, or for $(K_{Ic}/\sigma_{YS})^2$. Size effects are of the essence in fracture behavior, and to neglect them involves the penalty of incomplete information."

Buch (1970) writes (Abstract, p. I): "A new physical and analytical approach to the notch - and size effect is considered as an extension of Peterson's Theory. It was confirmed for round normalized steel specimens and for flat steel and aluminum alloy specimens with central holes and showed good agreement with fatigue-test results. A new notch sensitivity factor, suitable for comparing different materials, is proposed. This factor yields better results than Peterson's q-factor in the case of specimens with central holes."

Clark and Trout (1970) report the results of an investigation of the fatigue crack growth characteristics of a forging-grade Ni-Mo-V alloy steel in which particular emphasis was placed on the influence of temperature (-100°F to 75°F) and section size (1 in.- and 2 in.-thick specimens) upon the observed fatigue crack growth rate. They state the following conclusions (p. 122): "(1) The fatigue crack growth rate observed in Ni-Mo-V alloy steel at a given ΔK level [ΔK is the change in stress intensity in 1 cycle of loading] increases with increasing test temperature and decreasing specimen thickness. (2) The rate of fatigue crack growth in Ni-Mo-V alloy steel increases as loading conditions deviate from ideal plane strain behavior. (3) The influence of test temperature and section size upon the fatigue crack growth rate at a given ΔK level is more significant under non-plane strain loading conditions. (4) Fatigue crack growth rate data generated under non-plane strain laboratory test conditions provides a conservative prediction of fatigue behavior for a full-size component operating under plane strain conditions. (5) Fatigue crack growth rate data generated with test specimens larger than the proposed component or specimens tested significantly below operating conditions cannot be used to accurately predict realistic design data."

Cooper (1970), on the basis of measurements of absorbed energy in a miniature Charpy test on brittle-fiber ductile-matrix composites (tungsten wires in copper, copper 10 $^{w}/_{o}$ tin and phosphor bronze), states the following conclusions (p. 186): "It appears that well-bonded brittle-fibre ductile-matrix composites show a variation of work-to-fracture [W] with fibre volume-fraction [V_f] somewhere between W = const. $(1 - V_f)^2/V_f$ and W = const. $(1 - V_f)$, and the more ductile the matrix the more nearly the first equation is obeyed. An increase in work-to-fracture due to a nonhomogeneous fibre distribution is to be expected for ductile-matrix composites and this has been demonstrated experimentally for tungsten wires in copper 10 $^{w}/_{o}$ tin. The general form of the variation of work-to-fracture with fibre diameter below 150 μm fibre diameter demonstrated by Kelly and Cooper [Cooper and Kelly (1967)] has been confirmed and extended to 10 μm dia. fibres. The latter two considerations show that tensile strength and toughness are to some extent conflicting properties in composites and that an optimum fibre diameter and degree of nonhomogeneity should exist for a particular combination of strength and toughness."

Hamilton and Rawson (1970) write (Introduction, p. 127): "The strength of a glass specimen is largely determined by the number and severity of flaws on its surface. When strength measurements are made on a batch of apparently identical specimens there is usually a considerable spread in the results, the coefficient of variation being typically of the order of 15 per cent. One also finds that the measured strength values depend upon the size of the specimen and upon the distribution of stress over its surface, i.e. on the type of loading. Weibull (1939) first showed how these features of the strength of brittle materials (like glass) could be predicted quantitatively, once a function is specified which describes the flaw distribution on the surface of the material. A number of investigators have subsequently applied and further developed Weibull's approach to determine the flaw distribution function from strength measurements using glass as the experimental material.•••Although the analysis of strength data for brittle solids in terms of a flaw distribution function appears to be generally accepted as giving meaningful results, there is one type of fracture experiment for which, in spite of a number of careful investigations, there remains a lack of agreement as to the validity of the analysis. This is the experiment in which the glass is fractured by pressing a hardened steel ball against it to produce a ring crack or Hertzian fracture."

In addition to a discussion of the results of others, including Oh and Finnie (1967), the authors present the results of their own investigation, which they summarize as follows (p. 127): "Hertz fracture experiments were carried out on the two surfaces of specimens of float glass and on specimens of polished-plate glass in the condition as-received from the manufacturer and after various etching treatments in hydrofluoric acid. The results are analysed in terms of the theory of flaw statistics originated by W. Weibull. An acceptable degree of agreement is obtained between the experimental results and values predicted using simple flaw distribution functions which have been determined for each of the five surfaces investigated. Brief consideration is given to the objections which have been raised to the application of flaw statistics analysis to this type of experiment. The present results suggest that these objections are not justified." Weibull (1939a), Oh and Finnie (1967), and the authors all assume a Weibull distribution for the number of flaws per unit surface area. The authors' method is similar to that of Oh and Finnie except that they determine the values of the constants in the Weibull function which give the best fit to their results for the variation of the median fracture load, P*, with indenter radius R, thus using the P* - R relation to obtain the flaw distribution function.

Heavens and Murgatroyd (1970) consider three methods of determining the parameters of the Weibull distribution function from the results of brittle fracture tests. They write (Introduction, p. 503): "The methods currently used in analyzing the statistics of the stresses required to produce fracture in brittle materials are discussed in the present work. Since such problems involve a 'weakest-link' model, they can be treated by the methods of extreme-value statistics, which concern the distribution of smallest and largest values drawn from infinite populations of various kinds [Gumbel (1958)]. Such a treatment was first attempted by Weibull [(1939a)], who suggested, for surface-limited strengths, an empirical distribution function which is usually written in the form $P(\sigma) = 1 - \exp[-\int_A \{(\sigma - \sigma_u)/\sigma_t'\}^m dA$, $\sigma \geq \sigma_u$; $= 0$, $\sigma < \sigma_u$ (1). $P(\sigma)$ is the probability that a specimen will fail at a stress of σ or less, σ_u, σ_t, and m are parameters characteristic of the material, and A is the stressed area. In its present form, the exponent in Eq. (1) is not dimensionless. Following Weibull, this equation should be written as: $P(\sigma) = 1 - \exp[-\int_A \{(\sigma - \sigma_u)/\sigma_t\}^m dA/A_0]$, $\sigma \geq \sigma_u$; $= 0$, $\sigma < \sigma_u$ (2), where $1/A_0$ is the area density of

strength-impairing flaws. If Eq. (2) is used directly it must be remembered that: $\sigma'_t = \sigma_t A_0^{1/m}$ (3) and therefore the same units of area must be retained in any subsequent use of the parameters. The quantity σ'_t has the dimensions of force times length$^{(2/m-2)}$ and is not a 'stress' because A_0, the mean area per flaw, cannot be determined directly; although Eq. (2) emerges from the theory of extreme-value statistics [Freudenthal (1968)], Eq. (1) is used in practice."

Isherwood and Williams (1970) note that two of the most widely accepted theories that have been proposed to predict the load at which a cracked structure will break require the experimental determination of a fracture constant which is a property of the material under study and its thickness. Finite width effects are important because the free edges of the specimen alter the stress distributions. The authors give equations which take these effects into account.

Ivanov, Sinitsyn and Novikov (1970) write (pp. 881-882 of translation): "Assuming that in the dynamical loading of structures the process of their rupture is determined by the energy of crack formation, it should be expected that the rupture process will depend on a scale factor for similar changes in the structure and in the magnitudes of the specific impulses acting thereon.··· The influence of the scale effect has been investigated experimentally in tests with complete vessels of 22k steel during their rupture by explosions of high-explosive charges set at the center of the vessels. The outer radii R of the vessels equalled 5, 15, 75cm with the relative thickness of the vessel walls remaining unchanged at 0.214R. Charges whose weight was gradually increased until the vessel was ruptured were exploded successively in each vessel.··· Results of the research confirm the validity of the assumption of strong influence of the scale factor on the rupture of structures under dynamical effects. The hypothesis on the governing role of the crack-formation energy during dynamical rupture of structures permits satisfactory description of the experimental results."

Jensen (1970) writes (Summary, p. 4): "The course of failure in concrete is investigated on the basis of the conception of concrete as a three-component material, consisting of matrix, coarse aggregate particles and contact zone. A deterministic model is established which assumes rigid particles and rigid, perfectly plastic matrix and contact zone with no tensile strength. The model

is used to analyze the commencement and propagation of failure in a test specimen and motivates a division of the course of failure into three stages. The model further seems to substantiate the influence of the middle principal stress and the size of the test specimen on the ultimate stress."

Jones and Brown (1970) write (Abstract, p. 63): "Results are presented from a systematic investigation of the crack length and specimen thickness on the fracture properties of 4340 steel bend specimens heat treated to yield strength levels between 180 and 213 ksi. It is shown that K_{Ic} [plane strain fracture toughness] values determined in accordance with the present ASTM Tentative Method of Test for Plane Strain Fracture Toughness of Metallic Materials (E399-70T) can vary moderately within the specimen size and geometric limitations imposed by the Test Method. The magnitude of these variations will depend on the material properties and would be substantially increased by relaxation of the size requirements [as proposed by Rolfe and Novak (1970), Steigerwald (1970), and others]. The possibility of employing subsized specimens for screening materials regarding their plane strain fracture toughness was explored as well as several methods for relating K_{Ic} values to uniaxial tensile data. The results indicated that the use of subsized specimens with the ASTM Committee E-24 Test Method does not constitute a useful screening process."

Kaufman (1970) writes (Abstract, p. 3): "Progress in the development of fracture-toughness testing techniques by ASTM Committee E-24 is reviewed briefly. The evolution of the test method for plane strain stress-intensity factor, K_{Ic}, is described, including treatment of some of the problems dealing with fatigue cracking, thickness limitations and crack growth detection, and measurement." On page 7, he writes: "The broad use of more sensitive devices for following crack growth played an important role in further refinements of fracture testing, as it made it easier to recognize two things: (a) The fact that K_c [the critical stress-intensity factor] is a thickness-dependent property, as exemplified by the data for 7075-T6 sheet and T651 plate ···. Over a certain critical range of thickness for each material, the load and the crack length at instability (and, therefore K_c) decrease with increase in thickness, reaching a rather constant minimum value when plane strain conditions are approached closely. In the near fully plane strain situation, the instability leading to complete fracture almost immediately follows crack

initiation. This point had been observed earlier ···, but not fully appreciated. (b) The fact that the initial crack growth, whether stable or unstable, took place at about the same level of stress intensity for a given material, regardless of specimen thickness ···. Thus, it might be possible to learn something about a plane strain instability, even though complete fracture of the specimen did not take place under plane strain conditions. As a result of these two observations, the concept of a lower level of critical stress-intensity factor associated with plane strain conditions (K_{Ic}) and of the capability to approximate the plane strain stress-intensity factor from the conditions under which stable crack growth initiated were solidified."

Kudryavtsev (1970) presents the results of an experimental study of the size effect on geometrically similar notched specimens (20 to 92 mm. thick) of steel plate (16 GNMA) subjected to cyclic loading. He states the following conclusions (p. 444 of translation): "1. Under small-cycle low-frequency loading conditions, large specimens always fail earlier than small ones. In small-cycle fatigue tests the scale effect is observed at all stress levels as an increase in the number of loading cycles to crack formation or to failure as the specimen dimensions are reduced. 2. Under conditions of single static loading, the failure stress and deformation diminish with increase in specimen size.···3. In the region of very high cyclic loading levels (close to the ultimate strength), the effect of reduction in the accumulated deformation at a particular instant with increase in specimen dimensions is retained.···4. Smaller specimens pass from the region of quasistatic failure to that of transient and later fatigue failure at higher stress than large specimens.··· 5. Finally, in the case of life governed by the origination of fatigue small-cycle failure ···, for all specimen dimensions the scale effect is that with one and the same test basis small specimens ··· have a higher fatigue limit than larger ones ···."

Leichter and Robinson (1970) write (Summary, p. 197): "The fatigue behavior of a high-density graphite was investigated in reverse bending up to 5×10^8 cycles at room temperature. Fatigue life was analyzed statistically, using a Weibull distribution and a homologous stress. The homologous stress is the ratio of applied stress in fatigue to the expected first-cycle strength. The first-cycle strength was calculated from bending tests of similar materials or 'mate' specimens, using the Weibull statistical strength theory to correct

for the significant size effect. Homologous stress gives an essentially invariant fatigue correlation for many graphite grades and may be used to estimate fatigue life under a wide variety of operating conditions. The practical endurance limit for graphite corresponded to a homologous stress of about 47%." On pp. 198-199 they write: "The formula proposed by Weibull [(1939a)] for the probability that a specimen will survive an arbitrary applied tensile stress is $S = e^{-\int (\sigma_0/\sigma)^m dV}$ (1) [where] S = probability of survival, σ = applied stress, σ_0 = scaling factor, m = material constant (Weibull modulus), V = volume under tension, [and] $\int (\sigma/\sigma_0)^m dV$ is the 'risk of rupture'.···The results from the 50 mate specimens tested statically in three-point bending were used to estimate the strength distribution (or Weibull modulus) of the population from which the sample of fatigue specimens was drawn. This, in turn, allowed the estimation of size effect and the computation of the expected first-cycle strength.···It is well known that the strength of a specimen of a brittle material depends strongly on its size. The Weibull theory of brittle fracture based on a random distribution of flaws having random intensities establishes the importance of tensile stress as the failure mechanism. Stress concentrations tend to remain high in brittle materials because of the absence of localized plastic strains. Thus fracture may develop at lower values of nominal stress, supporting the 'weakest link' concept of brittle failure. It is evident that large specimens favor the presence of extreme flaws, and hence should be weaker. For specimens having different volumes and subjected to uniform tensile stress, the ratio of strengths is $\sigma_1/\sigma_2 = (V_2/V_1)^{1/m}$(3). Equation (3) shows that the strength depends on the specimen volume and decreases as the volume increases. Furthermore, the size effect decreases as the value of m increases. This relation holds even for nonuniform stress if the stress distributions in the specimens are geometrically similar."

May (1970) reviews the experience of a British working group on fracture testing of high-strength materials, and presents the results of a specific study on the influence of the specimen dimensions such as thickness and crack length on K_{Ic} (plane strain fracture toughness) measurement. He writes (pp. 51-53): "The requirement that the specimen thickness and crack length should be greater than $2.5(K_{Ic}/\sigma_{ys})^2$ ··· was considerably more stringent than previously adopted criteria and was based ··· primarily on specimen size studies

for maraging steels. Several members of the Working Group were of the opinion that these criteria were too conservative and certainly needed justification with regard to low-alloy, high-strength steels, titanium alloys, and aluminium alloys. A size effect test program, therefore, was initiated covering high-strength titanium and aluminium alloys and martensitic and maraging high-strength steels, in which specimens were successively machined from broken specimens of larger dimensions. This ensured that the influence of material variability was minimized.•••From the preliminary results obtained to date there is no justification for relaxation of the crack length and specimen thickness requirements."

Pfleiderer and Arendts (1970) write (English summary, p. 115): "At first, the necessity of carrying out extensive fatigue tests with small specimen is substantiated. The prerequisites which permit an assignment of the results of these fatigue tests with small specimen to the large-scale component are, on the one hand, an exact prediction of the stress condition in the large-scale component and, on the other hand, an exact reproduction of the constructive and technological parameters in the small specimen. Then, a systematic comparison of the two basic materials--metal and fibre-reinforced plastics--nowadays used in aircraft construction is given. Special problems in the development of structures of fibre-reinforced plastics subjected to dynamic stress are shown. These problems can only be solved by tests with small specimen. Finally, test results from the development of the hingeless rotor of the Bölkow system are presented."

Pilecki (1970) presents several formulae from which he infers that very thin monocrystals should display extremely high fatigue and static strength because, provided the diameter is less than a critical diameter d_{cr}, the process of the increase of dislocation concentration cannot be initiated in them. He uses the mechanism of this phenomenon to interpret experimental results which he summarizes as follows (p. 295): "It was experimentally established in 1956 that thin monocrystals whiskers actually display the tensile strength S_T almost equal to the theoretical value which is: 1337kg/mm^2 for iron, 300kg/mm^2 for copper and 225kg/mm^2 for zinc, while in thicker specimens of these metals the corresponding values were 30, 22 and 18kg/mm^2, respectively. It was also established that the strength of whiskers attains its maximum in the case of diameters of the order of 1-5μm. With increasing diameter the strength of the specimen decreases. With the diameter larger than 25μm for iron and 10-20 μm for

other metals the tensile strength of the specimens does not differ from that of ordinary specimens." He asserts that the fatigue strength is determined by the mobility of dislocations in the surface layer and that, consequently, the increase of the diameter of the monocrystal above the double thickness of the surface layer (the equivalent of d_{cr} in whiskers) should not result in lowering its strength. Consequences similar to those induced by decreasing the diameter of monocrystalline specimens will also occur in polycrystals if (1) the maximum size of the grains in the surface layer is decreased, which may be accomplished by comminution of grains in the surface layer with simultaneous increase of the initial concentration of dislocations in it, e.g., by applying surface cold work, or (2) the critical size of the grains is increased by introduction into the surface layer of additional elements capable of blocking the dislocations, e.g., by applying chemical-thermal processing.

Plutalova and Rotnitskiĭ (1970) report the results of an experimental study of the influence of sample dimensions on the strength of graphite material in which compressive tests were performed on cylindrical specimens and bending tests on specimens with square cross section. It was found that, in the case of compression, the strength decreases almost linearly as the cross section increases, while for bending the decrease is curvilinear (rapid for small cross sections and less so for larger ones). In the case of bending, the dispersion of the strength increases with an increase in specimen size. The height of the specimen has practically no effect on the mean or on the dispersion of the compressive strength.

Rolfe and Novak (1970) write (Abstract, p. 124): "The present paper describes the test procedures and analyses used in slow-bend K_{Ic} [critical stress-intensity factor] tests of steels having yield strengths from 110 to 246 ksi and Charpy V-notch impact energy absorption (+80F) in the range 16 to 89 ft.lb.···The results indicate that LEFM [linear-elastic fracture mechanics] can be used to analyze the fracture behavior of medium-strength high-toughness steels. K_{Ic} values ranging from 87 to 246 ksi in. [sic; ksi $\sqrt{\text{in.}}$] were obtained for the eleven steels investigated. Although the specimen sizes tested (generally 2 in. thick) did not satisfy completely the recommended ASTM Committee E-24 size requirements (in some cases, the suggested specimen thicknesses were greater than 6 in.), the K_{Ic} values are considered to be representative on the basis of general strength and Charpy V-notch toughness levels and expected

service performance. Therefore, the measured K_{Ic} values are estimated to be reliable for general engineering use, although some of the specimen sizes did not satisfy the current ASTM Committee E-24 requirements completely." On pp. 139-141, they write: "Although the primary purpose of this investigation was not to investigate minimum specimen sizes required for plane strain (by systematically varying the specimen geometry), some general observations on specimen size requirements can be made.•••Until the K_{Ic} values presented ••• are verified, either by service experience or by systematic variation of specimen thickness, no conclusions should be made regarding the minimum thicknesses or crack depths required for plane strain behavior of medium-strength steels. However, the K_{Ic} values obtained appear to be representative of the K_{Ic} values that would be expected for these steels on the basis of general strength levels, Charpy V-notch toughness levels, and service experience, even though the specimen thicknesses for some steels were only one third the recommended sizes. Therefore, it would seem possible that, for the medium-strength high-toughness steels, the general size requirements necessary to obtain valid K_{Ic} values may be less than those currently recommended by ASTM Committee E-24 for steels having yield strengths greater than 200 ksi. Investigation of the minimum size requirements for K_{Ic} testing of medium-strength high-toughness steels is currently in progress."

Schächter (1970) writes (translation of Summary, p. 27): "A qualitative conception of the mechanism of the fatigue of materials drawn from the examination of experimental facts is transposed into a group of four postulates. Starting from this base of postulates, a system of differential equations is then constructed, representing, as a function of two parameters, the evolutions of fatigue due to a unique series of identical cycles. For different series of cycles of stresses, there results a unified representation of phenomena of 'understressing' and 'overstressing' type, as well as a damage function. The examination of structural nonuniformities--random or ordered--in the spatial distribution makes possible a quantitative description of the influence of factors such as the type of excitation applied, the shape of the piece, the surface effects, the dimensional effect, etc. The comparison of these results with the mass of known experimental data makes possible verification of the proposed theory. Likewise, some critical experiments are defined."

Steigerwald (1970) writes (Abstract, p. 102): "The influence on fracture toughness of relative specimen thickness in the center-precracked specimen and crack length in the precracked bend specimen was evaluated.···The study of the influence of crack length on the frcture toughness obtained in the notch bend test indicated that valid values of K_{Ic} were obtained when the crack size (a) was 2.5 times the $(K/\sigma_y)^2$ ratio in accordance with ASTM recommendations. However, it appears that the crack size requirements could be reduced by a factor of 2.5 without producing variations of greater than 5 percent in the measured fracture toughness value provided the thickness requirement was satisfied." The discussion (comments by W. F. Brown, Jr. and M. H. Jones and author's closure) also deals in part with the validity of the ASTM criteria for crack length and specimen thickness. In response to protestations by Brown and Jones, the author writes (p. 123): "I do not recommend that the ASTM criteria for either thickness or crack length be reduced.···The purpose of indicating that in some materials the K value is not significantly altered by crack lengths in the range between $1.0(K/\sigma_{ys})^2$ and $2.5(K/\sigma_{ys})^2$ is to provide a proper perspective to engineers who often do not have the luxury of test data on very large specimens."

Steverding and Lehnigk (1970) consider the behavior of ceramic materials upon impulse, which is determined by a least action law that they interpret to obtain expressions for actual and theoretical dynamic strength, size sensitivity, effect of small and long cracks and the influence of grain size on dynamic strength. Concerning the size effect (dynamic scale-up), they write (p. 1059): "It is known that the strength of materials decreases as they increase in size. Based on the statistical distribution of flaws, Weibull has formulated a theory expressing the size sensitivity of materials in the following way: $\sigma_1/\sigma_2 = (V_2/V_1)^{1/m}$(4) where σ_1 is the mean strength of volume V_1 and σ_2 is the mean strength of volume V_2. The larger m the more size sensitive is the material. Completely independent of Weibull's theory a dynamic relation for the size sensitivity with respect to dynamic loads can be derived. As a simple model the head-on collision of ceramic bars with a wall of high rigidity is used. Bars of various length collide with this wall, all with the same collision velocity ν. The collision process produces a stress pulse in the bars which has the following properties: The amplitude of the stress pulse is independent of bar length, namely $\sigma = \nu E/\alpha$ [where E is the elastic modulus and α is the

velocity of sound]. The duration of impact and hence the pulse length τ depends on the bar length and we have $\tau = 2L/\alpha$, where L is the length of the bar. Inserting this value for τ into Eq. (2) $[\sigma^2\tau/E = \gamma/\alpha$, where γ is the surface tension] we obtain $L = E\gamma/4\sigma^2$. Comparing two bars, one of length L_1 and the other of length L_2 we obtain $\sigma_1/\sigma_2 = (L_2/L_1)^{1/2}$(5). Equation (5) has the same structure as Weibull's Eq. (4) with m = 2. Equation (5) may also be interpreted in the same manner. If the static fracture probability of Eq. (4) simply superimposes on the dynamic size effect, the exponent m of Eq. (4) should be increased by 2 when dynamic scale-up sensitivity has to be considered. However, for most ceramics, m is of the order of 20-30 and the addition of 2 would not have any great effect. As processing methods improve, the exponent m, which indicates the density of flaw distribution, may be substantially decreased and dynamic size sensitivity may then become decisive. For metals a law similar to Eq. (5) has been derived; however, the exponent is not equal to 2 but may be smaller or larger. It may come as a surprise that some metals like beryllium have a much higher dynamic scale-up sensitivity than ceramics."

Suh, Bhattacharyya and Grandage (1970) develop, by probabilistic argument, small sample and large sample properties of the bundle strength of parallel filaments studied earlier by Daniels (1945) and Sen, Bhattacharyya and Suh (1969). They write (Introduction, p. 712): "Consider a bundle of n parallel filaments of equal length and let the non-negative random variables X_1, X_2, $\cdots$, X_n denote the strength of individual filaments and $Y_1 \leq Y_2 \leq \cdots \leq Y_n$ represent the corresponding ordered random variables. Now if we assume that the force of a free load on the bundle is distributed equally on each individual filament and the strength of an individual filament is independent of the number of filaments in a bundle, then the minimum load B_n beyond which all the filaments of the bundle give way is defined to be the strength of the bundle. Now if a bundle breaks under load L, then all the inequalities $L/n \geq Y_1$, $L/(n-1) \geq Y_2$, $\cdots$, $L \geq Y_n$ are simultaneously satisfied. Consequently the bundle strength can be represented as $B_n = \max\{nY_1, (n-1)Y_2, \cdots, Y_n\}$." They proceed to consider small sample and large sample properties, as well as certain moment properties, of this statistic.

Vitvitskii (1970) writes (pp. 581-585 of translation): "Let brittle plates of unit thickness be strained in tension or compression in two mutually perpendicular directions by uniformly distributed forces p and $q = \eta p \cdots$. It is

assumed that the plates are made of a homogeneous material (i.e., that their resistance to the propagation of cracks is everywhere the same) in which crack-like defects of various (random) lengths 2ℓ and orientations α are distributed in such a way that no interaction between them takes place; here ℓ is the crack half-length and α is the angle between the orientations of the crack and force p. In a general case it may be assumed that $-\pi/2 \leq \alpha \leq \pi/2$, since both crack tips are under the same conditions, and that $c \leq \ell \leq d$, where c and d are material constants. The degree of imperfection of the plate material is characterized by a joint distribution function of ℓ and α, $F(\ell,\alpha)$, or by the density of probabilities $f(\ell,\alpha) = \partial^2 F(\ell,\alpha)/\partial\ell\partial\alpha$. It is assumed that these functions are known. An isolated crack of given length 2ℓ and orientation α present in a plate will not grow under the influence of a uniform stress field if the load is below a certain critical level described ··· by formulas in the form $p = (A/\sqrt{\ell})\phi(\alpha,\eta)$, $q = \eta p$, where A is a constant characterizing the resistance of the material to crack propagation, and $\phi(\alpha,\eta)$ is a known function the form of which may vary for different ranges of the variation in α and η and may depend on the approach used in finding a solution of the problem ···. Since ℓ and α are random values, the critical load p for an element with a defect is also a random value varying between $p_{min}(\eta)$ and $p_{max}(\eta)$. The distribution function of the critical load for an element with an isolated crack is found ··· from the formula $F_1(p,\eta) = \iint\limits_{A\phi(\alpha,\eta)/\sqrt{\ell}\leq p} f(\alpha,\eta)d\ell d\alpha$. Here the integration is carried out over all the admissible values of parameters $c \leq \ell \leq d$ and $-\pi/2 \leq \alpha \leq \pi/2$ for which the inequality shown under the integral sign is satisfied.··· If the plates contain up to N cracks (N may be regarded as proportional to the plate size S, i.e., $N = N_0 S/S_0$, where N_0 is the average number of defects per unit surface area S_0), the distribution function of the critical load of such plates is found ··· from the formula $F_N(p,\eta) = 1 - [1 - F_1(p,\eta)]^N = 1 - [1 - \iint\limits_{A\phi(\alpha,\eta)/\sqrt{\ell}\leq p} f(\ell,\alpha)d\ell d\alpha]^N$. If the function $F_N(p,\eta)$ or $F_1(p,\eta)$ is known it is possible to determine several statistical characteristics of the critical load of the plate. For instance, the average value of the critical load of plates with N cracks is ··· $\bar{p} = p_{min}(\eta) + \int_{p_{min}(\eta)}^{p_{max}(\eta)} [1 - F_1(p,\eta)^N]dp$, i.e.,

$$\bar{p} = p_{min}(\eta) + \int_{p_{min}(\eta)}^{p_{max}(\eta)} [1 - \iint_{A\phi(\alpha,\eta)/\sqrt{\ell} \leq p} f(\ell,\alpha)d\ell d\alpha]^N dp, \quad \bar{q} = \eta\bar{p}.$$ $\cdots$Data show that the critical stresses of a plate with a given N are lower in biaxial tension (η = 1) than in uniaxial tension (η = 0). Increasing N (i.e., the size of the plate, if the average number of defects per unit surface area is assumed to be constant) produces a reduction in the critical stresses, the magnitude of this reduction depending on η. Thus, an increase in N is accompanied by the manifestation of the size effect whose intensity depends on the form of the stress state, the effect being most noticeable under uniaxial stress conditions."

Zweben and Rosen (1970) note that Weibull (1939a) developed a statistical theory of material strength that assumes a distribution of "weak places" that cause propagating cracks, resulting in rupture, so that a material is only as strong as the volume element containing the worst flaw, while Gücer and Gurland (1962) proposed a model for "dispersed fracture" consisting of a chain whose links were treated as bundles of elements whose strength was analyzed by use of the statistical theory developed by Daniels (1945), with failure occurring when the weakest cross-section cannot sustain an increase in applied load. They point out that Gücer and Gurland compared the strength predictions of the dispersed fracture and weakest element modes of failure and found that the dispersed fracture failure mode is generally insensitive to size, in marked contrast to the weakest link model. Zweben and Rosen relate material strength to microstructure, using a geometric model based on the one suggested by Gücer and Gurland, and apply the results to composite materials, obtaining good agreement with available experimental data. They write (p. 192): "One can consider failure to occur either by the propagation of a crack through the material or by the inability of the overall cross-section to resist the applied load. These two failure modes are represented in Gücer and Gurland (1962) by the weakest link and dispersed fracture models, respectively. The first criterion, which is based on the concept of a propagating crack, is overly conservative, in general, since the fracture of a single microscopic element does not necessarily cause overall failure. The second is nonconservative since it does not consider the effect of load concentrations. However, these two criteria do provide lower and upper bounds on expected material strength,

although they may be too far apart to be of any practical use. We now propose alternate criteria for these two modes of failure that are based on an analysis of the sequence of events that precede overall material failure. For simplicity we assume that the failure surface lies in a single cross-sectional layer, perpendicular to the applied stress." They suggest the occurrence of the first multiple break for continuous fiber composites and the first triple break for whisker-reinforced composites as criteria for fracture propagation. They write (p. 204): "Although the dispersed fracture model of Gücer and Gurland (1962) shows a size effect, the cumulative weakening theories neglect this effect as unimportant for large specimens. The fracture propagation criteria inherently include the result that the strength varies inversely with size as is observed for brittle materials. The size effect predicted by the first multiple break criterion is demonstrated in Fig. 12 [not reproduced here]. ...The multiple break criterion was applied to two-dimensional and three-dimensional composites, the latter giving slightly higher failure [stress] predictions since the load concentration factors are smaller. The size effect on bundle strength and the results for cumulative weakening theories with and without load concentration effects are also shown. For the sake of comparison, the cumulative weakening theory including size effect, as given ... in the paper by Rosen (1964), is presented."

Allen (1971) describes an investigation whose objectives were to find the variation in fracture toughness with thickness for 7075-T6 and 7075-T73 aluminum alloys and to provide data upon which a theory could be established for the variation of fracture toughness with thickness. A test program was conducted on 60 centrally cracked specimens varying in thickness from 0.05 to 0.75 in. and in width from 2 to 32 in. Examination of the data shows that the fracture toughness of the 7075-T6 material is independent of thickness and width in the range investigated, while that of the T73 material varies with both. The difference is attributed to the fact that, owing to the low proportional limit of the T73 material, the net section stress was in the plastic range for most specimens, whereas for the T6 material it was always in the elastic range. A lumped parameter, redundant force analysis of three plates of varying thickness was made. Further work is needed to correlate the analytic results, which show the development of plane strain conditions with increased thickness, and the test results.

Bolotin (1971) considers a stochastic model of the propagation of a main crack in a body with an inhomogeneous structure. The main crack is the connected set of damaged elements or unit volumes whose growth leads to complete fracture of the body. The body is regarded as a system consisting of a discrete number of elements. The main crack propagates through the solid following the path of the weakest or damaged elements. As a physical analog of a one-dimensional model consisting of n elements the author takes a flat, single-layer sample of a unidirectional fibrous composite containing n fibers. He computes the number of possible states of this model assuming that each element is in one of the two possible states (intact or fractured). The number of system states m is then equal to the number of all possible combinations of n elements, i.e., $m = 2^n$. If there is only one main crack, which can originate as a result of fracture of any one of n elements, $m \sim n^3$; if on the other hand the sources of main cracks are side or internal concentrators or if it is assumed that cracks can originate at the surface only, $m \sim n^2$. Finally, if only a single crack that originates at the surface or at a concentrator is considered, $m = n+1$. The author has analyzed several examples with the aid of a computer, and has compared the results with those given by the Weibull and Daniels models. For the example considered in detail, he finds the Daniels model untenable. He notes that his own model, which treats fracture of consecutive elements along the path of crack propagation as a random process with a discrete set of states, describes, in a specific and flexible form, the effect of redundancy in composites.

Dukes (1971), in discussing the statistical approach to establishment of design stresses of brittle materials, writes (pp. 17-18): "Conventional structural design with metallic materials involves a number of basic assumptions....It is assumed that the mechanical characteristics of the material for any given set of conditions can be defined specifically by a single unique value, i.e. characteristics under tension loads by ultimate tensile strength. It is assumed that the strength of a structural component is determined by the stresses at some critical point and that the strength of the structure can be determined by an examination of the stress system at this single point.... With brittle materials, however, the mechanical characteristics are not expressible as a single number. Due to the wide variability in any particular mechanical property, the most that can be done is to predict the probability

of failure for any particular stress condition. Variability in mechanical properties is assumed to be due to the presence of flaws, which produce local stress concentrations....Whether a sample of material will fail under a given stress condition will depend, therefore, on the size and distribution of flaws, and since the latter is random the only statement that can be made about failure is the probability of its occurrence. Furthermore, since the probability of experiencing a flaw of critical size will increase as the size of the component or the volume of material under consideration increases, the strength becomes a function of component size. The introduction of a statistical approach to the expression of mechanical characteristics, and the dependence of failure probability on volume, also means that the strength of a component cannot be determined by the examination of a critical point. For example, an applied stress distribution may have a very localized peak, so that the probability of this peak combining with a flow sufficiently severe to produce failure is small. On the other hand the same stress distribution may subject a large volume of material to a lower stress, and although a more severe flaw is needed to produce a failure there may actually be a greater probability of meeting such a flaw. Thus the probability of failure of the entire component must be expressed, and this involves some type of summation of the failure probability at all points within the component. Again it must be pointed out that the dependence of failure probability on volume is based on the assumptions of the flaw concept. Its validity remains to be positively demonstrated, and experimental evidence to date is limited, and in some cases conflicting....The variability in mechanical properties of a brittle material, from supposedly identical specimens, implies that there is a distribution function which shows the relative frequency with which a particular value of some mechanical property may be expected to occur. If frequency of occurrence is plotted, for instance, against tensile strength, the plot will show the frequency rising to a maximum value at some strength level, and falling off on either side....Such a curve is called a frequency distribution; it is not presently known whether all brittle materials have a similar frequency distribution, nor is it possible to analytically determine such a function. The most that is presently possible is to assume various frequency distributions and seek a best fit with experimental data....There is no complete agreement on the type of curve to be used, though a relationship established by Weibull

is accepted as the most satisfactory to date....The Weibull distribution function is based on the flaw theory, with the assumption that the flaws are distributed at random with a certain density per unit volume. The strength of the specimen is then determined by the weakest point in the specimen. The result is a representation of the material as a series model, or a chain in which failure depends on the weakest link. This is perhaps the major limitation of the Weibull theory since a real material is perhaps better represented as a series of chains in parallel, the so-called series-parallel model. Again, however, a convenient, practical, mathematical representation of such a model is not available....The Weibull distribution function is expressed as the probability of fracture, which is given by $S = 1\text{-}\exp\{-V[(\sigma-\sigma_u)/\sigma_0]^m\}$ where V is the volume of the element under consideration, σ is the applied uniform tension stress, and σ_u, σ_0 and m are material constants. If the Weibull function is truly applicable to the material under consideration, and if the materials used for small test specimens and for the structural components are truly identical, then σ_u, σ_0 and m will indeed be material constants and will not change with volume....The above expression can be generalized by writing the exponent as an integral over the volume of the component, and expressing σ as a variable in terms of V. In the completely general case a component can be divided into a large number of elements, each sufficiently small that the stress within the element can be considered uniform. The above equation can be applied to each element, and by proper summation, the probability of failure of the component can be determined." The author proceeds to give analytical and graphical methods, based on the theory of order statistics, to estimate the material constants σ_u, σ_0 and m, by conducting bending tests on N bars and setting the failure probability corresponding to the nth smallest fracture stress σ_{bn} equal to $S_n = n/(N+1)$, $n = 1, \ldots, N$.

Dykhnov (1971) proposes an approximate method for calculating the scale factor in the presence of stress concentration, based on a statistical approach along the lines of Weibull's "weakest link" theory. His proposed method allows the nature of the state of stress at the concentration point and the minimum strength to be taken into account. Using equations developed by Bolotin, he computes the mathematical expectation of the failure stress from

the knowledge of the volume in which the stress level is high. He illustrates his method by an example, in which he finds that the strength of a steel plate 60 x 35 x 1 cm., under uniaxial tension, is 1.84 times as high as it would be in the presence of a circular hole 2 cm. in diameter. By contrast, a cast iron plate of the same dimensions shows no weakening effect of the circular hole, except that due to the decrease in the effective width from 35 to 33 cm., so that the ratio of strengths with and without the hole is 1.07.

Feddersen (1971) writes (Abstract, p. 50): "The residual strength of center cracked tension panels is considered from a phenomenological perspective. A simple, direct analysis technique is derived from experimental observations and stress intensity factor concepts. A smooth, continuous stress-flaw size curve is generated over the full range of crack lengths and panel widths. Plasticity and finite width effects are accommodated in a manner simpler and more consistent for engineering purposes than the iterative procedures implicit in current theoretical models. The technique is verified by an analysis of data from a variety of sources. It is concluded that toughness indexes, in the form of stress intensity factors, are reliable indicators of fracture instability, as well as other damage levels. They can be analyzed and interpreted on an elementary format. It is also shown that width can be uncoupled as a parameter in cracked panel behavior."

Grigory (1971) writes (Abstract, p. 161): "The 15 million pound tensile test facility at Southwest Research Institute has been used to test a series of large tensile specimens for the Heavy Section Steel Technology (HSST) Program. These specimens are A533 grade B, class 1, and are 6 in. thick, 18 in. wide, 4 ft. long and contain flaws. The procedures used to create the flaws are discussed and pertinent data for six tests at temperatures from 50°F to 220°F and requiring loads up to 7.5 million pounds for failure are presented." In discussing the test results, he writes (pp. 166-169): "Six of the six-in thick tensile specimens have been tested....The pertinent information for each specimen is presented...along with data from supplementary tests of scale model specimens. All specimens tested were obtained from the center 6-in of a 12-in thick A533 grade B class 1 plate with the longitudinal axis of the specimen parallel to the rolling direction. A cursory inspection of the data will reveal both an obvious temperature effect and a size effect. However, several anomalies exist which make it difficult to correlate the

data in a manner which adequately describes these effects at the present time....Regardless of the anomalies in the first test series of 6-in tensile specimens, one may note several interesting points. One point is that the temperature transition effect is very sharp in the large specimens. In fact, it may well turn out that the difference is toughness between specimens...may be within the normal scatter of data obtained using small specimens.... Perhaps the most significant trend in the data is the notable reduction in ductility as specimen size is increased for specimens tested at 200°F. In conclusion, it is believed that after some additional investigation of the material properties of the six large tensile specimens and perhaps one or two additional tests of six-inch specimens in the temperature transition range, it will be possible to derive a means of relating specimen size to ductility and the shift in transition temperature. Such a relationship could be used to predict the behavior of heavy section structures based on small specimen data, at least in sections with homogeneous material properties."

Hooper (1971) describes theoretical and experimental work on the failure of glass cylinders in diametral compression. According to conventional theory, failure initiates within the cylinder. For the case of point loads acting across a diameter of a thin disc, theoretically a constant tensile stress normal to the loading direction exists over the entire loaded diameter, except at the two loading points. For a load P per unit thickness compressing a disc of diameter D, this stress is given by $\sigma_x = 2P/\pi D (1)$. According to this theory, the failure load is directly proportional to both cylinder length and diameter for a given tensile strength. Experimental data, which the author gives in tabular and graphical form, show a limited proportionality, but the ratio of the slopes of the curves of average failure load against cylinder length for constant cylinder diameter and against cylinder diameter for constant cylinder length is not equal to the ratio of cylinder length to diameter. There is also a wide variation in the central tensile stress σ_c, as deduced from (1). This suggests the possibility of an alternative mode of failure. A closer look at the stresses in the contact region and an examination of failed specimens shows that fracture always initiates at the contact surface, not within the cylinder as predicted by conventional theory.

Kudryavtsev & Besman (1971) attempt to establish quantitative criteria for evaluating fatigue fracture formation conditions by studying different macro-, micro-, and submicroscopic features of their structure, using fatigue fractures in small samples of St. 3 and St. 45 steels containing different concentrators and having different diameters. Some samples had as many as four notches. The fracture occurred along one of these notches, while fatigue cracks developed in the others. The fractures and cracks that were produced were investigated visually and with the help of optical and electron microscopes. One of the main problems in analyzing fractures is to determine the load level of the broken part. During the analysis of fatigue fractures caused by bending loads, it is necessary to take into account not only the size of the prefracture zone, but also the configuration of the entire fracture as a whole, while looking for the most hazardous portion of the sample at the prefracture moment with respect to the external load. This position is reached when the moment of resistance of the section containing the crack takes on a minimum value and the stress at the most heavily loaded point in the notched cross section reaches its maximum. For samples belonging to the same series, this stress is close to some constant value, regardless of the level of the stresses imposed at the beginning of the loading and the acuteness of the original concentrator. Under these circumstances the authors discovered that the sample's cross sectional dimensions has a considerable effect on the prefracture stress: as the sample's diameter increases, the prefracture stress converges on some maximum value. For St. 45 and St. 3, this size effect is manifested for sample diameters of less than 15 mm and 35 mm, respectively, the prefracture stresses for larger diameters coinciding, for all practical purposes, with the actual stresses at the moment of fracture in tensile tests (83.5 kg/mm^2 and 72.5 kg/mm^2, respectively). The authors hypothesize that it is precisely this characteristic--the value of the prefracture stress--as determined for large samples, that is most closely related to the material's resistance to direct tension.

Sandorff & Sandifer (1971) write (Abstract, p. iii): "Constant amplitude axial load fatigue tests at a stress range ratio of R = 0.02 were conducted on various sizes of unnotched sheet specimens of Ti-6Al-4V titanium alloy and 2024-T3 aluminum alloy, to evaluate size effects and to determine a minimum size suitable for orbital fatigue test work in the Skylab spacecraft....Strong

size effects were found at stress levels below the proportional limit, which were in accordance with the statistical theory of strength ['weakest link' theory--see McClintock (1955]."

Shah & Vijaya Rangan (1971) write (Abstract, p. 126): "Mechanical properties of concrete and mortar reinforced with randomly distributed smooth steel fibers were investigated to understand the mechanism of fiber reinforcing. Different volumes, lengths, orientations and types of fibers were used. Fibers were compared with conventional reinforcements in flexure, tension and compression. It was observed that the significant reinforcing effect of fibers is derived after the cracks are initiated in the matrix, just as with conventional tensile and stirrup reinforcement. The post-cracking resistance of fibers is considerably influenced by their lengths, orientation and stress-strain relationship. The spacing of reinforcement appears to have little influence on crack propagation below a certain length which in this investigation was about 1 in. The reinforcing action of fibers was analytically predicted by using the composite materials approach based on the properties of individual components."

Shur (1971) proposes a comparatively simple statistical model of a solid that makes it possible to appraise the danger of failure depending on the type of stressed state, which is not possible for statistical theories of strength based, as most such theories are, on the model of a solid as proposed by Weibull (1939a). On the basis of Shur's model, which considers a polycrystalline solid as a manifold of "physical points" [Volkov (1960)], it is possible to obtain not only all the usual qualitative results for statistical theories of strength, but also some new results. In particular, it is possible to predict the existence of a dependence of the dispersion of strength properties not only on the absolute dimensions of bodies, but also on the form of the stressed state.

Steverding (1971) writes (Summary, p. 342): "A relationship between size and fracture probability for metals as well as ceramics [see Steverding & Lehnigk (1970)] is developed. This dynamic scale-up relationship is formally identical with Weibull's law. However, it is not a consequence of the statistics of flow [sic; flaw] distribution but follows from a least action law for dynamic fracture. For Hookean bodies this law can be derived from

theoretical concepts. For metals, an empirical relationship is used to arrive at similar results."

Vorlíček (1971a) analyzes the strength σ of a brittle body according to the statistical theory of the "weakest link." He characterizes the probability function of the strength of a body with volume V in a uniform stress state by the Weibull cumulative distribution function $P(\sigma) = 1-\exp[-\beta V(\sigma-\sigma_0)^{\alpha}]$ for $\sigma \geq \sigma_0$. He expresses the three parameters σ_0, βV and α as functions of the mean $\bar{\sigma}_V$, the standard deviation s_V and the coefficient of skewness a, and then proceeds to estimate them by the method of moments. He extends the probability function $P(\sigma)$ to the case of nonuniform states of stress in which the dependence of the stress $\sigma(x)$ on the coordinate x is piecewise linear or parabolic, and derives approximate relations between the Weibull distribution parameters and the moment constants $\bar{\sigma}_V$, s_V and a. The relation between the skewness a and the shape parameter α is the same in every case, viz., $a = \{\Gamma[\alpha+3)/\alpha]-3\Gamma[(\alpha+2)/\alpha]\Gamma[(\alpha+1)/\alpha]+2\Gamma^3[(\alpha+1)/\alpha]\}/\{\Gamma[(\alpha+2)/\alpha]-\Gamma^2[(\alpha+1/\alpha]\}^{3/2}$, where the symbol Γ designates the Gamma function.

Vorlíček (1971b) considers the prediction of failure probabilities for a body composed of brittle elements connected in parallel. He assumes that the strength follows a Weibull distribution. He takes account of the number of elements (or the volume) and considers various stress fields ranging from uniform to concave or convex cubical parabolas. He makes calculations showing, for a particular element strength distribution, the effects of the number of elements and the stress distribution on mean strength and on lower 1- and 0.1-percent confidence limits, and tabulates the results. Since exact calculations are complicated for the parallel case, he makes use of the fact that strength statistics reach asymptotic values for infinite numbers of elements to evolve an approximate method which uses a correction factor to relate the parallel element case to the simpler series element case. He applies the theory to provide a method for relating the strength distributions of bodies with different volumes and stress distributions (scaling effects) for series elements, and uses the approximate analysis to extend the results to parallel element systems.

Witt & Berggren (1971) investigate the effect of size on the energy-absorbing capacity (and hence on the strength) of impact specimens of ASTM A 533, grade B, class 1 steel. They write (Abstract, p. 193): "A series of

impact-type specimens, ranging in thickness from 0.1 to 12 in., have been tested. The effects of size on the impact energy per unit volume of the plastically deformed material of the specimens tested are investigated using the laws of similitude....It is shown that significant size effects on impact energy exist at all temperatures and, in particular, a size effect of around five exists even at upper-shelf temperature for 12-in.-thick impact specimens, say a Charpy impact specimen. This is to say that, on the upper shelf, the impact energy of a 12-in.-thick specimen is equivalent to between 25 and 30 ft-lb Charpy impact energy. To lend further support to the behavior thus defined, it is shown that the nil-ductility type behavior in 12-in. thicknesses is exhibited at around 145°F as compared to the similar behavior exhibited by regular drop-weight specimens at around 0°F. That is, the nil-ductility temperature, if redefined as a type of behavior and not confined to a given size specimen for 12-in.-thick plate, is about 145°F."

Anthony (1972) discusses, in closing remarks, a colloquium in which several papers are relevant to the size effect on material strength and are summarized in the sequel, while several others deal with extreme-value theory and are included in the bibliography on that subject given at the end of this report. He writes (p. 528): "...A number of points seemed to stand out: 1. The fracture of composites and the fatigue of both metals and composites appears to be statistical in nature. 2. Of prime importance in statistically defining material behavior is the low probability of failure tail of the strength distribution. ..." He proposes an approach to the definition of the low strength tail which involves the use of the three-parameter Weibull distribution or some other distribution allowing a non-zero threshold strength. On a subject more directly relevant to the size effect on the strength of composite materials, he writes (pp. 531-532): "One last observation might be made regarding the overall approach discussed in several of the papers concerned with composite materials and in my own remarks above. The test data have been fitted with statistical models that essentially assume series behavior. In the case of the data presented by Waddoups and Robinson, good fits with the experimental data were obtained. However, it has been shown by many investigators that composite material assemblies, even fiber bundles, contain multiple load paths. Therefore, despite the good fits obtained

with a series probability model, one cannot help being somewhat apprehensive, particularly as small specimen data is utilized for large assemblies. Fortunately, scaling based on a series statistical model tends to be conservative if the real situation involves parallel behavior. However, unduly large conservatisms may result in serious penalties."

Buch (1972) gives the results of a survey of the literature (32 references) on test results and analyses of the effect of the size of specimen (unnotched) on the fatigue strength in the case of fatigue tests under reversed bending-, reversed twisting- and reversed tension-compression stresses; the maximum value of the plain geometrical effect of size, effect of the surface hardening, the surface roughness and the statistical material defects; and estimation of the fatigue strength of structural components. The test results show that rotating-beam fatigue strength decreases first rather rapidly with increasing diameter of the specimen, then more slowly for larger diameters. The various authors who have analyzed the data do not agree as to whether the decrease is purely a geometric size effect, a surface-condition effect, a stress-distribution effect, or a material characteristic. A size effect is also present in torsional fatigue.

Cruse, Konish & Waszczak (1972) attempt to adapt metals technology [in particular, linear elastic fracture mechanics (LEFM)] for use with composite materials. They point out that recent work has shown that scaling of failure loads for geometrically similar coupons, which can be done for metals, is not always possible for composite materials. They state that the development of sufficient experimental data to correct for size effects should improve prediction capabilities, though conservative strength estimates can be and have been made without such corrections. They write (p. 227): "At least three characteristic material lengths may be important in fracture testing of composites: Ply thickness, fiber diameter, and the size of a zone of initial damage. Testing to date has not been directed toward the effects of ply (or laminate) thickness; however, some observations may be made concerning the other characteristic lengths of interest. The fiber size is important because the stresses are characterized by K (the stress intensity factor) in only a small portion of the specimen, the elastic singularity region. For LEFM to apply to composites, this elastic singu-

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larity region must be large enough that K characterizes the stresses in 'many' fibers. While the exact value of 'many' is not known, this requirement is clearly a lower bound, in terms of fiber diameters, on the acceptable size of the elastic singularity region. However, the size of this region is proportional to crack length..., so a lower bound on acceptable crack lengths in a composite fracture specimen may be stated in terms of fiber diameter. Another characteristic dimension of interest in composite fracture specimens is the size of the 'initial damage' zone, a region surrounding the crack-tip in which...large stresses...cause material damage prior to crack growth. As in the somewhat analogous plastic zone in metals, linear elasticity is not valid within the damage zone in composites. Thus, LEFM may be applied to composites only if the damage zone is not excessively large. At present, no means of restricting the size of the initial damage zone by varying specimen dimensions, analogous to the procedures used to control the size of the plastic zone in metals, is known for composites. However,...the size of the damage zone correlates quite well with the size of that portion of the elastic singularity region in which the stresses...exceed fiber and/or matrix strength in the individual plies. This correlation may provide a means of analytically predicting the size of the initial damage zone; consequently, the present inability to control the size of the initial damage zone via specimen dimensions may be less critical than it first appears to be."

Das (1972) writes (Abstract, p. 783): "A theoretical analysis and practical verification of Vicker's hardness formula in relation to applied load and the size of the diagonal of impression is made for a number of coal samples. Vicker's hardness (H_V) is found to be related to the length of the diagonal of impression (d) by the equation $H_V = C \cdot d^{n-2}$ where C and n are constants (n is around...2, i.e. 2.32 and 1.81). The value of $n<2$ is more frequent and is also in agreement with the concept $H_V \to \infty$ for point impression at low load to a definite value of H_V for a measurable impression at high load. The value of n around 2 shows that by suitable rectification of the formulation of Vicker's hardness to correspond to reality, H_V may be rendered independent of applied load." These results on the size effect on hardness are of interest because of the analogy that is known to exist between hardness and brittle strength [see, for example, Auerbach (1891, 1896) and Frenkel´ (1948)].

Eisenman, Kaminski, Reed & Wilkins (1972) write (Introduction, pp. 359-360): "Up to the present time, design with composite materials has generally followed the conventional rules developed over the past forty years for metallic structures. ...Two aspects of design in composite materials are not addressed at all by the conventional approach: scaling and complexity. Geometric scaling (changing all dimensions by a constant factor) results in changes in mean static strength as well as mean lifetime in bonded joints... . Complexity, such as the number of holes in a component, or number of airplanes in a fleet, has pronounced effect on the time to first failure of the weakest member of the population... . Observations of this nature coupled with actual design experience for composite elements and components using conventional design approaches have proved to us that a more appropriate set of design rules is required for composite structure." They recommend a design procedure based on a characterization of loads and material or element behavior. This procedure involves the establishment of a reliability goal for a individual vehicle or the entire fleet and apportioning this goal to the components of the vehicle or the planes in the fleet. The authors note that the effect of complexity on establishment of a reliability goal must be considered in apportionment. They write (p. 376): "When a single stress concentration is placed in a laminate, the distribution of strengths is shifted to the left by a factor determined by the magnitude of the stress concentration. If a number of identical, independent stress concentrations are placed in a laminate and stressed equally, the distribution of strengths is shifted further to the left. This is a result of the linear stress-strain behavior of the material in that failure of the multiple hole component occurs when the weakest hole fails. Thus the mean value of the multiple hole component strengths lies in the vicinity of the weakest member of the single hole strength distribution. An analogous effect is seen in the time-to-first-failure in fatigue of single-hole and multiple-hole aluminum coupons... ."

Ekvall, Brussat, Liu & Creager (1972) present design criteria and analysis procedures for potential fracture resistant aircraft structures. They include graphs showing the effect of panel length on crack tip stress intensity for an elastic analysis of center cracked panels and the effect of thickness on fracture toughness properties for structures containing one or more cracks. They also include, in both tabular and graphical form, data on the effect of thickness on fracture toughness of Ti-6Al-4V (mill annealed)

and of 7075-T6 aluminum, with transverse and with longitudinal grains. In the case of Ti-6Al-4V, the fracture toughness is somewhat greater for loading rate > 100 ksi/sec than for loading rate ≤100 ksi/sec. For both materials, it is greater for longitudinal than for transverse grain. The fracture toughness of Ti-6Al-4V increases as thickness increases up to about 0.12 inch, then decreases, while that of 7075-T6 aluminum is essentially constant for thicknesses of 0.05-0.12 inch, then decreases as thicknesses increase above about 0.12 inch.

Farkas (1972) considers the optimum design for bending and compares thin-walled beams with box (rectangular), tube (circular) and oval cross-sections. He writes (English summary, p. 388): "The oval cross-section investigated in this paper consists of two semi-circular flanges and of two vertical straight webs. The sizes of cross-sections are determined by means of the following conditions: criterion of minimum cross-sectional area, constraint of maximum bending stress, conditions of elastic local buckling of flange and of webs. It is supposed that the buckling of the semi-circular flange of an oval cross-section is similar to the buckling of a circular tube. The connections between the webs and the semi-circular flanges are recommended to stiffen with longitudinal ribs. The comparison shows that the oval section is more economical than the box [or tube] one."

Girenko, Kir'yan, Deinega & Koval'chuk (1972) write (p. 1171 of translation): "The choice of metallic materials with the use of the criteria of fracture mechanics can be effected, proceeding from their resistance to the beginning or propagation of a brittle fracture. Without completely negating the second approach, we nevertheless note that the first approach [preventing the start of a fracture] is more rational, and sometimes the only one applicable. ...With such an approach to the maintenance of safety of structures, we must take into account the effect of the so-called geometrical factors (dimensions, the configuration of the possible defects, dimensions of the cross section, etc.) and the local variations in the ductility of the material at the tips of the real defects." They proceed to investigate the fracture behavior of a number of different steels under a variety of conditions. They use two basic geometries of test specimens: (1) a finite plate containing (a) a machined notch and (b) a fatigue crack; (2) a plate similar

to the first except that the central portion contains a weld seam. They measure net stresses, gross stresses, and crack openings and plot them against length-to-width ratios. They also represent these same quantities graphically for a variety of thickness for each type of steel.

Hancox (1972) describes the use of a torsion apparatus, in which a solid rod specimen is subjected to a shear stress field only, to measure the shear modulus and strength of unidirectional carbon fiber reinforced plastics. He concludes that the shear modulus, but not the shear strength, can be measured accurately, using either square or circular cross-section specimens. He plots the shear modulus as a function of the specimen diameter for two specimens made from 60% of type 1 (high modulus) fibers treated by the Harwell process to give improved adhesion between resin and fiber. Both specimens were originally 1.27 cm in diameter, and they were reduced to 0.3 or 0.4 cm in steps, the shear modulus being measured each time. The modulus was found to be significantly lower for the largest diameter, but almost constant for the other diameters. The author suggests that the low results for the 1.27 cm diameter may not represent a true size effect, but that they may be due to a poor fiber distribution in the outer portions of the specimens.

Hoshiya (1972) notes that there have been extensive studies of the reliability of structures, taking account of the statistical nature of the problem, since it became evident that both structural resistance and external load are of random nature. He points out that the exact calculation of the reliability of redundant structures is a difficult problem, since the failure of a particular component does not necessarily result in the overall collapse of the structure. He proposes to replace the exact calculation by a Monte Carlo approximation, which he illustrates for a redundant cable system. He writes (pp. 147-148): "Consider a structural system of initial m components...--a parallel cable system with single load S. ...Consider the case where the component resistances are mutually independent but identically distributed random variables with lognormal distribution function and share equally the external load S with lognormal distribution. ...For a ductile system, the Monte Carlo experiment starts with the generation of S and the resistances R_i, i = 1,2,..., m of each component from the given probability distributions of S and R_i. ...Arrange the generated R_i in the ascending order. Then examine whether the member force S/m exceeds the resistances R_i for i =

1,2,..., m. If k component fail, the load, $S - \sum_{i=1}^{k} R_i$ is to be equally redistributed over the remaining components. Since the material is ductile, the failed k component still carry at least the maximum yielding capacities R_i, i = 1,2,..., k respectively. Thus each remaining component is now subject to the member force $(S - \sum_{i=1}^{k} R_i)/(m-k)$. Comparison between the remaining resistances and redistributed member forces is then made repeatedly until the end of all failure paths. Repeat the above experiment many times, keeping the tally of each failure path. Then, the probability of failure of i members can be obtained by n_i/N where n_i is the total number of failure [sic; number of failures] of i members out of N experiments. For a brittle system, failure occurs through the fracture and the failed components become inactive." The author tabulates the results of 2,000 trials each for ductile and for brittle systems with m = 2(2)14. He also considers two other examples: an indeterminate truss and a tower truss subjected to lateral joint loadings.

Kececioglu (1972) outlines a fifteen-step methodology for "design by reliability". One step is determination of the failure governing strength distribution, $f_1(S)$, which is defined as the distribution of the stress at failure. The procedure for determining $f_1(S)$ is as follows: 1. Establish the applicable failure governing strength criterion. 2. Determine the nominal strength. 3. Modify the nominal strength with appropriate strength factors to convert the nominal strength determined under idealized and standardized test conditions to that which will be exhibited by the component in the actual use geometry and environment it is designed for. 4. Determine the distribution of the nominal strength and of each strength modifying factor and parameter in the failure governing strength equation. 5. Synthesize these distributions into the failure governing strength distribution. As an example, the author considers a steel shaft designed for 2.5×10^6 cycles of life. Its strength distribution for stress ratio r is given by $f_1(S) = f_1[f_3(S_e), f_4(S_u), r]$, where $f_3(S_e)$ = endurance strength distribution at 2.5×10^6 cycles, $f_4(S_u)$ = static ultimate strength distribution of standardized specimens, and r = ratio of alternating failure governing stress to mean failure governing stress. The endurance strength is given by $S_e = k_a\, k_b\, k_c\, k_d \ldots S_e'$, where S_e' = endurance strength of a rotating beam fatigue specimen tested to failure under idealized, controlled laboratory conditions, k_a = surface finish factor, k_b = size factor, k_c = fatigue stress concentration

factor, k_d = temperature factor,... . No information is given as to how the size factor, k_b, should be determined. The strength distribution, $f_1(S)$, and the associated stress distribution, $f_2(s)$, may be of various types, including normal (Gaussian), lognormal, Weibull, and extreme value. When only data from strength tests conducted so that the weakest fail first are used to determine the strength distribution, the Weibull distribution best represents the data. The best distribution to use to represent each failure governing stress component should be the extreme value distribution of the maxima, because the highest stresses will cause failure and not the lowest.

Lenoe & Baratta (1972) write (p. 448): "In the commonly used Weibull form the probability of fracture can be expressed as $P_f = 1.0\text{-exp}\{-\int_{vol} [(\sigma-\sigma_u)/\sigma]^m du\}$, $\sigma \geq \sigma_u$; $P_f = 0$, $\sigma < \sigma_u$. The three distribution parameters m, σ_u, σ_0 are taken to be material constants which are supposedly independent of size or loading of the structural element. A number of researchers...have integrated the risk of rupture $R_W = \int_{vol}[(\sigma-\sigma_u)/\sigma_0]^m dv$ as well as the 'reduced' risk of rupture $R_W' = \int_{vol}(\sigma/\sigma_u)^m dv$[sic; σ_u should be replaced by σ_0] for simple loadings. Table 1 [not reproduced here] lists the results for both forms of the risk of rupture, where, for instance, we have introduced the constants C as follows, for a rectangular beam of height (h) [length L] and width (b). $R_W' = CV(\sigma_B/\sigma_0)^m = \int_0^{h/2} (\sigma_B 2y/\sigma_0 h)^m bLdy$. Using this information it is possible to compare results for different test loadings and volume of specimens." Various methods of estimating Weibull parameters are discussed.

Lifshitz & Rotem (1972) write (pp. 861-863): "Strength of unidirectional fibrous composites when a tensile load is applied in the fibre direction has been studied by several investigators. ...Most fibres used in high strength and high stiffness composites are brittle, and their strength can generally only be characterized statistically due to distribution of flaws and imperfections in the brittle fibres. ...Rosen [(1964)] presented a theoretical and experimental treatment of the failure of fibrous composites. ...The failure model introduced by Rosen...assumes statistical accumulation of fibre fractures with increasing load until a sufficient number of fractures occur at some cross sectional region of the composite, resulting in composite failure. The fracture process, thus, takes place at a single cross section of the specimen. Some composite materials do in fact fail in such a manner (e.g. some carbon/epoxy composites). Experimental results with glass

reinforced plastics (e.g., epoxy of polyester), however, show very complicated fracture surfaces... . The test specimen does not fail at a single cross section. Instead, bundles of fibres, grouped together, seem to fracture at different locations along the specimen, followed by new fractured surfaces running parallel to the fibres. ...It was therefore decided to propose a new failure model that agreed with the observed failure geometry. The proposed model is based on the cumulative weakening model of Rosen...but it is not assumed to fail in a single cross section. Size effect in brittle materials has been studied previously, and in particular the effect of length on the strength of glass fibres has been investigated by Metcalfe and Schmitz [(1964)]. It was found that strength of glass fibres exhibits a significant size effect. When glass fibres were used in a composite material the size effect is drastically reduced since failure of single filaments does not cause immediate failure of the composite. Investigation of size effect in unidirectional glass reinforced composites is rather difficult to conduct since the size effect is quite small. Some experimental results have been reported, however, showing a slight reduction in strength for longer specimens [McKee and Sines (1969)]. The results of the present analysis show that a small decrease in strength is expected when the size is increased by a few orders of magnitude. ...When some fibres fracture, Rosen...assumes that the load is distributed uniformly among the unbroken fibres in each layer. Increasing load produces an increasing number of fibre fractures. Composite failure occurs when the remaining unbroken fibres at the weakest layer are unable to sustain the applied load. The result of this analysis for fibres characterized by a strength distribution of the Weibull type [Weibull (1951)] is the definition of a statistical mode of the composite strength $\sigma_c^* = (\alpha\delta\beta e)^{-1/\beta}$ where α and β are constants describing fibre strength, δ is the thickness of a layer (or ineffective length) and e is the base of natural logarithms. When a unidirectional fibrous composite (glass/epoxy) is subjected to a slow strain rate test, in the fibre direction, it is observed that as the applied load approaches the value of the composite's strength, some groups of fibres (with the matrix material holding them together) break away from the specimen. (These groups of fibres are termed 'strands' in this report.)...The specimen is assumed to be composed of a series of strands connected in parallel, all having the same size. The length of a strand is equal to the

specimen's length and each one of the strands is divided into layers of length δ (the ineffective length). The analysis of a strand is identical to the analysis of Rosen's model... . Once this analysis has been performed and the statistical strength characteristics of the strands are known, the analysis of the new model continues by considering the specimen to be composed of a bundle of such strands and using the statistical theory developed by Daniels [(1945)] for calculating the strength of bundles."

Lundborg (1972) writes (Abstract, p. 617): "Weibull's statical theory of strength [used by Weibull (1939a) to explain the volume effect and polyaxial stress on the tensile strength of materials], extended to compressive stresses, is used in calculating the influence of the intermediate principal stress on the strength. The effective shear stress $\tau_e = |\tau_n| - \mu\sigma_n$ is calculated and integrated over the solid angle where $\tau_e > 0$, and the probability of rupture in polyaxial compression is calculated. The results are in good agreement with experimental results."

Markström (1972) writes (Abstract, p. 593): "The effect of thickness delamination on fracture toughness has been studied. The test material used was a martensitic-austenitic pressure vessel steel. Tests were performed on compact tension and three point bend specimens of varying sizes. All specimens were machined from plate material of 100 mm thickness. It was found that the ASTM practice of test evaluation leads to a size dependence of fracture toughness although data for all specimens fulfill the ASTM size requirements. This size dependence has been explained theoretically and it was shown that an evaluation method based on a fixed absolute amount of apparent crack extension would lead to coinciding data for all specimen sizes."

Munro & Adams (1972) write (Abstract, p. 705): "Maraging steel [200 ksi grade] containing 18 percent nickel offers apparent advantages of high strength, weldability, corrosion resistance and toughness. In view of the suitability of the material for bridge construction, a study of the fatigue crack growth and fracture properties was undertaken. A proposed bridge design contained hinged sections of 4 in. and 2 in. thickness..., and thin welded girders manufactured from plate material 0.18 in. thick. Plane strain fracture toughness tests were carried out on samples taken from the 4 in. and 2 in. sections using 3 point-bend and compact tension specimens. K_Q [provisional value of plane strain fracture toughness] values of approximately 75 ksi $\sqrt{\text{in.}}$

for the 4 in. samples and 110 ksi $\sqrt{\text{in.}}$ for the 2 in. samples were obtained. Plane stress fracture toughness tests were conducted on center cracked sheets up to 11 in. in width. Using critical crack lengths determined by compliance measurements, K_c [critical stress intensity factor] values in excess of 400 ksi $\sqrt{\text{in.}}$ have been obtained in specimens of up to 0.180 in. thickness. Fatigue crack growth rates were determined from these specimens prior to fracture testing. In both plane strain and plane stress failure modes [prevalent for thick sections and for thin plate girders, respectively], laboratory results are in agreement with those estimated from the failure of experimental structures." In their summary (p. 715), the authors write: "...Grain boundary precipitation can occur during air cooling of thick sections which will degrade the plane strain fracture toughness properties. ...Within the range of specimens tested, plane stress fracture toughness was found to be independent of specimen thickness but highly sensitive to specimen width. ..."

Murthy & Swartz (1972) review several publications concerning the application of extreme value theory to problems of fracture and fatigue, including two papers by Epstein (1948a,b) dealing specifically with the size effect on material strength.

Nelson, Schilling & Kaufman (1972) write (Abstract, p. 33): "The effects of specimen geometry, notably thickness and crack length, on the values of plane-strain fracture toughness, K_{Ic}, of five aluminum alloys have been determined. Notch-bend specimens of 2024-T851, 6061-T651, 7075-T7351 and 7079-T651 and compact tension specimens of 2219-T851 were tested... . Constant values of K_Q (candidate values of K_{Ic}) are generally obtained for specimen thicknesses and crack lengths greater than $2.5(K_{Ic}/\sigma_{YS})^2$, while for thinner specimens values above or below the true K_{Ic} value may be obtained, dependent upon other dimensions of the specimen. ..." In their summary and conclusions, the authors write (pp. 48-49): "...Several interesting features of the dependence of K_Q values on specimen size are indicated: (1) When viewed as a function of thickness, values of K_Q generally reach a nearly constant level for thicknesses equal to or greater than 2.5 times the square root of the ratio of K_{Ic} to the tensile yield strength of the material. For specimens with thicknesses which are less than this stated limit, and other dimensions proportional to the thickness..., values of K_Q generally increase with increasing specimen thickness. For specimens with other

dimensions not in proportion to the thickness, K_Q values did not follow a consistent pattern with respect to thickness. Thus, there does not appear to be any basis for reducing the thickness limitation in the ASTM method. When the specimen thickness requirement cannot be met, the use of relatively wide specimens will increase the likelihood of approximating the 'true' value of K_{Ic}. Data for one alloy and temper, 7075-T7351, like those published previously for 7178-T76, ...indicate that a specimen thickness of $2.5(K_{Ic}/\sigma_{YS})^2$ does not always assure constant K_{Ic} values. (2) For materials for which the load-deformation curve in tests of notch-bend or compact tension specimens rises through the region representing 2 percent of crack growth..., K_Q values increase slightly with increase in crack length, introducing some variability into the results. ..."

Obert writes (pp. 121-122): "It can be readily demonstrated that the uniaxial tensile strengths of specimens of a given rock type decrease as the dimensions of the specimens are increased. This observation has led to associating the strength of rock to the 'weakest link' theory. Related to rock, this theory assumes that the rock contains mechanical defects and, as the dimension of specimens is increased, the probability of including a weaker defect is increased; hence, the specimen's strength should decrease with the specimen size. Weibull (1939[a]) considered the size effect on a statistical basis, and Evans and Pomeroy (1958) evaluated the size effect by measuring the strength of cubical coal specimens. The 'weakest link' theory can be stated as follows: if a cube of side a contains r elements (defects) of the kind to which the weakest link theory applies, and P_0 is the probability of one of the r elements surviving a given stress, then the probability P_a of the cube surviving this stress is $P_a = P_0^r$ (27). Correspondingly, for a cube of side b containing s elements, the probability P_b of survival is $P_b = P_0^s$ (28). Thus, $P_b = f(P_a^{s/r})$ (29). As s/r should be related to some dimensional characteristic of the cube, i.e. $(a/b)^\beta$, Eq. (29) can be written $P_b = P_a^{(b/a)^\beta}$ (30) or $\beta \ln(b/a) = \ln(\ln P_b/\ln P_a)$ (31) where β will be 1 if the characteristic is related to length, and 3 if it is related to volume. Evans and Pomeroy (1958) found the value of β to be approximately 1 for cubical coal specimens; hence, it can be assumed that the characteristic that affects breakage is related to the length of the cube. As given, this theory is oversimplified, because each element or defect will not have the same

probability of survival; hence, a statistical distribution for the strength of the element should be considered. ...The weakest links have been identified with Griffith's cracks..., and, in this case, the distribution for the orientation of the cracks and the crack length should also be considered. Gaddy (1956) determined the crushing strength of cubical coal specimens of side a subjected to uniaxial compression and found them to be proportional to a $^{-\frac{1}{2}}$. On the other hand, Greenwald et al. (1939) measured the in situ crushing strength C_0 of approximately cubical coal pillars of cross-sectional dimension L and found that C_0 increased as $L^{\frac{1}{2}}$. Unfortunately, very little data are available from corresponding tests in rock (other than coal). Obert et al. (1946) reported very little difference in the compressive strengths of diamond-drill core specimens having a length-to-diameter ratio of unity for diameters ranging from 7/8 to 2-1/8 in."

Ohuchida, Nishioka & Nagao (1972) note that they previously found that size effect on fatigue could be detected in annealed carbon steel by means of X-rays and that fatigue damage in a corrosive environment could also be detected by X-rays. In the present paper, they report the results of an attempt to detect the effect of size on the fatigue strength of machined steel specimens with the help of the X-ray method. They write (Synopsis, p. 100): "X-ray observations were conducted on rotating bending fatigue test specimens having diameters of 100 mm, 20 mm and 10 mm. It was found that changes in residual surface stress depend only on the fatigue life, irrespective of specimen size and surface finishing condition. X-ray data obtained from small size specimens can be applied to the evaluation of large machine parts. The X-ray diffraction method is considered to be useful for direct nondestructive fatigue damage detection." They plot the S-N curves for fatigue tests of various specimens, and write (p. 103): "From these results, it is evident that the fatigue strength (at 10^7 cycles) varies with the size: while the fatigue strength for the 10 mm ϕ specimen was $\sigma_W = 26.5$ kg/mm^2, the fatigue strength for the 20 mm ϕ specimen decreased by 3.8% [to 25.5 kg/mm^2] and for the 100 mm ϕ specimen by 17% [to 22.0 kg/mm^2] compared with the 10 mm ϕ specimen. The fatigue strength of these specimens was only 1 ~ 2 kg/mm^2 higher than that of annealed specimens, as reported previously... . There was only a slight difference between the strength of normally machined specimens and that of lightly machined ones."

Parratt (1972), in Chapter 2 (The Pursuit of High Strength), writes (pp. 28-29): "One way of avoiding many of the defects which cause real strengths to fall short of the theoretical is to use fine fibres of single crystals or glasses. This minimizes the incidence of gross defects, and unidirectional stressing can avoid the weakest planes in a crystal if these are parallel to the fibre axis. Here one should expect to achieve high strengths, up to 10% of the moduli. Table 2.2 [not reproduced here] shows the extent to which this has been done. ...Technologically, metallic whiskers are not as important as ceramic whiskers and so there is less information available about them, but early studies suggested an equivalent level of strength. ...Strength levels of 3% E[E = Young's modulus] are found in the case of asbestos fibres carefully prepared from the rock, whereas silica glass fibres remain an anomaly, attaining 20% E when tested at low temperatures." He adds (p. 33): "Some varieties of asbestos show a slight size effect (i.e., bigger fibres have bigger defects) but this is much more severe in crystals of more isotropic materials." In Chapter 3 (Mechanics of Reinforcement), he writes (pp. 53-54): "The knowledge that most reinforcing fibres, long or short, possess weak spots along their length, has worried a number of workers for some years. Because of these defects, most fibres show a pronounced length-strength effect; the shorter they are, the stronger they are. At the same time the strength of a bundle or group of fibres will be less than the average fibre strength, since immediately before failure of a bundle, a number of fibres are already broken, so that the stress on the remainder is increased. Coleman [(1958)] evaluated bundle strength for different amounts of strength scatter. Thus, from a knowledge of the length-strength effect and the corresponding bundle strengths, one would propose a crude theory for maximum reinforcing strength...by defining a critical length at which fibre bundles broke and relaxed their stress completely... . Rosen [(1964)] treated this idea mathematically, retaining the assumption of random fibre breaks but allowing very short fibres to continue to carry small loads... .Riley [(1966)] proposed a simple graphical method of predicting effective reinforcing strength and basing it on the same assumptions. However, Rosen's experimental model gave lower effective strengths than predicted and final fracture was due to associated groups of broken fibres. Riley had also noted this effect in discontinuous composites by using a massive model in which

fibres were fitted with strain gauges. He found the additional stress-raising factor in fibres adjoining a broken one to be the reciprocal of the number of surrounding fibres. A typical co-ordination number for fibres is five, and gives a stress-raising factor of 1.2. Meanwhile, Wadsworth [& Spilling (1968)] had explained this apparent ambiguity when observing carbon fibres imbedded in a resin. Fibre fractures in a well-adhering resin matrix eventually cause adjoining fibres to fail, but poor adhesion allowed the extra load from a broken fibre to be transferred over a greater length through the surroundings. In this way, the load was spread over many other fibres. ...A paper by Zweben [(1968)] modified Rosen's original theory to take account of local multiple fractures with good adhesion; in summarizing the experimental evidence, Zweben also noted the existence of some inefficient composites in which the first fibre failure may be catastrophic. ...This multiple fracture theory also predicts a size effect; i.e., larger bundles will be weaker. The effect may be offset in practice by subdividing the bundles and making a component from a number of small bundles."

Phillips (1972), in discussing the general fracture behavior of glass, notes that specially prepared silica glass fibers have exhibited tensile strengths exceeding two million psi, which are higher than for most materials, and that strength generally increases as the size of the area tested becomes smaller. Because of the strength of their chemical bonds, silicate glasses should theoretically be very strong. In order to account for observed strengths which are typically much lower than the theoretical values, Griffith (1920) postulated that all glasses contain numerous minute flaws, in the form of microcracks, which act as stress concentrators. Concerning statistical failure theories, Phillips writes (p. 17): "The flaw theory has led to a number of statistical theories, all based on the reasoning that 'a chain is only as strong as its weakest link.' The best known is that of Weibull (1939[a]), and this theory and others have been compared and summarized by Epstein (1948[a]). All predict certain results which often appear to be true, but they fail in other ways... . Because flaws of widely varying severity may be present in supposedly identical specimens, the statistical theories predict a wide range of breaking stresses, and this is found to be the case, at least for commercial samples or for strengths not above several hundred thousand pounds per square inch. For the same reason, average

strength should, in general, decrease as the area under load decreases. Thus, all other things being equal, glass tested in straight tension should be weaker than in flexure, and short glass samples should be stronger than long ones. Small-diameter fibers should be stronger than rods. There is a tremendous mass of evidence, accumulated over the past 40 years, that these predictions are correct, again at commercial and somewhat higher levels." He adds, however (p. 32): "Statistical theories based on the presence of numerous flaws can readily explain the wide scatter usually observed in strength measurements. They can also explain the normally pronounced effect of the area under maximum or near maximum stress. In this case, they correctly predict that, as area increases, average strength decreases. However, they can offer no plausible explanation for the observed fact that, for very small, very strong specimens, such as fibers, strength is independent of loaded area. On the contrary, they incorrectly predict that dispersion must increase as the median failure strength increases, whereas, with fibers, precisely the opposite effect is observed. Statistical theories, as presently constituted, must be used with caution."

Phoenix & Sexsmith (1972) write (Abstract, p. 322): "A probabilistic model for the tensile strength of brittle fibers is presented and related to present conventional testing. The relationship of fiber flaw structure to the statistical strength [as a function of length and diameter] is discussed. The probabilistic effects of the clamp on fiber tensile testing at various gauge lengths are treated. Examples using Weibull flaw structure and simulated clamp conditions are provided to demonstrate the marked effects of the clamps on expected tensile test results at 'short' gauge lengths. It is shown how the Weibull strength distribution can be modified to include clamp effects when used in conjunction with tensile test data. The effect of discarding the data from fibers that fail within the clamp is also examined. Suggestions for future testing are made."

Randall & Merkle (1972) report the results of an experimental study of the effects of crack size and specimen size on the gross-strain crack tolerance of A-533-B steel. They write (Abstract, p. 935): "Gross-strain crack tolerance is a measure of fracture toughness that is useful where yielding precedes fracture yet flaw size must be treated quantitatively. In these tests, rectangular bars containing surface fatigue cracks were pulled in

tension at several temperatures. Strains were measured at the edges of the specimen on a gage length spanning the plane of the crack, and also in the gross section above and below the crack. Gross-strain crack tolerance is the strain at maximum load on the gage length designated. It is shown to be a function of crack size) constraint, and temperature." In summarizing earlier results and the study to be reported, they write (pp. 936-937): "The initial test objective was to measure the gross-strain crack tolerance of the material as a function of crack size, temperature, and constraint, for strains ranging from 0.2-2.0 percent. However, the initial test program...revealed that the plot of critical strain values versus temperature exhibited a sharp transition in which the strain increased from less than 1.0 percent to 5-10 percent between two tests that were only a few degrees apart. Consequently, our test results will be given in terms of a transition temperature at which a certain strain was achieved, as well as in terms of the strains that were tolerated at a given temperature in the presence of cracks of certain sizes. ...With regard to test-specimen design, the situation is as follows. The study of the effects of specimen width and thickness, relative to crack size, revealed two constraint effects. Proximity of the back-face free surface to the crack tip lowered the transition temperature when the ratio B/a [B = specimen thickness, a = crack depth] fell below 2.5. Reduction of width relative to crack-length raised the transition temperature, apparently because of net-section effect, an inhibition of gross-section yielding when stress in the gross section was much less than the net stress. We see now that the tests for proximity effects should have been run at a constant ratio of net to gross area. In a practical test specimen, this ratio will be 0.90 to 0.94. Use of the data in design can be made with the knowledge that the full-scale part should exhibit a transition lower than for test specimens with the same flaw size. How much lower? Some test results bearing on this question are to be reported. ...Test results for 0.10-in-deep cracks in both large and small bars will be reported in answer to this question of net section effects." Later in the paper (p. 939), the authors report a net-section effect of about 20°F (transition temperature 20° higher for the small specimen than for the large one with the same crack size), with reduced scatter in the small specimen.

Kameswara Rao & Sridhar Rao (1972) write (p. 55): "A phenomenon normally observed with testing of concrete is a decreased mean strength with increased specimen size. Further the related scatter in strength values also decreases with increased specimen size. For further developments in the paper an expression to characterize the scatter associated with strength values as dependent on the size of test specimens is required. The earliest attempt towards a solution of this problem is due to Weibull [(1939a)], who showed that the probability of failure as dependent on size (length, area or volume) could be given by the following distribution function: $\Phi^*(\sigma) = 1\text{-}\exp\{-V[(\sigma - \sigma_u)/\sigma_L]^m\}$, $\sigma > \sigma_u$; $= 0$, $\sigma \leq \sigma_u$ (1) where σ is the applied stress, σ_L and m are material parameters and σ_u is the lowest mean strength. It was shown recently by the authors...that a suitable form of failure distribution function can be constructed for any size-mean strength relation obtained from tests. The most general form of the distribution function $\Phi^*(\sigma)$ can be of the following type: $\Phi^*(\sigma) = 1\text{-}\exp[-Vf(\sigma)]$ (2) where volume V characterizes the size and $f(\sigma)$ is such a function that it increases with σ and satisfies suitable conditions to make $\Phi^*(\sigma)$ a cumulative distribution function. It could be observed from Eq. (2), with such a form of $f(\sigma)$, the probability of failure increases with increase in applied stress for a given specimen size and the probability of failure increases with specimen side for a given applied stress. Besides, any number of material constants can be accommodated in $f(\sigma)$."

Robinson (1972) writes (pp. 13-17 of JPL Memorandum): "The conventional form of the so-called two-parameter Weibull distribution is $S = \exp[-\int_V(\sigma/\sigma_0)^m dV]$ (8) where S is the probability of survival, σ is an arbitrary applied stress in a volume V, and m and σ are parameters. To see the implication of this formula, consider a specimen under uniform aniaxial [sic; uniaxial] tension. The specimen is assumed to fail according to the weakest link model; that is, upon the initiation of fracture anywhere, gross failure occurs. If the stress is everywhere the same, Eq. (8) becomes $S = \exp[-(\sigma/\sigma_0)^m V]$ (9). ...Large specimens, just as the long chain is weaker than the short one. If specimens of sizes V_1 and V_2 are tensile-tested and are characterized by similar values of m and σ_0, Eq. (9) may be used to compare the strength at the same probability of failure (when m and σ_0 are the same this also compares the mean strengths of both volumes), leading to $\sigma_1{}^m V_1 = \sigma_2{}^m V_2$, and the well-known

strength size effect $\sigma_1/\sigma_2 = (V_2/V_1)^{1/m}$ (11). This equation immediately suggests a method for estimating m. Rewriting it in logarithmic form gives $\log(\sigma_1/\sigma_2) = (1/m)\log(V_2/V_1)$, and a plot of strength ratios vs volume ratios on logarithmic coordinates should yield a straight line of slope 1/m if the Weibull distribution is appropriate. ...We can express this size effect in a general way (for constant m and σ_0) as follows. The applied stress is written in terms of a reference stress (say the maximum tensile stress) and a geometric function which describes the stress distribution..., that is, $\sigma_i[f_i(V)]$. The exponential terms of Eq. (8) for two arbitrary cases being compared become $\sigma_1^m \int_{V_1} [f_1(V)]^m dV = \sigma_2^m \int_{V_2} [f_2(V)]^m dV$, and the general expression for size effect becomes $\sigma_1/\sigma_2 = \{\int_{V_2} [f_2(V)]^m dV / \int_{V_1} [f_1(V)]^m dV\}^{1/m}$ (13). So far the flaws in the material have been tacitly assumed to be distributed throughout the volume and the integrations have been carried throughout the specimen volume. The strength-controlling flaws may, however, be confined in many cases to the surface (e.g. in glass) or even to the edges, and along the length of reinforcing fibers. For these cases the form of the stress distribution and the region of integration are different. ...Equation (13) was used to calculate the size effects in tension, simple bending, and uniform bending for flaws distributed throughout the volume, area, or length. The resulting formulas are collected in Table 2 [not reproduced here]. The size effect helps to explain why, even in brittle materials, the influence of stress concentrations, at holes or notches, is not as great as would be expected from the theoretical stress concentration factor and a maximum stress failure criterion. The large theoretical stresses are operative in very small volumes within which the probability of finding weakness is greatly diminished."

Rosen & Dow (1972) begin their review of the mechanics of failure of fibrous composites with a characterization of filaments. They write (pp. 613-614): "Perhaps the only common characteristic of the various filaments used for composites is their small diameter. 'Small' here is a loose term; by and large, it means less than 0.13 mm (0.005 in.). The reasons for small diameters are several. First, of course, is the important one that many materials--glass being a prime example--exhibit extraordinary strengths in the form of fine filaments. Historically, the determination of this size effect is generally attributed to Griffith (1920)... . It is not clear that the subsequent development of fiber glass was motivated by his work, however.

The high strengths attained in fine glass fibers may be only a happy by-product of the development of an article of manufacture. Subsequent developments of filamentary material, on the other hand, have been specifically directed toward the utilization--or at least have been cognizant of the importance--of the size effect for obtaining high-strength characteristics. Thus, we find that all current filamentary composites comprise filaments of small diameters, whatever the material. We find boron filaments of diameters of 0.10 to 0.13 mm (0.004 to 0.005 in.); beryllium filaments of approximately the same diameters; carbon and glass filaments, however, run to one-tenth of the above diameters." Concerning the strength of filaments, they write (p. 620): "Unlike the familiar structural metals, fine filaments cannot be characterized by a single number representative of the 'ultimate tensile strength'. Rather, they are characterized by a statistical variation of strengths, accountable on the basis that flaws of various magnitudes occur along the length of the filament, that failure occurs at the biggest flaw, and so the strength is a function of the probability of a flaw of that size being located within the test length. Thus, one would expect that the longer the test length, the greater the probability of encounter of a large flaw, and hence the lower the test strength. Such is indeed the case." After a characterization of matrix materials they discuss failure modes. They write (pp. 625-626): "The failure analyses that will be brought forth herein in general begin with a consideration of the strength of a unidirectionally reinforced lamina subjected to tension, compression, or shear loads, and then build up to laminates having their reinforcements in a multiplicity of directions and subjected to combinations of loads. ...For the unidirectionally reinforced lamina, [tensile] strength is determined along and transverse to the reinforcing filaments. In the direction of the filaments, the failure is developed as an accumulation of fractures in the filaments according to the statistical approach of Rosen (1964) until the material is incapable of further redistributions of loads to undamaged portions of filaments. Fracture transverse to the filaments is indicated to take place in the matrix independently of the reinforcement. For the laminate, stress components along and across the filaments in each lamina are found and related to the strength found for the individual laminae. Contributions

of 'failed' laminae to the overall strength are found by a limiting-type analysis, and progressive failures of laminae are noted until overall failure is encountered." They note that the tensile strength of individual fibers may be either uniform (represented by the Dirac delta function) or may follow a statistical distribution. For brittle fibers, they assume a Weibull distribution of the form $f(\sigma) = L\,\alpha\sigma^{\beta-1}\exp(-L\alpha\sigma^{\beta})$ where α,β are two parameters characterizing the distribution function, L is the fiber length, and σ is a fiber stress. For brittle-fiber composites under tension, they discuss the models of Parratt (1960), Gücer & Gurland (1962), Rosen (1964) and Fridman (1967) and use the latter to develop a strength theory. They use the results of Daniels (1945) to develop a model for cumulative damage failure which yields a measure of the potential for fiber composite strength. They also develop the theory for strength under shear, compression and combined loads, and compare the results of theoretical and experimental evaluations.

Serensen, Strelyaev & Bolotnikov (1972) present theoretical and experimental results on the determination of rated strength characteristics of fiberglass in zones of stress concentration. They write (pp. 1154-1155 of translation): "According to experimental data, the failure of fiberglass with stress concentration is chiefly brittle. This indirectly points to the absence of leveling-off of the stress curve with increased load because of inelastic deformation. However, due to progressive failure under static loading, the effective coefficients of stress concentration, $k_\sigma = \sigma_{t_u}/\sigma_{t_c}$, where σ_{t_u} and σ_{t_c} are the tensile strengths under nominal stresses for specimens without stress concentration and with stress concentration, are less than the theoretical values... . In analyzing the resulting data in published papers [see, for example, Serensen & Strelyaev (1962)], authors have not distinguished the effect of absolute sizes and the inhomogeneity factor of the stress state. Taking into account the fact that the effect of absolute sizes and the effect of stress concentration have statistical characters, the further treatment of the effect of stress concentration is considered in view of the irregularity of stress distribution. ...For quantitative determination of the conditions of failure in fiberglass as a quasibrittle material, we used the hypothesis of the weak

link in the formulation of Weibull [(1939a)]. The distribution function of maximum rupture stresses of links of material may be obtained if we calculate the integral of the probability of rupture in the zone of stress concentration: $P(\sigma_{max}) = 1-\exp\{-(1/v_o)\int[(\sigma-u)/\sigma_o]^m dv\}$, where $\sigma = \sigma_{max}$ $f(y)[f(y)$ is a dimensionless function of the coordinate y, deriving from the elastic distribution of stress in the cross section]; m, σ_o, and u are the distribution parameters of Weibull, characterizing the distribution function of static strength of fiberglass considered as a quasihomogeneous medium...; v_o is a characteristic volume being stressed; and dv is an element of the volume being stressed." They note (p. 1159 of translation) that experimental data indicate that the value of u may be taken as zero, and that (Cramer-von Mises) goodness-of-fit tests show that the normal distribution of maximum rupture stresses may also be adopted for smoothing experimental data. They assert that for m $\leq$ 10, characteristic for fiberglass, the Weibull and normal distributions differ insignificantly.

Shinozuka (1972a) discusses reliability-based optimum design involving the quasistatic approach to structural analysis. He presents a method combining the concept of reliability analysis with that of the minimum weight design to achieve an optimum trade-off between safety and cost. He defines structural failure in accordance with the "weakest-link" hypothesis, which makes the probability of failure dependent on the size of the structure or the number of components.

Shinozuka (1972b) writes (pp. 1434-1435): "Among a number of possible methods to deal with the statistical scatter of material strength, the most satisfactory approach appears to be the one in which the strength at each point of the material is assumed to have a statistical distribution. The statistical size effect then follows logically from the weakest link hypothesis. The effect of statistical anisotrophy is disregarded for simplicity at this time, and the statistical scatter of X_0, a measure of the strength of a hypothetical reference volume, dv, of microscopic magnitude is considered. The quantity, X_0, may then be interpreted as the strength at a point within the material. Due to the weakest link hypothesis, if X_n denotes the strength of an aggregate of the material consisting of n such microscopic volume elements, it has to

be the minimum among the strengths of these volume elements constituting the aggregate. Assuming that these strengths are identically and independently distributed with the distribution function $F_{X_0}(a)$, the distribution function $F_{X_n}(a)$ of $X_n = F_{X_n}(a) = 1-[1-F_{X_0}(a)]^n$ (1) in which, by definition, the distribution function $F_X(a)$ of a random variable X = the probability that X will be $\leq$ a. As n increases ndv = v becomes finite and X_v, written for X_n, represent the strength of a piece of material of finite volume v. It is usually postulated that the distribution function of X_v approaches the Weibull distribution, one of the asymptotic distribution functions of smallest values, of the following form: $F_{X_v}(a) = 1-\exp[-v(a/A)^k]$ (2) in which k and A = positive constants depending only upon the material property. The analytical form of Eq. (2) is such that the distribution function of the strength associated with volume v_1 is located to the left of that associated with volume v_2, if $v_1 > v_2$... . This indicates the statistical size effect implying that an aggregate of smaller volume exhibits on the average a greater strength than that of larger volume, as long as they consist of the same material. The validity of such a postulate depends, among other things, on the mutual independence of the strengths of reference volume elements. In this study, a general interpretation of the statistical strength distribution will be used which includes the classical interpretation previously mentioned as a special case. The essence of this new approach lies in the interpretation that the strength is a multidimensional random process." This interpretation is used, together with a recently developed method of digital simulation of a random function, to demonstrate the statistical size effect in terms of a numerical example.

Sobolev, Morozov, Markochev, Gol'tsev, Sapunov & Bobrinskii (1972) write (p. 817 of translation): "The above methods of crack-length measurement [eddy-current method, method of ferrography and method of difference in electric potentials] were used for constructing fracture diagrams of a wide range of materials... . They were also used for investigating the effect of specimen dimensions on the fracture characteristics of the material... . For instance, the relationship shown in Fig. 1...was obtained while studying the effect of the thickness on the ability to resist fracture. [Fig. 1, not reproduced here, shows a sharp

increase in the coefficient of stress intensity K of a steel VKS-1 specimen 180 x 35 x 10 mm (sic) as the thickness increases from 0.2 to 0.8 mm, then a slower decrease as the thickness increases to 1.6 mm.] A reduction in the value of K_c, on reducing the thickness of the material when the fracture has a viscous character, indicates the danger of premature fracture. This reduction is apparently caused by an increase in the reserve of elastic energy for a unit of plastically deformed volume. A similar relation (K_c - t) was obtained for steels VKS-1, 40Kh5MVFS, 20Kh5MVKS, EP142, and the aluminum alloy V-95. Curves of crack propagation, under cyclic loading conditions with different maximum cyclic stress values, can be drawn as fracture diagrams of the form K + K ($d\ell/dN$) [ℓ = crack length, N = number of cycles]... . It has been established ...that these diagrams are weakly dependent upon the maximum stress in a cycle and the specimen dimensions."

Stephens (1972) writes (Synopsis, p. 119): "Fatigue behavior, fracture toughness and residual stresses were investigated using Hadfield steel in the as-received, shock-hardened at 160 and 235 kbar pressure level, and cold-rolled conditions. Surface hardness for the cold-rolled and for the shock-hardened conditions were identical. Both shock hardening and cold rolling reduced fatigue properties, however, cold rolling caused the largest decrease. The as-received material was quite notch sensitive compared to the shock-hardened or the cold-rolled material. Only slight differences existed in K_c for the shocked or cold-rolled conditions and residual compressive stresses in the cold-rolled material were substantially less than the shocked or as-received material." In his discussion of the results he writes (p. 129): "The average fracture toughness, K_c, for the cold-rolled condition was just slightly greater than the shock-hardened conditions. This slight increase could be due to the smaller thickness (0.165 inches) of the cold-rolled specimens compared to the 0.19 inch thickness of the shock-hardened specimens. Load versus crack opening displacement curves along with fracture surfaces of SEN [single edge notch] specimens were similar for both shock-hardened and cold-rolled specimens, and apparently no substantial size effect existed. Thus fracture toughness of the two shock-hardened and the cold-rolled conditions appear to be quite similar."

Taylor (1972) describes a series of tests which was the first to be carried out specifically to investigate the problem of scale effect on the shear strength of concrete beams over the full practical design range. He states the following conclusions (p. 2489): " 1. A series of tests on true to scale model beams of depth varying between 250 mm and 1,000 mm showed that slight reductions in the strength of the larger beams occurred, when compared with the relative strength of the small beams. If the maximum size of the aggregate used in the concrete is scaled correctly, the loss of strength is less significant. A satisfactory design approach in the case of large beams, regardless of whether the aggregate size is scaled correctly or not, is to insist that nominal stirrups (the code minima) be provided in all cases. 2. A series of tests carried out by Kani, in which the depth of the beam was varied between 150 mm and 1,220 mm while the width remains constant at 150 mm, shows that the loss of strength of the large beam is 40% of the strength that would be expected from the test results on the small beams. A satisfactory design approach for these large deep beams, with depth-width ratios greater than 4, would be to reduce code stresses 40% and to insist on nominal stirrups. 3. Tests on shallow beams, of depth less than 250 mm, show that there is a consistent increase in relative strength as the depth is decreased. The increase is significant enough to be exploited in the design of shallow beams and slabs and a satisfactory design approach would be to have a multiplying depth factor which may be applied to code nominal shear stresses."

Terasawa, Yoshioka & Asami (1972) report the results of fatigue tests of tufftrided steel conducted under various conditions of specimen diameter and tufftriding time. They write (p. 92): "The S-N curves of non-trufftrided and 20-, 90- and 180-minute tufftrided specimens of 12 mm diameter are shown in Fig. 2 [not reproduced here]. The fatigue limit of the non-tufftrided specimen was 18.2 kg/mm^2. However, when the specimen was tufftrided for 20 minutes, the fatigue limit remarkably increased to 30.7 kg/mm^2, although it only reached 38.9 kg/mm^2 even when the tufftriding time was 180 minutes. In order to observe the size effect of specimen diameter, S-N curves of various diameters are shown in Fig. 3 [not reproduced here]. Fatigue limit decreases with increasing

specimen diameter, [the fatigue limit was approximately 45, 34 and 31 kg/mm^2 for 90-minute tufftrided steel specimens of diameters 6, 12 and 18 mm, respectively], and the rate of decrease of the fatigue limit is larger than that of non-tufftrided or non-surface hardened specimens reported on by many other researchers [see, for example, Ouchida (1962)]."

Vorlíček (1972) notes that in an earlier paper [Vorlíček (1971b)] he formulated the statistical theory of strength without taking account of strain, but that sometimes one must consider the strain, as he does in the present paper. He analyzes strength and strain for two idealized types of bond between the basic particles of the body--the serial connection (modelled by a chain, which fails when the first link fails) and the parallel connection (modelled by a rope, which fails when all strands fail). He distinguishes between two idealized kinds of materials--perfectly plastic and perfectly brittle. In each case, important roles are played by the extent of the stressed zone (size effect) and by the theory of extreme values--the Weibull distribution, which he calls the type II distribution of smallest values [in western countries, it is called the third (or type III) distribution of smallest values], and/or the corresponding distribution of largest values.

Waddoups (1972) writes (pp. 535-536): "The strength and stiffness performance advantages of advanced composite materials are derived from the filament dependent behavior. A statistical model [based on the results of Weibull (1939a) and Daniels (1945)] for the behavior bounds is shown in Figure 1 [not reproduced here]. The issue is whether the composite will fail from fracture of the weakest filament or whether an accumulation of failures as represented by a classical bundle will occur. The engineering properties of the material have a first order relationship to the failure process. A weakest link failure will result in volume (area) dependent scaling problems and the material scatter will equal fiber scatter. Since the advanced fibers have large scatter (covar [coefficient of variation = standard deviation/mean] from 10 to 25 percent) no advantage over monolithic ceramics would be achieved. From an engineering standpoint a continuum with bundle reproducibility would be ideal (covar $\rightarrow$ 0 percent). As would be expected in the real materials neither bound is achieved. Models for the material behavior

have been proposed [Rosen & Dow (1972)] and correlate the data qualitatively. Figure 2 [not reproduced here] illustrates a recent possible solution to a problem in determining a length effect on filament strength. The data is test method dependent. Effects such as this further complicate the computation of filament direction structural response from constituent behavior. The fact that the structural response of the composite reflects some of the statistical behavior of the filaments suggests that the composite reproducibility characteristics may be optimized. Practical experience has not supported this conjecture yet the qualitative aspects of the proposed failure models may be useful."

Watanabe (1972) writes (p. 153): "The shape of the stress-strain curve for concrete has been investigated by many researchers as of fundamental importance in theories regarding the mechanical behavior of reinforced concrete members. Recently, investigation has been focused on the complete stress-strain curve involving the falling branch, because the ultimate strength design calculation takes this branch into consideration... . Though numerical expressions of complete stress-strain curves have been proposed by some investigators..., research papers describing available experimental data are rare. This is mainly due to the fact that there are some difficulties in avoiding the uncontrolled release of energy stored in the testing machine during the test and in estimating the influences of such factors as the scale of the specimen, the strain measuring length and, above all, the end restraint of specimens... . The objective of this research was to develop the complete stress-strain curve of concrete under uniaxial compressive loading from the experimental

data on cylindrical specimens. The effects of the diameter/height ratio of the specimens, the gage length of the strain measurements and the degree of end restraint of specimens on the shape of the stress-strain curve were also investigated." It was found that the shape of the complete stress-strain curve is strongly dependent on the size of the specimen, strain measuring length and frictional restraint at the end surface.

Weeks & Assur (1972), in a section on scale effects, write (p. 956): "It has been known for sometime that the measured strength of ice is some function of the size of the sample. Small samples give high strength values relative to values determined on larger samples. Finally, the failure of large ice masses under loads can be explained only by assuming still lower strength values. An adequate understanding of these scale effects is quite important in order that proper design values can be used in problems involving the strength of ice." They review the results of several earlier authors, including Jellinek (1958) and Lavrov (1958). Jellinek assumed that ice contains a distribution of defects and that each of these defects can withstand stresses up to a certain size, from which it follows that the probability of the weakest imperfection occurring in the volume subjected to stress is proportional to the size of this volume, so that tests that force failure on a small volume should yield higher strength values. Lavrov assumed that the strength of ice is determined by the strength of the molecular bonds which are, of course, independent of sample size, and that the scale effect results from the fact that the larger the sample, the smaller the relative elongation at which failure is presumed to occur and hence

the smaller the ultimate strength. Weeks & Assur note that Lavrov's interpretation of the scale effect has not been widely accepted [see Bartenev & Tsepkov (1960)]. In their summary, they write (p. 970): "One of the most important problems concerned with the gap between science and application in ice engineering is the scale effect in test results. Small ice volumes show considerably higher strengths than the large ice volumes involved in applications. This could lead to unjustified pessimism in the design of structures to withstand ice forces. The scale effect has been known for many years, but attempts can be made to develop a coherent picture. For large volumes of ice, the basic physical strength properties become less important than the effective distance between major defects such as cracks. This distance is governed by brittleness which is high for cold fresh-water ice and low for warm ice or normal sea ice. The strength of small samples increases with lower temperature, but the scale effect produces the opposite result when large ice volumes are involved in failure."

Yang & Knoell (1972) write (pp. 412-413): "Suppose the thickness of a composite vessel consists of N laminae, and the vessel is divided into M finite elements; thus the total number of elements of a vessel is NM. Each lamina in one element is assumed to consist of fibers in one direction, i.e., the lamina is orthotropic. Let V_{ij} be the volume of the ith lamina in the jth finite element, referred to as the volume of i-j element. ...Let τ_{1ij}, τ_{2ij} and τ_{3ij} represent, respectively, the analyzed normal stresses parallel to the fiber direction, transverse to the fiber direction and the shear stress in the i-j element. ...The strength of ductile laminated composites has received extensive theoretical and experimental consideration because of their importance in the design of composite structures. Tsai's work...was based on the assumption that yielding and strength are synonymous. Therefore, the yield criterion of Hill..., based on the distortional energy theory, can be used as the failure criterion with the yield stresses replaced by the ultimate stresses, $(\tau_{1ij}/X_{ij})^2 + (\tau_{2ij}/Y_{ij})^2 + (\tau_{3ij}/Z_{ij})^2 - \tau_{1ij}\,\tau_{2ij}/X_{ij}^2 = 1$ (1) where X_{ij}, Y_{ij} and Z_{ij} denote the ultimate stresses in the fiber direction, transverse to the fiber direction, and the ultimate shear stress, respectively, of the i-j element. Extensive experimental data

indicates that ultimate strengths of composite materials are random variables, and their statistical distribution can be represented by the Weibull distribution... . Assuming that the probability of failure of each element is statistically independent and each layer is identical, the probability of failure of the vessel, denoted by p_f, can be derived as $p_f = 1-\exp\{-\sum_{i=1}^{N} \sum_{j=1}^{M} (V_{ij}/\nu)(\tau^*_{ij}/\gamma)^k\}$ (2) where ν is the unit volume, k and γ are, respectively, the shape parameter and the scale parameter of the Weibull distribution of material strength X_{ij} obtained from specimen tests, and $\tau^*_{ij} = [\tau^2_{1ij} + \alpha^2\tau^2_{2ij} + \beta^2\tau^2_{3ij} - \tau_{1ij}\tau_{2ij}]^{\frac{1}{2}}$ (3) is referred to as the 'equivalent stress' acting on V_{ij}. α and β are, respectively, the ratios of X_{ij}/Y_{ij} and X_{ij}/Z_{ij}. Equation 2 indicates that the survival of the vessel implies the survival of each i-j element. Note that the assumption of independent failure of each element is conservative... . Furthermore, the probability of failure can easily be obtained when the applied loads...are random variables. It should be emphasized that the failure process of a composite structure is extremely complicated, and little quantitative results for the failure process are available to date. Equation 2 represents a general formulation for the upper bound of the failure probability of orthotropic structures, and it is very reasonable for some structures such as composite plates. ..."

Yusuff (1972) reports the results of a theoretical and experimental study of the effects of yielding and size upon fracture of plates and pressure vessels. He writes (Abstract, p. 129): "The size effect for plates of high strength material is expressed by a formula relating the Griffith stress to the applied stress, critical crack length and the width of specimen: but in the case of more ductile material different formula is derived on the basis that in different sizes of plates the yielding zones have the same width for crack lengths proportional to the width of specimens. The size effect obtained for cylinders not only yields the solution for failure mechanism of cylinders but furnishes the means of obtaining the rate of strain energy release when the formula is applied to test results. ...The test data used to substantiate theory consists of four materials. They are two aluminum alloys, stainless steel and a titanium alloy. The width of flate plate specimens in tests is in the range from

2.25 in. to 30 in. and the radii of cylinders are from 3.6 in. to 15 in."

Almási (1973) points out that prestressing was first applied to compressive members very recently, and that not all questions can be answered with assurance from previous investigations. He seeks to answer the following questions: How are the deformation and strength properties of fresh concrete influenced by prestressing? How should the load carrying and deformation data of concrete supports be determined? After a brief survey of the literature on prestressed-concrete columns, he describes his own experiments on centrally prestressed hinged columns. His tests confirm previous findings that for slenderness ratios $\ell/h > 26$ (ℓ = column length, h = width of cross section) prestressing increases the ultimate strength of columns, but for smaller slenderness ratios the ultimate strength is reduced. In the subsequent theoretical analysis, the author determines ultimate loads and gives some formulas for dimensioning of centrally and eccentrically prestressed beam-columns.

Armenàkas and Sciammerella (1973) write (Discussion and Conclusions, p. 58): "A number of the results obtained in this investigation confirm conclusions reached in previous theoretical and experimental investigations. However, other results obtained in this investigation provide new information which will contribute to a better understanding of the failure mechanism of fiber-reinforced composite materials. For a systematic discussion of the results, let us consider the two best-known statistical models employed in predicting the strength of fiber-reinforced composite materials subjected to uniaxial tension. The first model is referred to as the 'cumulative-weakening model', whereas, the other model is known as the 'crack-propagation model'. In the cumulative fracture model, the composite is divided into a series of n layers of length ℓ_e. ℓ_e must be such that a break in a fiber inside a layer affects negligibly the fiber load-carrying capacity in the adjacent layers. Each layer consists of N parallel fibers, uniformly loaded. Thus, the composite becomes a chain of bundles. Each fiber of length ℓ_e will fail independently, whereas the composite will fail at a critical load wherein a sufficient number of the fibers in a layer have broken so that the sur-

viving fibers can no longer support the applied load. A crack-propagation barrier is assumed so that the aggregate body fails by independent, simultaneous or consecutive fracture of individual fibers. The mathematical analysis of this model entails the following steps: (1) Establishing the effect length, ℓ_e. (2) Establishing the strength distribution for fibers of length ℓ_e. (3) Finding the strength distribution of a bundle containing N fibers of length ℓ_e. (4) Applying the weakest-link theorem. In order to take into consideration the effect of stress concentration in the fibers adjacent to the broken fibers, the crack-propagation model was introduced [see Zweben (1968)]. This model is similar to the cumulative-weakness model. The composite made of N fibers of length ℓ is considered to be composed of n layers of thickness ℓ_e. The basic element is a fiber of length ℓ_e. The total number of such basic elements is nN, ($n = \ell/\ell_e$). The mathematical analysis of this model entails the following steps: (1) Establish the effective length, ℓ_e. (2) Establish the strength distribution for fibers of length ℓ_e. (3) Determine the stress - (or load) - concentration factors. (4) Analyze the crack growth. (5) Introduce a fracture criterion. The assumption of both theories, that randomly distributed fiber breaks occur well below the ultimate strength of the composite, has been substantiated by the experimental results. ... The assumption of the crack-propagation theory of stress concentration in the fibers adjacent to the broken fiber was confirmed by the experimental results... . The effective length ℓ_e is a basic quantity in both theories inasmuch as the strength of the fibers is a function of their length." The authors compare the results given by the two theories with available experimental results, and conclude that there is a great need for an extensive, systematic experimental program to evaluate these theories.

Asami, Yoshioka and Terasawa (1973) write (English summary, p. 359): "The authors previously observed that the tufftriding treatment was effective to the improvement of fatigue limit of iron and steel. However, it is a complex task that the designer obtains the fatigue limit of tufftrided steel from the fatigue test. In this paper, the prediction of fatigue limit was discussed from the viewpoints of the increase of hardness due to the tufftriding treatment and the cyclic stress and the existence of residual stress due to the tufftriding treatment which contributes to the improvement of fatigue limit. The following prediction equation was proposed. $\sigma_{w \cdot t} = C_1[h/(a-h)]\sigma_{w \cdot nt} + (1+C_2)\sigma_{w \cdot nt}$ where, $\sigma_{w \cdot t}$: fatigue limit of tufftrided specimen; $\sigma_{w \cdot nt}$: fatigue limit of non-tufftrided specimen; h: hardened depth; a: distance from the surface to the center of specimen; C_1: a constant which indicates hardening due to cyclic stress; C_2: a constant which indicates the degree of improvement of fatigue limit due to residual stress. The validity and the appli-

cability of the equation was investigated by the experiment." For rotational bending fatigue tests, it was found that $C_1 = 2.35$ and $C_2 = 0.38$ for low carbon steel, while $C_1 = 1.08$ and $C_2 = 0.31$ for medium carbon steel.

Bölcskei and Mistéth (1973) report the results of tests one of whose objects was to investigate the variation of the strength of wires with their length. They point out at the outset that if the material is homogeneous, the ultimate tensile strength must decrease with increasing length, according to the 'weakest-link' theory. They write (Conclusions, p. 19): "The experiments conducted on 1500 specimens produced from 5 mm diam tension wire of 10 cm, 100 cm and 1000 cm basic lengths, gave the following results: 1. The distribution function of the nominal value [in kp/mm^2] of the tensile strength is $F(\sigma_n) = 1 - \exp\{-n[4.489(\sigma_n - 132.51)/168.613]^{13}\}$. The above extreme distribution was followed only in the case of the rejection of 20 results which indicates that the material is not homogeneous. 2. The nominal value of the tensile strength of the specimens decreases with the growth of the basic length, which was logically to be expected. This may be represented by the formula $\bar{\sigma}_{nL} = 16861(0.7859 + 0.2141L^{-1/13})$. This formula is only true up to the length of $L_{min} = 0.40$ because the subsequent 0.10 mm[sic; 0.10 m] segments are not independent. This independence is only true in the case of 0.4 m long segments."

Broek and Vlieger (1973) report the results of residual strength tests which were carried out on center cracked aluminum alloy panels of various thicknesses. They discuss, among other things, the thickness effect, concerning which they write (Conclusions, p. 32): "To investigate the effect of specimen thickness on the plane stress fracture toughness residual strength tests were carried out on center cracked panels of 2024-T3 and 7075-T6 aluminium alloys having thicknesses of 2, 4, 6 and 10 mm. From the investigation the following conclusions can be drawn: a. It turns out that in general the values of the plane stress fracture toughness, K_c, gradually decrease with increasing thickness. The 4 mm 7075-T6 and the 6 mm 2024-T3 specimens showed an anomalous behaviour in this respect. This is probably due to the fact that the panels used for the investigation were not cut from the same plate material and therefore had slightly different properties. b. Of the models that exist to predict the effect of thickness on the fracture toughness in the transitional region between plane strain and plane stress none gives a satisfactory agreement with the test results. For the time being the approximate method of Anderson ... is probably the best for an engineering judgement of the thickness effect. ..."

Burck and Rau (1973) write (Conclusions, p. 50): "The stress intensity factors for cracks extending from one or all holes in periodic linear arrays have been

approximated and used to analyze fatigue crack propagation in these arrays. The fatigue crack propagation life-times have been calculated and shown to increase with decreasing hole spacing for holes aligned parallel to the stress axis but to decrease with decreasing hole spacing for holes aligned normal to the stress axis. In both cases the effect is larger when each hole is cracked than when only one is cracked. For both types of arrays, hole size is shown to affect the relative lifetimes over and above the size effect associated with a single hole."

Carlsson (1973) proposes a model which allows determination of the critical stress for fracture of pipes with surface flaws of different depths. The theoretical curves are obtained by relating the yield strength of the cracked region σ_{yi} to crack depth b and wall thickness t through $s_i = \sigma_{yi}/\sigma_{ye} = 2(t-b)/t\sqrt{3}$, where $\sigma_{ye} > \sigma_{yi}$ is the yield strength outside the cracked region, which is assumed to take some value between the yield strength σ_Y and the ultimate tensile strength σ_U. Theoretical and experimental results agree fairly well for pipes with diameter 760 mm and wall thickness t = 9.5 mm, also for pipes with diameter 610 mm and wall thickness 41-44 mm.

Carman (1973) reviews three experimental techniques for developing the crack resistance curve and plane stress fracture toughness using center cracked panels. He presents experimental data, for a variety of materials and thicknesses and two panel widths (4 and 20 inches), which show the size effects and demonstrate the equivalence of the various methods for determining the crack resistance curve.

Cruse (1973) discusses the effect of notch size on the tensile strength of laminates. He writes (p. 218): "The particular problem considered is the effect of circular notches which has received considerable experimental and analytical attention recently. The discussion focuses on a rational means for understanding and predicting notch size effects. The key findings are that the notch size effect is a result of the stress distribution in the quasi-brittle laminate and that free edge effects are not important."

Davidge, McLaren and Tappin (1973) write (Summary, p. 1699): "The concept of the SPT [strength-probability-time] diagram for ceramics is introduced as an essential aid to the design engineer when using engineering ceramics. For given conditions of stress state and distribution, environment, temperature and component size, the diagram enables estimates of a safe working stress to be made for specific component lifetimes and survival probabilities. The theory underlying the SPT diagram is reviewed. This involves the merging of the concepts of statistical variations in strength, with sub-critical crack growth which leads to delayed fracture. It is shown how the SPT diagram can be generated by simple measurements of the strain rate

dependence of fracture strength. Data for the delayed fracture of alumina are used to demonstrate the reliability of the SPT diagram." Concerning statistical variations in strength, the authors write (pp. 1699-1700): "The strength (σ_f) of ceramics is controlled generally by the stress to propagate small microstructural flaws, which are invariably present, according to a modified Griffith equation, $\sigma_f = (1/Y)(2E\gamma_i/C)^{1/2}$ (1) where Y is a geometrical constant, E Young's modulus, γ_i an effective surface energy, and C a flaw size. Because a given ceramic will have a range of flaw sizes there will be a corresponding variation in strength. In principle, one could rationalize this in terms of the variables in Equation 1 but this has yet to be achieved. Fortunately, an empirical statistical analysis is available and that of most widespread (but not universal) applicability is the weakest link model due to Weibull [(1939a, b, 1951)]. $P = \exp\{-D[(\sigma_f-\sigma_u)/\sigma_0]^m\}$, (2) where P is the survival probability, D the stressed volume, σ_u the zero probability stress, σ_0 a normalizing constant, and m the Weibull modulus [shape parameter]. Thus $\ln \ln (1/P) = m \ln (\sigma_f-\sigma_u) - m \ln \sigma_0 + \ln D$.(3) Intuitively, $\sigma_u = 0$ and this is often found experimentally. A plot of ln ln (1/P) versus ln σ_f thus gives the Weibull modulus. ..." The statistical effects are combined with a time-dependent failure analysis to give the SPT diagram.

Eisenmann, Kaminski, Reed and Wilkins (1973) write (Abstract, p. 298): "The current deterministic design approach is reviewed, and an alternate reliability-based design procedure is described which is more appropriate to the distinct behavioral characteristics of composite materials. The proposed procedure treats: load generation, material characterization, reliability apportionment, scaling, and complexity. They continue (Introduction, p. 298): "... Two aspects of design in composite materials are not addressed at all by the conventional approach: scaling and complexity. Geometric scaling (changing all dimensions by a constant factor) results in changes in mean static strength as well as mean lifetime in bonded joints ... . Complexity, such as the number of holes in a component, or number of airplanes in a fleet, has pronounced effect on the time to first failure of the weakest member of the population ... . Observations of this nature coupled with actual design experience for composite elements and components using conventional design approaches have proved to us that a more appropriate set of design rules is required for composite structure."

Ellis and Harris (1973) write (Abstract, p. 76): "The work of fracture of composite materials has often been measured without sufficient attention, for example, to specimen geometry and test conditions. In this work a range of testing techniques has been used to investigate how the measured work of fracture of uni-

directionally-reinforced carbon fibre/epoxy composites, containing fibres of types 1 and 2, changes with various specimen and test variables. The measured work of fracture is seen to be independent of specimen width, and shows but slight dependence on notch root radius and test rate. More significant variations occur with orientation, specimen height and notch depth, however. Attempts are described to measure other fracture parameters which, for this class of materials, might prove to be of more value than the total work of fracture. In particular, an effort has been made to adapt the methods of Linear Elastic Fracture Mechanics, and some degree of success has been achieved in relating K_{Ic}, the critical stress intensity factor, measured by double edge notched plate techniques, and G_{Ic}, the critical strain energy release rate, with those measurements discussed previously."

Fearnehough (1973) writes (Abstract, p. 77): "For many years there has been a search for small-scale tests which will predict the propagation behavior of cracks in engineering structures. Inadequate prediction can be ascribed in some cases to an incomplete knowledge of crack propagation characteristics in full size structures; however, it is usually associated with unrealistic small specimen behavior. This paper reviews the development of small-scale tests and demonstrates how their limitations may be sometimes overcome either by due consideration of the characteristics of propagation in engineering structures or by appropriate analysis of the small specimen behavior. Laboratory tests are usually inadequate for one or more of four basic reasons: (a) the strain rate does not reproduce that of propagating cracks; (b) the small size does not allow the full constraints appropriate to thick sections to be developed; (c) general yielding of the specimen before fracture necessitates energy absorption which is not directly associated with the propagation process; (d) inappropriate choice of specimen compliance, thus allowing premature fracture arrest. Small-scale tests are reviewed in the light of these limitations and recent developments designed to eliminate these difficulties are discussed."

Gamble (1973) reports experimental results on the strength of large steel reinforcing bars, #14(1.693 in. diameter) and #18(2.257 in. diameter). Average yield stress and average ultimate stress were lower for the #18 bars (55.1 ksi and 96.7 ksi, respectively) than for the #14 bars (59.8 ksi and 100.4 ksi, respectively). An adjacent length of a #18 bar which had one of the lowest yield stresses found was sawn lengthwise and then one half was split again. Tensile test specimens of 0.75 and 0.505 in. diameter were machined from the pieces and tested. The smaller specimens were stronger than the full bar sections. The yield stresses were 52.0 ksi, 55.5 ksi and 55.0 ksi for the full bar, the 0.75 in. diameter specimen and the 0.505 in. diameter specimen, respectively. The corresponding ultimate stresses

were 92.2 ksi, 102.6 ksi and 104.3 ksi, respectively. The author writes (p. 33): "The increase in ultimate stress was obviously much greater than the increase in yield stress, but it is clear that in this case the small smooth specimens were appreciably stronger than the large rough bar. While this is only a single item of data on the effects of specimen size on the strength of the material, the results are consistent with those obtained by Sanders and Siess [(1965)]. These results represent a strong argument in favor of requiring testing of full sections of bars, as is required by ACI 318-71, rather than allowing testing of small machined down specimens, as is permitted under ASTM A 615-68."

Hardy, Hudson and Fairhurst (1973) present a method for deriving the complete force-displacement curve associated with the failure process in a laboratory test with special reference to tests of rock beams. They discuss the size and shape effects. They point out that a fixed initial crack length has a disproportionate effect on differently sized beams -- the crack will have a greater weakening effect for smaller beams. They tabulate the variation in tensile strengths that would occur if the beam dimensions were doubled at various length: depth ratios. Their tabular values indicate that the size effect can take the form of either an apparent strength increase or strength decrease, which reflects the fact that the tensile strength is not a material property. They note that in laboratory tests, no unique trend can be established for the size: strength relationship. They write (Summary and Conclusions, pp. 65-66): "... If the beam geometry is changed, the complete force-displacement curve is not proportionately scaled because the rock microstructure is not similarly changed and the work of fracture associated with a relative increment in crack length is not constant. ... The theory predicts that no unique trend is evident for the size: strength relationship. ... The material property associated with rock failure is the unit work of fracture; the tensile strength varies with the specimen geometry and is not a material property."

Hudson, Hardy and Fairhurst (1973), in a sequel to the above paper, report the results of experimental studies conducted to verify the theoretical ideas presented in the earlier paper. They write (pp. 80-81): "The experimental results have established the fundamental validity of the theory: complete force-displacement curves should be analyzed on the basis of the amount of energy absorbed during an incremental reduction in the specimen modulus as failure occurs. In the past, the tensile strength has been the (assumed) material property used to characterize the failure of rock in tensile stress fields. When failure is analyzed by energy-absorption considerations, however, the material property is the unit work of fracture; the tensile strength, or maximum tensile stress at failure is a function of

the type of test and specimen geometry and is not a material property."

Joint ASCE-ACI Task Committee 426 (1973) discusses the effect of the size of cross section of reinforced concrete members on their shear strength. The Committee writes (pp. 1119-1120): "Leonhardt and Walther ... studied the effect of beam size by testing two series of similar specimens without web reinforcement. The first series consisted of four completely similar specimens in which the cross section varied from 2 in. x 3.1 in. (5 cm x 8 cm) to 8 in. x 12.6 in. (20 cm x 32 cm). In this series, the bar diameter was proportional to the external dimensions and the number of bars was constant. In the second series, the cross section varied from 4 in. x 7 in. (10 cm x 18 cm) to 9 in. x 21.3 in. (22.5 cm x 67 cm), the ratios of depth were different from the ratios of widths, the bar diameter was constant, and the number of bars was varied to maintain the same steel percentage. The shear stress at failure decreased 37% between the smallest and largest specimens in the first series and decreased 21% in the second series. The greater strength decrease in the first series was explained on the basis of poorer bond quality with increasing bar diameter. The high shear strength of small scale beams has also been reported by Swamy ... who attributed it to the influence of the high value of the modulus of rupture of the material and the increase in strength produced by extreme strain gradients in small specimens. Kani ... tested beams of various depths and the same concrete strengths, steel percentage, and a/d ratio [a = shear span, distance between concentrated load and face of support; d = distance from extreme compression fiber to centroid of tension reinforcement]. The shear stress at failure decreased with increasing beam depth. In these cases the beam width was held constant at 6 in. (15 cm) as the depth was varied. ... Tests by Taylor ... have shown much less size effect if the size of the coarse aggregate is changed in the same proportion as the beam size. Taylor also showed that if large beams with normal b/d ratio ($d/b < 4$) [b = width of compression face of member] are tested, the loss of strength is not as serious as that reported by Kani. For beams with $d/b > 4$ Taylor has proposed that the design value of v_c, the shear stress carried by the concrete, should be reduced by 40%. The beams tested by Leonhardt and Walther, Kani and Taylor to study the effect of size on shear strength did not have web reinforcement. It appears reasonable to expect that while V_c [shear carried by concrete] has a scale effect, V_s [shear carried by shear reinforcement] has not, so that the effect of size on beams with web reinforcement is small. Statistical analysis ... of test results confirms that the beam depth has no significant effect on the ultimate strength of beams with web reinforcement. This is confirmed in part by tests of two 4 ft (122 cm) deep 'wall beams' with web reinforcement ... ." Results are also reported on the

effects of flange width and web thickness in T-beams and I-beams.

Judy and Goode (1973) write (Abstract, p. 48): "New procedures for characterizing the fracture extension resistance of ductile metals have been established. The fracture extension resistance curve (R-curve), which delineates the increasing rate of plastic work to cause crack propagation in nonbrittle metals, is determined by using dynamic tear (DT) test procedures. The resistance parameter is the slope of the R-curve. The effect of thickness on the R-curve slope was investigated for three high-strength steels of high, intermediate, and low resistance to ductile fracture. R-curves were determined for each steel in the full thickness (1 in.) and for thicknesses of 0.625 and 0.325 in. The R-curve slopes showed good agreement for each section size, and a transition in the fracture mode from flat fracture at short crack extensions to the metal's characteristic degree of shear fracture for long extensions was observed for each steel. The data can be described by an exponential equation involving fracture energy, specimen cross-section dimensions, and a constant, R_p, which is proportional to the R-curve slope. For each steel, R_p is unaffected by specimen geometry, thus indicating it to be a material property."

Kanazawa (1973) reports that, in recent studies on brittle crack propagation in Japan, the effects of specimen size (aspect ratio) have been investigated by systematic experiments and have been found as expected from theoretical analysis. He writes (pp. 574-575): "Specimens of finite size are usually used for various tests performed to estimate brittle fracture initiation and propagation arrest characteristics of structural steels. The effects of specimen size, however, cannot be neglected if the proper value of material characteristics of brittle fracture is to be obtained experimentally. Strictly speaking, it is a controversial problem to deal with the brittle fracture propagation arrest (which is intrinsically a dynamic phenomenon) in the similar light with linear-elastic fracture mechanics based on static calculation. As far as the problem of elastic strain energy which contributes to crack growth is concerned, however, a series of studies ... on this problem conducted by the present author and his co-workers have been carried out with successful results. In most Japanese works, the effects of specimen size and loading condition on the stress intensity factor K were discussed on the basis of the static elastic solution ... ."

Knott (1973) reports in Chapter 5 the results of experiments which were carried out on 7075-T6 aluminum alloy to determine the effect of specimen thickness on fracture toughness. The results [see also Tetelman and McEvily (1967)] show a large variation in toughness with specimen thickness. For very thin specimens, the toughness increases with thickness. The peak of the toughness curve occurs at a thickness

of approximately 2 mm. For thicker specimens, the fracture toughness drops as the specimen thickness increases up to about 15 mm, then remains almost constant (approaching a limit asympotically) for further increases in thickness. These results on the effect of specimen thickness for an aluminum alloy and similar ones for maraging steels have been used in setting ASTM standards [see Brown and Srawley (1970)] which impose limitations on specimen thickness in plane strain fracture toughness (K_{Ic}) testing. The effects of specimen thickness and notch depth on the fracture of notched specimens are discussed in Chapter 7, and the effect of specimen thickness on crack propagation is discussed in Chapter 9.

Kowal and Lemiesz (1973) discuss the safety of curved ties transverely loaded in such a way that tie tension depends upon the position of the point on the tie. They point out that the probability of tie plasticization under constant tension would be equal for sections of equal length, and we would be dealing with a homogeneous process, and that this problem is also well-known as the statistical effect of the scale [Weibull (1939a)]. In the case under consideration, the tension is a function of the position on the tie, and therefore the probability of plasticization of individual sections of the tie is different, and the process is a non-homogeneous Poisson process. The safety in this case is equal to the probability of no plasticizations of the elastic-plastic tie or of no elasto-plastic tie ruptures. The authors present the method of determining the safety and the probability of plasticization, as well as the expected value and the variance of the number of plasticizations.

Lawrence (1973) writes (Conclusions, p. 219-s): "An analytical model has been developed to calculate the fatigue crack propagation life of arbitrarily shaped and loaded weldments containing an external crack of assumed initial size. With the model, the effects of weld geometry, material properties, stress level, and initial flaw size were considered. Differences in weld geometry [flank angle, θ, or height to width ratio, h/w, and edge preparation angle, $\emptyset$, or width to thickness ratio, w/t] were found to influence the fatigue crack propagation life by as much as a factor of three while material properties and initial flaw size can have a much larger effect. ..."

Moses (1973) surveys some applications of the reliability of systems that are used as tools in the reliability-based optimization of structures. For yield, 'weakest-link', models he gives curves of the normalized central safety factor against the number of members to show how much an element size must be increased for the same overall failure probability when it is part of a system rather than isolated. He also considers collapse models ('fail safe') and brittle members, con-

cerning which he writes (p. 250): "In redundant brittle structures, members that reach their capacity and fail carry little or no load thereafter. This greatly complicates the reliability analysis, and only a few analyses have been given. Several factors suggest, however, that 'weakest-link' analysis would be applicable to brittle elements. Unless strength variability is large, failure of one element will usually 'trigger' consecutive failures of other members following load redistribution. Also, redundant structures usually have some elements, such as foundations or a weld point, which are statically determinate and thus also are part of a 'weakest-link' model. Only in brittle yarn-type systems is the 'fail-safe' probability significantly larger than 'weakest-link' analysis [see Gücer and Gurland (1962)]." The author gives numerical examples of the analysis of 'weakest-link' and collapse ('fail-safe') structures.

Newman (1973) writes (Abstract, p. 667): "The Neuber stress-concentration relation for notches in an elastic-plastic material subjected to shear loading was generalized for a crack in a finite plate subjected to tensile loading ... . An equation was derived which related the linear elastic stress-intensity factor, the applied stress, and two material parameters. The equation was then used as a two-parameter fracture criterion for surface- and through-cracked specimens. Fracture data from the literature on surface- and through-cracked sheet and plate specimens of steel, titanium alloy, titanium weldment, and aluminum alloy tested at room and cryogenic temperature were analyzed according to the proposed equation. For surface cracks, wide ranges of crack-depth to crack-length ratio and crack-depth to specimen-thickness ratio were considered. For through cracks, wide ranges of crack length and specimen width were also considered. An empirical equation for the elastic magnification factors on stress intensity for a surface crack in a finite-thickness plate was also developed. The fracture stress predictions computed from the two-parameter fracture criterion for both surface- and through-cracked sheet and plate specimens are consistent with experimental failure stresses."

Owen and Bishop (1973) write (Introduction, pp. 146-147): "The possibility of failure of glass reinforced plastics (GRP) structural components in a manner similar to brittle fracture in metals is now receiving attention ... . Irrespective of the details of micro-fracture processes, if the structural element is large enough the loaded element may store sufficient elastic energy to propagate a fracture from a defect, accidental local damage, or a design detail. Two structural failures which have been brought to the attention of the authors on a confidential basis have revealed a different mode of fracture from that observed in testing small laboratory samples. In small specimens tensile fracture is accomplished by progressive damage (debonding

and resin cracking) extending through the stressed region of the specimen whereas in the larger structures the failures took the form of narrow cracks, with the material almost entirely damage free on either side of the fracture surface. A similar form of failure has been reproduced in the laboratory ... when a considerable adverse size effect was obtained for geometrically similar specimens containing a circular hole. The large specimens also demonstrated the same absence of damage away from the fracture surface. The study of the fracture mechanics of composites has tended to separate into two areas, micro-fracture mechanics and macro-fracture mechanics. In the first area several workers ... have associated fracture with such mechanisms as debonding, fibre fracture and fibre pull out. Generally these workers cast doubt on the applicability of conventional linear elastic fracture mechanics to composite systems due to the different nature of the crack tip in a composite to that in a metal. In the second area other workers ... have used the linear elastic fracture mechanics approach to investigate the effect of a crack on the failure of reinforced plastics. ... If it could be shown that the stress intensity approach can be applied to composites then it may well be possible to further exploit the considerable volume of theoretical and experimental work which has been carried out on isotropic materials. An alternative approach is to determine critical strain energy release rates from considerations of the energy balance in the fracture process. ... The authors have carried out fracture toughness tests on a polyester resin containing various forms of glass reinforcement as part of a Ministry of Defense (Navy) program to investigate the effect of stress concentrators on the failure of reinforced plastics. The effect of crack length has been determined by testing geometrically similar specimens of increasing size and the results converted to K_c [critical stress intensity factor] values. Where K_c values were not independent of crack length a method similar to the plastic zone correction factor proposed by Irwin ... has been used to adjust the K_c value to be independent of crack length. Using the adjusted values of K_c the failure stress for a typical structural member in the form of a plate containing a circular hole was established and compared with that measured experimentally."

Phoenix and Taylor (1973) determine the asymptotic distribution of tensile strength of a bundle of parallel fibers, as the number of fibers in the bundle grows indefinitely large, in terms of the statistical characteristics of the individual fibers. Versions of this problem were previously treated by Daniels (1945), Sen et al. (1969, 1973) and Suh et al. (1970).' The present authors extend their model to cover the case of bundles composed of a mixture, in fixed proportions, of several fiber types. They show that such bundles can be weaker than similar bundles of any single component.

Plath (1973) writes (English summary, p. 397): "The bases of testing procedures are frequently not known concerning testing standards which were developed some decades ago or originated from such models. Special occasions are then necessary to revise them. With regards to the German standards for bending properties DIN 52352 (fibre boards), DIN 52362 (particle boards) and DIN 52371 (plywood) difficulties arise when they are applied to boards with a thickness of 4 mm or less. Since such boards are used for supporting or stiffening purposes in building, the present testing standards must be revised. For the most important characteristics of standardized bending tests, namely specimen size, distance between supports and diameter of supporting rolls, mathematically supported proposals were elaborated. They do not only deviate from the existing, partly older standards, but deviate also from the most recent drafts for an ISO standardization. Not only the relations of support, i.e., the relation between width of support and specimen thickness, but the roll diameters must be reduced as well. They affect the magnitude of evaluation errors, which occur by using Naviers bending formula."

Pogosian (1973) writes (Summary, p. 175): "In order to simulate on a laboratory model the processes of friction and wear in a crane brake under conditions of rapidly-repeated loading, the scale factor must be taken into account. A method is described for obtaining scale coefficients for the transfer from a full-size to a model system on the basis of similarity theory and dimensional analysis. The validity of the coefficients has been confirmed experimentally and the accuracy of the simulation estimated."

Raske (1973) writes (Abstract, p. 394): "A means to determine the fatigue notch factor, K_f is devised by identifying the zone of metal at the notch root which is thought to govern the fatigue process. Corrections are then developed for the effects of the stress gradient and critical volume in this zone. [These corrections depend on the dimensions of the specimen and on the notch radius.] The resulting equation for K_f embodies these corrections and the theoretical elastic stress concentration factor, K_t. Experimental verification is obtained by testing notched plate specimens and by utilizing data from the literature. Predictions of K_f over fatigue lives of from 10^2 to 10^6 cycles are within 4 percent of experimental values when calculated in terms of stress and within 9 percent in terms of strain."

Røren, Solumsmoen, Tenge and Søntvedt (1973) summarize methods available at Det norske Veritas (DnV), Oslo, Norway for determining the fatigue strength of marine propeller blades. With regard to the size effect, they write (p. 156): "In interpreting the results from fatigue test series, it is desirable to have the scatter as small as possible. This criterion can best be achieved with test coupons limited

in size giving a uniform micro-structure throughout its volume. ... Both the geometrical and the metallurgical size effects contributed to the reduction in strength. The latter is governed by the grain size, while the geometrical effect is dependent on the volume of the specimens subjected to the maximum stress. A Japanese investigation on the Cu-alloys ... revealed a 30% reduction in the corrosion fatigue strength at 10^8 cycles when the thickness of the test coupons was increased from 25 mm to 250 mm. ... DnV results ... from fatigue tests on a X-welded Cu-propeller blade (t=95 mm) also revealed a 30% reduction at 10^8 cycles compared with the results from tests with the 30 mm thick coupons. ... From the foregoing and data available from other test laboratories a size effect in the region of 30% at 10^8 cycles looks realistic for Cualloys. However, in our case the fatigue strength is required at $20 \cdot 10^8$ cycles. The size effect may then attain smaller values. At these long endurances the main factor determining the fatigue strength will probably be the resistance of the materials against corrosion attack."

Sawaki and Yokobori (1973) write (Introduction, pp. 25-26): "In the case of time dependent fracture, thermal agitation will help in attaining to both critical local stress condition and energy unstable condition with equivalent effect enhancing the local concentrated tensile stress. From this reason it can be understood that fracture phenomena have microscopically statistical nature since separation between neighboring atoms may be helped by thermal agitation. The statistical aspect of this type may be called microscopical nature. For this case the distribution function of fracture strength derived [see Yokobori (1952)] by the stochastic approach in terms of kinetic theory reduced to formally quite the same as Weibull's distribution function [see Weibull (1939a,b)]. On the other hand, the size of the stress raiser such as crack-like flaw has variability and rather random distribution, and thus fracture phenomena have a statistical aspect of another type, say, macroscopical nature. In this article an attempt is made to extend the stochastic theory of fracture to solids in such a case that the materials have both microscopic and macroscopic statistical natures mentioned above. It is shown that the stochastic theory of fracture phenomena proposed in this article also covers as a special case formally the Weibull's extremal probabilitic [sic] expression of fracture stress."

Sen (1973a) writes (Abstract, p. 526): "The present paper deals with the asymptotic theory of sequential confidence intervals of prescribed width $2d(d>0)$ and prescribed coverage probability $1-\alpha(0<\alpha<1)$ for the (unknown, per unit) strength of bundle of parallel filaments. In this context, certain useful convergence results on the empirical distribution and on the bundle strength of filaments are established and incorporated in the proofs of the main theorems. The results are the sequential

counterparts of some fixed sample size results derived in a concurrent paper by Sen, Bhattacharyya and Suh [(1973)]." Sen (1973b) writes (Abstract, p. 586): "Based on a Wiener process approximation, a sequential test for the bundle strength of filaments is proposed and studied here. Asymptotic expressions for the OC [operating characteristic] and ASN [average sample number] functions are derived, and it is shown that asymptotically the test is more efficient than the usual fixed sample size procedure based on the asymptotic normality of the standardized form of the bundle strength of filaments, studied earlier by Daniels (1945), and Sen, Bhattacharyya and Suh (1973)."

Sen, Bhattacharyya and Suh (1973), in a paper which is essentially the same as their earlier report [Sen et al. (1969)], write (Introduction, p. 297): "Let $X_{n,1} \leq \cdots \leq X_{n,n}$ be the n ordered values of $X_1,\ldots,X_n$, representing the strengths (nonnegative random variables) of individual filaments in a bundle of n parallel filaments of equal length. If we assume that the force of a free load on the bundle is distributed equally on each filament and the strength of an individual filament is independent of the number of filaments in a bundle, then the minimum load B_n beyond which all the filaments of the bundle give way is defined to be the strength of the bundle. Now, if a bundle breaks under a load L, then the inequalities $nX_{n,1} \leq L, (n-1)X_{n,2} \leq L,\ldots, X_n \leq L$ are simultaneously satisfied. Consequently, the bundle strength can be represented as $B_n = \max\{nX_{n,1},(n-1)X_{n,2},\ldots,X_{n,n}\}$. When the X_i are i.i.d. rv (independently and identically distributed random variables), Daniels [(1945)] investigated the probability distribution of B_n and established the asymptotic normality of the standardized form of B_n by very elaborate and complicated analysis. We observe that if $S_n(x)$ be the empirical distribution function for $X_1,\ldots,X_n$, then $n^{-1}B_n$ can be written as $\sup_{x>0} x[1-S_n(x)]\ldots$. This leads us to consider a general class of statistics of the form $\sup_x \psi(x,S_n(x))$ to which B_n belongs, and by probabilistic arguments on the fluctuations of S_n, we are able to study the distribution theory even in a more complicated situation when the $\{X_i\}$ forms an m-dependent process, not necessarily stationary." They apply the theory specifically to the study of the limiting distribution, strong convergence and convergence of the first moment of the strength of a bundle of parallel filaments (which they show to be the extremum of a function of the empirical distribution).

Sorkin, Pohler, Stavovy and Borriello (1973) write (pp. 338-339): "The resistance of a structure to cyclic loading (fatigue) is of paramount importance to the designer of high performance ships. One of the most aggravating problems to be faced is the lack of correlation between laboratory tests and the service life of welded structures. A recent study by Gurney [private communication] ... indicates that

... the majority of tests suffer from the following limitations: ... The tests have been conducted on specimens of relatively small size, typically less than 3 in.^{2} in cross sectional area (due to machine limitations). ... Effective correlation between testing and service performance can only be secured by using realistic load histories with proper regard for size and environmental effects. ... To account for size and residual stress effects, it will be necessary to require confirmation tests of full size structures in a realistic enviroment."

Sullivan, Freed and Stoop (1973) report experimental results, from which crack growth resistance curves (R-curves) have been obtained, for center-cracked tension specimens (CCT) of three aluminum alloys (including 7075-T6 and 2024-T3), one titanium alloy, and two steels (including 15-7 PH). A study is made of the influence of specimen dimensions (especially width) on K_c [critical stress intensity factor] and on the R-curve. On page 102, the following conclusion is stated: "Specimen width does not influence K_c for 2024-T3 when W>12 in. Earlier work indicated a similar independence of K_c and width for 7075-T6 for specimens as narrow as 6 in. A possible R-curve width dependence was observed for PH-15-7 stainless for CCT specimens as wide as 20 in."

Swanson (1973) writes (p. 113): "To establish values of K_{Ic} [plane-strain fracture toughness], which is as close to a material property as any quantity in fracture-toughness testing, one must obtain fractures under plane-strain conditions. The amount of material which yields at the crack tip must also be small. To ensure this, specimens must be of sufficient thickness so that a triaxial state of stress can exist at the flaw tip. K_{Ic} can vary over a range of three to one if the influence of crack length is eliminated. Crack-toughness values vary from a high in thin plates to a low in thick plates. The low value appears to be an asymptote...; further reductions in toughness do not occur in thicker plates." His Figure 5.11 (not reproduced here) shows the size effect in K_{Ic} measurements.

Tedford, Carse and Crossland (1973) present the results of a series of block program fatigue tests performed on notched steel cantilever specimens and automobile front wheel spindle bodies, both having approximately the same material specification and stress concentration factor. Various test programs were investigated, most of which were based on a normally distributed stress histogram profile. Results indicated that 8 stress levels were sufficient to simulate accurately the histogram profile. Eight small specimens and five full-scale specimens were tested at each nominal stress level and the results were analyzed to provide Weibull mean lives [see Weibull (1961)]. The nominal stress was plotted (on log-log paper) against the number of cycles to failure and the best straight line was drawn through the

mean lives. The authors write (p. 255): "The results presented show that the small specimens are satisfactory for program development and the conclusions deduced are applicable to full-scale specimens. The question then arises whether the performance of the full-scale components can be predicted from small specimen tests. Figure 28 [not reproduced here] shows the results of an eight stress level program on both small and full-scale specimens, in which the r.m.s. stress allows for the stress concentration at the notch, which had been found to be 1.75 for the small specimen and 2.03 for the full-scale specimen. There is a significant difference and the full-scale specimen appears weaker, but these specimens were subjected to a mean load to simulate the portion of the vehicular weight carried by the spindle body. Some additional tests were carried out on full-scale specimens with this mean load removed and from Fig. 28 it will be seen that the agreement between these full-scale specimen tests and small specimen tests is excellent. It would appear that this agreement must be fortuitous since the fillet radius on the full-scale specimen has been roll-burnished, and this should have improved the fatigue performance. However, to offset this it might be expected that the full-scale component would be weaker on account of size effect. It would appear that these two effects counteract one another to make it appear that small and full-scale specimens have the same fatigue lives."

Toor (1973) writes (pp. 841-842): "The critical stress intensity factor, K_c, is very much dependent on material thickness. Figure 2 [not reproduced here] shows the trend for 7075-T6 aluminum alloy. There are three distinct regions which exhibit three different types of failure modes, namely, plane stress, mixed mode, and plane strain. Therefore, to correctly apply the value of K_c in the design of structural components it is very important to properly account for the material thickness. ... Derivation of the stress intensity factor is based on the original elastic analysis of Inglis and Westergaard for an infinite plate. It is obvious that if one wants to use this factor for the analysis of finite plates, some sort of width correction factor is needed. Paris ... has suggested that to ensure applicability of the elastic analysis used in determining K, the panel length should be at least twice its width, W, which in turn should be at length twice the crack length, 2a." Toor discusses the tangent and secant corrections and gives references to more detailed discussions of finite width correction factors.

Tuset, Pera and Cubaud (1973) present the results of tests to failure of reduced 1/5-scale models of a reinforced concrete beam loaded at the third points of the span by two equal concentrated forces. They report good agreement with the prototype for the load-deflection relationship and failure load, with only 3 percent error, but

with fewer cracks on the model. They interpret their results as tending to prove that the similarity principle is applicable in this case. The reviewer (E. Nicolau) believes, however, that the scale factor has a much greater influence on the behavior of the model for reinforced concrete and that the authors have missed at least two issues: (1) same or different dimensions of the samples used for the determination of ultimate strengths for prototype versus model and (2) the embedment length and adhesion of steel to the concrete as a function of all dimensions involved.

Virdi and Dowling (1973) report the results of a series of nine tests on concrete encased steel H section columns (10 in. x 10 in. in cross section) of varying lengths and varying biaxial eccentricities -- all combinations of 3 lengths (6, 12 and 24 ft.) and 3 eccentricities. The test results showed predictable loss of strength with increasing lengths and applied eccentricities. The failure loads for the 12-ft. columns ranged from 74% to 88% of those for the 6-ft. columns with the same eccentricities, while those of the 24-ft. columns ranged from 53% to 62% of those for the comparable 6-ft. columns.

Wells (1973) reviews the history of fracture mechanics. He begins (p. 401): "Creative man has always been interested in fibers and their enhanced strengths. The ancient Egyptians made glass fibers in 1500 BC and Leonardo da Vinci's descriptions of tests on iron wires are well known." The high strength of wires was exploited in suspension bridges during the 19th century and in aircraft during World War I. An attempt to explain the high strength of fibers led to the work of Griffith (1920,1924), who demonstrated unstable fracture in glass, with the well known inverse-square-root dependency of strength upon crack length, and gave an almost convincing demonstration of a quantitative relationship between strain-energy release rate and surface tension, a concept which can now be seen to be greatly oversimplified. The author notes that by the end of World War II, working hypotheses had been established to describe the ductility of engineering materials at two widely separated extremes. He writes (p. 402): "On the one hand, the behavior of very crack-resistant materials could be described quite simply in terms of a loss of cross section by cracking, leading to an average fracture-stress criterion for the remaining ligament, following the concept of Galileo. On the other hand, very brittle materials reflected the inverse-square-root relation between fracture strength and crack length, restricing the latter to be small in comparison with other dimensions of a body. This naturally led to recognition of a size effect, in that the second extreme of behavior could be approached as the leading dimensions of a component increased. Nevertheless, most engineering structures were made of materials that would permit the designer to accept the first hypothesis and, at the time of the Second World War, no other

alternative was generally recognized." This state of affairs continued until publication in 1954 of an exhaustive historical study by Shank. The author continues the history up to date, discussing crack-propagation studies, the battle between the stress-intensity (K) criterion for crack extension and that of crack-extension force (G) or strain-energy release rate, as well as fatigue-crack studies. Lastly, he discusses the present position and future prospects of fracture mechanics, closing with the statement (p. 410): "The conclusion to be offered from this review is that the challenge of unsolved problems in fracture mechanics is as great as ever, and the topic is worthy of the continued attention of the proponents of experimental mechanics."

Wilkins and Jones (1973) write: "Graphite is brittle and fractures by the propagation of the most suitably orientated Griffith-type flaw [Griffith (1920,1924)]. The material contains a range of flaw sizes and this leads to a large variability in both fracture and fatigue strength. Fracture strength depends on the distribution of flaws and applied stress and, hence, must also depend on material volume, shape and mode of stressing." Their results are mostly qualitative in nature, and no information is given as to how the strength depends on material volume.

Wright and Iannuzzi (1973) write (pp. 430-446): "The strength of individual carbon fibers was measured, and the results were used to calculate the lower bound of strength expected from carbon-fiber reinforced epoxy matrix materials. ... We suggest that the strength of these carbon reinforced epoxy specimens is best described in terms of a lower bound. The stress level necessary to cause failure is then simply the strength of a bundle of fibers. ... Individual carbon fibers representative of those used to reinforce the epoxy were supplied with the specimens. Tensile tests were then carried out on those fibers and on fibers extracted from specimens by dissolving away the matrix in fuming nitric acid. Fiber bundles 1, 2 and 4 inches long were prepared by dissolving the appropriate length of matrix from a specimen in fuming nitric acid. ... The mean failure loads, $\overline{P}$, failure stress, σ, standard deviation, S, and coefficient of variation, c, computed from data obtained from as received fibers, 1 in., 2 in., 4 in. and 8 in. long and from extracted fibers 1.0 in. long is shown in Table 1 [not reproduced here]. The mean strengths of the as received fibers was identical to [not significantly different from] the strength values exhibited by the extracted fibers at the 5% significance level. It was therefore assumed that the nitric acid leaching treatment did not damage the fibers. It is quite apparent that the shorter fibers exhibited the larger mean strengths in agreement with the accepted probability failure criteria for brittle materials. The strengths of bundles of fibers fabricated by the process described previously are

also shown in the table. It is interesting to note that the strengths of these bundles also increased as the fiber length decreased; in addition, the bundle strengths were always less than the mean strength of the individual fibers. ... The strength and moduli of the unidirectionally reinforced materials are shown in Table 2 [not reproduced here]. ... The tensile strength of a composite was greater than the strength that could be expected by assuming that the fibers functioned as a simple load carrying bundle. In effect, a synergistic strengthening effect was observed when the matrix surrounded the fibers. This effect is best appreciated by comparing the failure loads of a bundle of fibers, as shown in the table, with the failure loads exhibited by a composite. In every case the composite material withstood the higher load; and that load was much greater than that expected by treating the matrix as a simple tensile load bearing component. ... We have shown that: 1. The strength of carbon fibers (Hy-E-13116) could be described by a Weibull expression. 2. The strength of carbon reinforced X-904 resin could be predicted ... from a knowledge of the strength of individual carbon fibers. ..."

Yokobori (1973) describes an attempt to combine macroscopic fracture mechanics [stress concentrator as crack or notch] with microscopic fracture mechanics [stress concentrator as dislocation], and to relate it to a thermal agitation process approach to time dependent fracture. In a related paper, Yokobori and Sawaki (1973) point out that in the microscopic case the distribution function of fracture strength derived by the stochastic approach in terms of kinetic theory reduces to the Weibull distribution function. In both papers, the stochastic theory is extended to the case in which the materials have both microscopic and macroscopic statistical nature, and it is shown that the resulting distribution of strength contains the Weibull distribution as a special case.

Argon and Bailey (1974) write (pp. 201-204): "It has been shown earlier ... that if the surface flaw distribution function in the brittle reinforcement layers of a laminate can be described by a simple power function as was originally done by Weibull [(1939a)] then the ratio of the average strength of the laminate to the average strength of the isolated element (for a given laboratory test length) is greater than unity for flaw distribution function exponents $m<10$. When such laminates with elements having exponents m [Weibull shape parameter] of the flaw distribution function less than 10 are subjected to tensile loading, isolated fractures can occur in the elements from bad flaws at stress less than the laminate strength. Because of the relatively high variability of the strength of elements, however, the stress concentrations produced by such early fractures can be tolerated by the surrounding elements which have a high probability of being strong in these areas of

local overstressing. Hence, many isolated element fractures are observable in laminates prior to the final fracture of the laminate as a whole [Rosen (1964)]. Alternatively, when the exponent of the flaw distribution function m of the elements exceeds about 10, the elements possess a high average strength with low variability, so that when, finally, an individual element fails from a bad flaw, it has a high probability of overstressing the surrounding elements at the site of the initial break leading to progressive fracture of adjacent elements and final fracture of the whole laminate. In such cases, the average strength of the laminate is less than the average strength of the elements, and very few, if any, stable element fractures are observable in the laminate prior to total fracture of the whole laminate. In nearly all cases of reinforcement using glass, boron, or carbon fibre (unless extreme precautions are taken), the former case holds, the exponent is less than 10, and the laminate is stronger that the elements. ... If the expected cummulative distribution function of flaws $\xi(\sigma) = C\sigma^m$ as used by Weibull [(1939a)] where C has the dimensions of stress^{-m} area^{-1} then the cumulative probability of fracture of a specimen of length L and width w at a stress of σ or less is the well-known expression ... $Q(\sigma) = 1-\exp(-s)$ where s is a nondimensional stress $s = LwC\sigma^{-m}$. The exponent, m, and the coefficient, C, of the flaw distribution function can be readily determined from a double logarithmic plot of $\{-\ln[1-Q(\sigma)]\}$ against σ, where the slope of the straight line immediately gives m and evaluation of the function with this exponent, at any point, gives C." The authors give an example of the use of this method to estimate m and C. They compare their theory with other theories for tensile strength of composites, in which element coupling methods are considered, including that of Rosen [(1964)] for large numbers of parallel elements (of the order of 100 or more).

Bader and Ellis (1974) write (Summary, p. 253): "Charpy impact tests have been conducted on a number of composites consisting of uniaxial carbon fibres in an epoxy resin matrix. The specimen geometry has been varied so as to obtain different length-to-depth ratios in the test piece both for plain test pieces and those containing notches. The results show that there is a critical length-to-depth ratio which marks a change of fracture mode from a relatively brittle transverse cracking to a tougher mode involving extensive delamination. The materials are not notch sensitive provided tests are always conducted at similar length-to-depth ratios. The impact performance of the composite and the critical geometric factors are both governed by the elastic strain energy potential of the composite and by its interfacial shear strength." The measured specific impact energy rises sharply below the critical length-to-depth ratio.

Barker, Dana and Pryor (1974) write (Introduction, p. 477): "The importance of free-edge interlaminar stress in laminates under axial extension has been considered by ... [previous authors]. These interlaminar stresses, as well as tensile stresses, become even more important when the laminates are pierced by holes or cutouts. An evaluation of the stress concentrations near these holes and at the free edges is necessary in order to better understand how the hole size and shape affects the overall strength of the composite. In this paper a three-dimensional analysis, based on the finite-element method, is presented to measure these effects."

Batdorf (1974) writes (p. 45): "A statistical theory for the fracture of isotropic brittle materials was recently developed [see Batdorf and Crose (1974)] in which the flaws are assumed to be cracks and, therefore, to have the directional sensitivity of cracks to the applied stresses. This theory permits a unique and accurate conversion from uniaxial to polyaxial fracture statistics. In this theory, the fracture characteristics of a material are completely determined by a function, $N(\sigma_{cr})$, which represents the number of cracks/unit volume that will fracture when the tension normal to the plane of the crack is $\leq \sigma_{cr}$. According to this theory, the probability of fracture is given by $P_f = 1\text{-}\exp\{-V\int_0^\infty [\Omega(\Sigma,\sigma_{cr})/4\pi][dN(\sigma_{cr})/d\sigma_{cr}]d\sigma_{cr}\}$ (1), where V is the volume of material, Σ the macroscopic stress in the material, Ω the solid angle enclosing the normals to crack-plane orientations such that the tensile component of stress, Σ, normal to the crack plane exceeds σ_{cr}, and σ_{cr} the critical normal stress of a particular crack. It can be shown that in the cases of uniaxial tension $(\sigma,0,0)$, equibiaxial tension $(\sigma,\sigma,0)$, and equitriaxial tension (σ,σ,σ) the values for $\Omega/4\pi$ are $\Omega/4\pi = 1 - \sqrt{\sigma_{cr}/\sigma}$ (uniaxial) (2); $[\Omega/4\pi] = \sqrt{1 - \sigma_{cr}/\sigma}$ (equibiaxial) (3); $[\Omega/4\pi] = 1$ (equitriaxial) (4). When σ_{cr} is $> \sigma$, $\Omega = 0$. If it is assumed that $N(\sigma_{cr}) = \bar{k}\sigma_{cr}^m$(5), Eqs. (1)-(4) lead to the following probabilities of failure: $(P_f)_{uniax} = 1\text{-}\exp[-V\bar{k}\sigma^m/(2m+1)]$ (6); $(P_f)_{eqbiax} = 1\text{-}\exp\{-V\bar{k}\sigma^m m!/[(m+0.5)(m-0.5)\ldots 1.5]\}$(7); $(P_f)_{eqtriax} = 1\text{-}\exp[-V\bar{k}\sigma^m]$(8). ... As the preceding considerations show, the 2-parameter Weibull equation retains its form exactly in going from the uniaxial to the polyaxial stress state, and all equations are identical when suitably normalized. However, Dukes [(1971)] has shown that this is not true of the more commonly used 3-parameter equation, which assumes that there is a uniaxial tension, σ_u, below which fracture cannot occur."

Batdorf and Crose (1974) write (Concluding discussion, p. 464): "The present paper introduces a new physically based statistical theory of fracture for isotropic brittle materials, which in large measure eliminates the long-standing rift between weakest link theories on the one hand and the main body of fracture mechanics on the other. It is incomplete since a theory which assumes that fracture is determined by

the tensile stress normal to a crack plane cannot account for fracture in pure compression. In effect, this assumption takes account of the direct stresses acting on a crack, but neglects the effect of shear, which is only justifiable when the maximum principal compressive stress is less in absolute value than three times the maximum principal tensile stress... . Thus the theory is not applicable when the stress is overwhelmingly compressive. Since the theory has only recently been formulated, it has not, at the time of writing, been used to analyze a large amount of test data. However, it is in very satisfactory agreement with the limited data to which it has been applied. For materials with low dispersion in fracture stress, the new theory reduces to a form similar to that of Weibull, but without the empiricism characteristic of the latter. It is a very practical theory in that, with uniaxial fracture data as input, it permits the calculation of the probability of failure for polyaxial stresses. Used in conjunction with a finite-element code, it can thus be utilized to determine the probability of failure of an arbitrary structure employing isotropic brittle materials under arbitrary (but not predominantly compressive) loading conditions."

Baumgartner and Richerson (1974) write (pp. 376-381): Based upon the results of ... fractography studies, a series of hot-pressed silicon nitride materials were prepared under a variety of techniques to produce a more homogeneous microstructure. The resulting strengths of these materials are shown in Figure 6 [not reproduced here] in the form of Weibull plots. The individual strength value for each sample of the given material is plotted versus its corresponding failure probability, P. P is defined as n/(N+1), where n is the ranking of the strength value placed in ascending order and N is the number of strength values in the test series. Material SN1 has an average 4-point bend strength of 104,300 p.s.i. and represents typical Norton HS130 silicon nitride. Material SN2 has an average strength of 123,500 p.s.i. and has improved homogeneity. Material SN3 has an average strength of 139,900 p.s.i. and represents the best experimental material thus far produced. ... Using McClintock's method [see McClintock (1955)], the Weibull parameters m, σ_0 and σ_u were calculated from the experimental strength data. These parameters were then inserted into the theoretical Weibull expression for the probability of failure, P, derived from a bulk flaw distribution: $P = 1-\exp\{-kV[(\sigma-\sigma_u)/\sigma_0]^m\}$, where σ = applied stress, V = volume subjected to tensile stress, k = load factor for fourth point loading = $(m+2)/[4(m+1)^2]$. The generated, theoretical Weibull curves are compared with the experimental curves in Figures 8-10 [not reproduced here]. The agreement is quite good for SN1, indicating that the calculated Weibull parameters adequately describe the experimental data. It further implies that the strength was controlled by a vol-

umetric Weibull distribution of critical flaws; that is, a random distribution of defects both in size and location. The theoretical curves for SN2 and SN3 do not fit the experimental data as well, indicating that the data cannot be properly treated by a single Weibull distribution function. ... The strength data for [SN2 and] SN3 ... indicate bimodal behavior. It is suspected that the bimodal behavior results from fractures originating from two distinct sources, for example from bulk and surface origins. Efforts are currently underway to resolve this point."

Buch (1974) writes (Summary, p. 75): "The work is based on the assumption of a size-independent critical thickness h or a surface material layer subjected to stresses exceeding some constant critical value, and is an extension of Peterson's theory. Some shortcomings of Neuber's, Peterson's and Stieler's one-parameter formulas for the fatigue notch factor K_F are reported. A new two-parameter formula of K_F is derived and proved for internally-notched aircraft sheet specimens with and without cladding in the cases of tension-compression and pulsating tension." He states the following conclusions (pp. 84-85): "1. The assumption of size-independent critical depth h of the surface layer with stresses exceeding some critical level can be used for derivation of a formula of the size effect for smooth specimens and of a formula for the notch-size effect. 2. The ratio of K_F/K_T is a better characteristic of the notch sensitivity of aircraft sheet material specimens than the ratio $(K_F-1)/(K_T-1)$ because of the greater regularity of the dependence of the notch radius in the former case. 3. The formulas of Neuber, Peterson and Stieler-Siebel ... overestimate the negative notch-size effect and do not take into account the influence of the cyclical strain hardening and of the cyclic stress ratio R. 4. The two-parameter equation fits well the experimental points for internally notched bar and Alclad aircraft sheet specimens and may be used for prediction of their notch factor in the case of $K_T \leq 4$ to 5 and $r \geq 0.5$ mm. It does not overestimate the notch-size effect. 5. The parameter h of the two-parameter equation takes into account the stress gradient support effect, and the parameter A, the cyclical strain hardening support effect. 6. In the case of a large notch ($r \geq 10$ mm) and $K_T \approx 2$ the fatigue notch factor K_F of Alalloy sheet specimens is in general smaller than K_T. The ratio K_T/K_F for large notch radii has in general a higher value for pulsating tension than for tension-compression, which may be explained by the stronger cyclical strain hardening at the higher fatigue limit level in the first case."

Bullock (1974) writes (pp. 200-201): "Ratios of flexural strengths to tensile strengths for wide varieties of brittle materials have been found to agree very well with Weibull's statistical strength theory..., which has been widely used in the ceramics field... . *Systematic studies to determine* if Weibull theory correctly pre-

dicts such strength ratios for fibrous-reinforced epoxy composites have not been reported as yet, although much data exist for making such strength comparisons. The purpose of this note is to report data indicating that Weibull theory does correctly predict strength behaviors for certain composite materials and to suggest that other composite systems be tested against this theory. ... According to the Weibull theory..., the probability that a specimen containing a distribution of flaws throughout its volume can survive the application of a stress distribution $\sigma(x,y,z)$ is given by $S=\exp\{-\int_V[\sigma(x,y,z)-\sigma_u)/\sigma_0]^m\}$ dx dy dz (1), where m is the flaw-density exponent that determines the scatter of strength for the material, σ_0 is the normalizing scale parameter that locates the strength distribution, σ_u is the threshold stress below which the material will never fail (usually taken to be zero), and V is the volume of the specimen that is being stressed. For tensile tests where the stress is uniform throughout the specimen volume and $\sigma_u=0$, Equation 1 gives $S_t=\exp[-V_t(\sigma_t/\sigma_0)^m]$(2), and for the non-uniform stress distribution of a rectangular specimen under three-point bending it gives $S_f=\exp\{-V_f(\sigma_f/\sigma_0)^m[1/2(m+1)^2]\}$(3), where σ_f is the maximum tensile stress produced at the central load point. The ratio of the median failure stress in three-point flexure to that in tension is obtained from Equations 2 and 3, by setting $S_t = S_f$, as $\sigma_f/\sigma_t = [2(m+1)^2(V_t/V_f)]^{1/m}$(4). ... For quadrant-point loading in four-point flexure, the counterpart of Equation 4 is $\sigma_f/\sigma_t = [4(m+1)^2V_t/(m+2)V_f]^{1/m}$. Therefore, for equal volumes of a given brittle material, strengths should always be greater in three-point flexure than in four-point flexure [by $(m/2+1)^{1/m}$] and greater in four-point flexure than in tension... ."

Colombo, Reina and Volta (1974) write (Abstract, p. 179): "The paper introduces the concept of a cumulative stochastic process and derives the general mathematical expression of the distribution corresponding to such processes when they can be assumed to be Markovian. The behavior of such a distribution in correspondence to accumulation functions of the type $u(t) = at^b$ and $u(t) = \ell \ln(1+t)$ is explored. It is shown how the exponential, Weibull, gamma, normal and lognormal distributions are particular cases of the general distribution. Next, the characteristics of the extreme values of n independent observations coming from such a general distribution are investigated. The central characteristics of the extreme value distributions are related to the hazard rates of the initial distribution. In particular, a simple method for relating the modal smallest value and the modal largest value to the sample size using the asymptotic expression of the hazard rate is given. The tail characteristics of the extreme values distributions are investigated numerically or analytically. The mathematical findings are applied to the volume effect on the failure probability of materials." On the volume effect they write (p. 183): "As

an example of the practical application of these mathematical findings, let us consider the problem of the 'volume effect' on the strength distribution of inhomogeneous and brittle materials such as graphite. The experimental results show a volume dependence of the mean strength of the material of the type [single maximum; skewed to the right] shown in Fig. 8 [not reproduced here]. Three problems are raised: how to interpret the non-monotonic volume effect; given the mean strength and the strength distribution of a volume V, how to evaluate the strength of a volume nV, $(n>1)$; given a volume nV, what is the acceptable stress corresponding to, say, probability of failure of 10^{-3} (such acceptable stresses are defined with reference to the mean strength measured on volume V)." The authors suggest how one can use the theory developed earlier in the paper to solve these three problems.

DiBenedetto, Salee and Hlavacek (1974) use a phenomenological model combining a Weibull distribution function with a kinetic equation for flaw growth to describe the static tensile strengths and fatigue lives of short graphite fiber reinforced nylon 66 sheet materials. They use a simple Weibull function of the form $P(\sigma_b) = \exp[-(\sigma_b/\hat{\sigma})^{9.5}]$ to describe the distribution of static strengths, the scale factor $\hat{\sigma}$ varying with the annealing treatment and being, in general, a function of environmental variables. The cumulative distribution of breaking times in fatigue is characterized by a translated three-parameter Weibull function $P(t_B) = \exp\{-[(\sigma_{max}/\hat{\sigma})^{16} + t_B/\hat{t}]^{0.594}\}$. The average time to break is related to the time scale factor $\hat{t}$ and appears to be a function of the flaw growth rate. The authors write (pp. 3-4): "The breaking stress of a unit volume of solid can be defined in terms of the probability of the rupture occurring at a given stress σ_b. If $(1-P_0)$ is the probability, then $(1-P_0) = f(\sigma)$ is a monotonously increasing function of the stress and P_0 is the probability of surviving the stress σ. If the material contains two elemental volumes coupled in either series or parallel, the probability of the two units of volume surviving simultaneously is $[P_0(\sigma)]^2$. Generalizing this to V units of volume, one obtains $P_V(\sigma) = [P_0(\sigma)]^V$ where $P_V(\sigma)$ is the probability of a material of volume V surviving a stress σ. One may define a quantity $B = -\ln P_V(\sigma)$ as the 'risk of rupture' which varies between zero and infinity so that $B = -V \ln P_0(\sigma) = V\, n(\sigma) = \int n(\sigma)dV$. Thus the risk of rupture is proportional to the product of volume and a function of stress related to the probability of survival of an elemental volume. The probability of rupture $(1-P_V)$ is given by $(1-P_V) = 1-\exp(-B) = 1-\exp[-\int n(\sigma)dV]$. ... Probabilistic formulae have been used to model the tensile strength of fibers..., fiber bundles... and fiber composites. A popular form of the stress function $n(\sigma)$ for the elemental volume is the so-called Weibull function $n(\sigma) = k(\sigma-\sigma_1)^m$, $\sigma\geq\sigma_1$; $n(\sigma) = 0$, $0\leq\sigma\leq\sigma_1$. The probability of rupture is thus given by:

$(1-P_V) = 1-\exp[-k(\sigma-\sigma_1)^m V]$, $\sigma \geq \sigma_1$; $(1-P_V) = 0$, $0 \leq \sigma \leq \sigma_1$. This simple unimodal function only approximates experimental data for most large populations and more accurate analysis usually requires multimodal functions for $n(\sigma)$."

Donald and Shannon (1974) write (Abstract, report documentation page): "Aluminum, titanium and steel alloys were selected on the basis of their strength/toughness properties for the purpose of investigating the performance of a proposed fatigue-cracked fracture toughness screening specimen. Specimens of varying thicknesses and widths were tested with the largest specimen of each alloy satisfying the minimum size requirements according to ASTM-E-399-74 specifications. The results of this program demonstrate the feasibility of the proposed specimen configuration with lowest width to thickness ratio showing the best correlation between the net-fracture strength and plane strain fracture toughness K_{Ic}. ..."

Finnie and Vaidyanathan (1974) write (Abstract, p. 231): "After a brief review of the literature it is concluded that the Weibull probabilistic treatment of brittle strength explains, very adequately, the condition under which Hertzian ring cracks initiate on glass surfaces. The case of cracking by a flat cylindrical punch is also considered and is compared with that due to a sphere. It is shown that ring cracking data obtained with a punch, or less conveniently with a sphere, may be used to deduce the parameters of the Weibull strength distribution." Concerning prior work, they write (p. 233): "The first experiments on Hertzian fracture by Auerbach [(1891)] yielded the intriguing result that the mean fracture load varied with indenter radius R as $\overline{P} \sim R$. This is in striking contrast to the expression $\overline{P} \sim R^2$ which would be expected from the Hertz equations if glass failed when the maximum tensile strength reaches critical value. The result $\overline{P} \sim R$, often referred to as 'Auerbach's Law' represents a strong size effect on strength in that it leads to a value of the maximum tensile stress at fracture varying inversely as the cube root of indenter radius. This observation and the large variability in strength observed in Hertzian fracture tests led Weibull [(1938,1939a,b)] to propose his probabilistic treatment of brittle strength. ... This approach is well known and has been applied to Hertzian cracking in earlier work [see Oh and Finnie (1967)]... ."

Friedrich, Pompe and Kopjov (1974) write (Abstract, p. 1911): "The brittle boundary layers often caused during the production of composites or by their treatment at higher temperatures, may change the mechanical properties. On the 'steel wire/aluminum' system the growth of the intermetallic boundary phase and its influence on the strength of the composite were investigated. Hence followed a maximum strength at smaller layer thicknesses. By means of fracture investigations new models were developed which allow the calculation of the dependence of strength behavior on

layer thickness."

Gurney, Mai and Owen (1974) write (Abstract, p. 213): "Cracks in large structures of materials of relatively high fracture toughness and low yield stress propagate before disseminated yielding occurs, whereas in geometrically similar small structures of materials with the same mechanical properties disseminated yielding precedes cracking. The size parameter which controls the cracking-yielding transition is ER/σ_y^2, where E is the Young modulus, R fracture toughness and σ_y yield stress. The paper describes a laboratory-size testing rig which enables the large structure fracture mode of materials with large ER/σ_y^2 to be simulated using small laboratory test pieces in conventional testing machines so that the appropriate fracture toughness can be determined. Some experimental results are quoted for thin warm mild steel and aluminum alloys. The rig also has interesting possibilities in producing stable cracking in $dR/d\dot{A}$ negative materials, where $\dot{A}$ is the time rate of change of crack area."

Hahn and Tsai (1974) write (Abstract, p. 160): "Design of composite laminates requires the determination of changes in stiffness and strength resulting from varying the ply orientation and total laminate thickness. Graphical determination can be made simple by use of laminate stiffnesses in conjunction with failure surface in a special strain space. A set of generalized graphs for all practical laminates of a given composite can now replace the current limitation of one set for each discrete laminate."

Harris and Lee (1974) write (Abstract, p. 101): "Examination of existing fatigue data on metal matrix composites obtained in tension-tension and plain reverse bending shows that a considerable amount of variability exists between systems. Particular attention has been drawn to the rather large difference between the cyclic behavior of aluminum-boron and aluminum-carbon fibre systems. The low fatigue strength obtained for the latter system is rather surprising since theory would suggest that small diameter fibres should give superior properties. Controlled fatigue experiments on the weakly-bonded copper-tungsten system show that fibre diameter does not appear to affect [sic] significantly the fatigue behavior of these composites. Closer examination of the data obtained indicates that very high fatigue stresses may be maintained at 10^6 cycles and that matrix hardening plays a significant part in maintaining this property level."

Heller (1974) writes (Introduction, pp. 335-336): "It is no news to anyone concerned with the fatigue of materials that fatigue life tests exhibit a large amount of scatter. This is true for constant, as well as for variable amplitude (and frequency) loading, for simple laboratory specimens and large structures, and

for all materials. The statistical variation in constant amplitude results has been traced through the physical mechanisms of fatigue to the dispersion of lattice defects and impurity atoms at the sub-microscopic scale and to the variations in crystal size and orientation in metals at the submicroscopic scale. In newer, so called, composite materials defects in the form of holes and discontinuous fibres as well as irregularities in the orthotropic arrangement of laminae are the basic causes of statistical variability. While size effects can also be explained on the basis of defect density, crystal size and preferred orientation, damage accumulation under the influence of service loads requires consideration of the statistical description of such loads as well as an understanding of nonlinear interaction effects produced by structural response, load history, stress redistribution and crack propagation. Considerable effort has been devoted by researchers to establish statistical distribution functions for fatigue life and strength." The author discusses statistical distribution functions (especially Weibull and lognormal) for fatigue and their relation to cumulative damage rules and environmental interaction effects.

Jones, Liebowitz and Eftis (1974) write (Abstract, p. 639): "A method has recently been developed for determining a nonlinear fracture toughness parameter defined by the relation $\tilde{G}_c = \tilde{C}\, G_c$ where G_c is the critical elastic strain energy rate as defined by Irwin. The $\tilde{C}$ term is a function of the nonlinearity of the load-displacement test record... . For the case of a linear load-displacement record $\tilde{C} \to 1$ and $\tilde{G}_c$ reduces to the linear elastic result. The toughness parameter $\tilde{G}_c$ has been evaluated for a number of high strength aluminum alloys and compared with published G_c values for these materials. The tests were conducted on center-cracked sheets of 2014-T6, 2024-T81, 7075-T6 and 7475-T61 aluminum alloys under conditions of varying specimen geometry and displacement gage length. It was found that the values of $\tilde{G}_c$ obtained from displacement readings with a gage length of 2 in. generally agreed with published values of $G_c = K_c^2/E$ [K_c = plane stress single cycle critical stress intensity factor (psi$\sqrt{\text{in}}$); E = Young's modulus of elasticity (psi)]. The $\tilde{G}_c$ values were found to vary inversely with gage length and a/w ratios [a = half crack length, w = specimen width]. The variation in values for $\tilde{G}_c$ is of the same order of magnitude as the scatter in published values for G_c. However, $\tilde{G}_c$ appears to be less sensitive than G_c to changes in a/w."

Kanazawa, Machida and Miyata (1974) investigate the effectiveness of COD (crack opening displacement) tests and Charpy V notch impact tests and their relation to the size effect and the transition temperature. They conclude that the COD bend test is of practical importance because the critical COD, ϕ_c, a measure of fracture

toughness, can be measured using small sized specimens, which usually fracture under large scale yielding conditions in the realistic temperature region, at a considerably lower load than that of a tension test. They give empirical formulae relating the COD bend test and the Charpy V test and use them to plot the effect of plate thickness on transition temperature. The results agree well with published results from the Welding Institute (U.K.) up to thicknesses of about 35 mm.

Karpenko, Pogoretskii and Matseiko (1974) write (pp. 157-158 of translation): "The absolute dimensions, occurrence of stress concentrations, and the media in which they are used affect the fatigue limit of many important machine parts. However, the combined effect of these factors is unknown. Using a range of the same type of machines, the authors subjected notched specimens and some without notches, 5-200 mm in diameter, to pure bending tests, with rotation in a corrosive medium. The examined metal was 35 steel ($\sigma_y \approx 30$, $\sigma_v \approx 60\text{kg/mm}^2$) and 38KhNMA steel ($\sigma_y \approx 50$, $\sigma_v \approx 75\text{kg/mm}^2$). ... 3% solution of NaCl in tap water was used as the corrosive medium. ... The investigation showed...that an increase in the diameter of a smooth specimen (without notch) from 5 to 200 mm considerably decreased the fatigue limit, i.e., the scale factor $\varepsilon_\sigma = \sigma_{-1}^{200}/\sigma_{-1}^{5} = 0.7$, where σ_{-1}^{200} and σ_{-1}^{5} are the fatigue limits of a specimen with 200 and 5 mm diameter, respectively. If these curves are extrapolated to a diameter 1000 mm, we obtain $\varepsilon_\sigma \approx 0.6$. On testing similar specimens in a corrosive medium, on the other hand, the decrease in endurance was the greater, the smaller the diameter of the specimens. ... The results of fatigue tests carried out on specimens with stress concentrations showed that their endurance in air was sharply reduced owing to the mechanical notches... . Moreover, the dimensions of the specimens made of both steels had practically no effect on this decrease in endurance. ... Stress concentration reduces the corrosion fatigue strength. ... An increase in the dimensions of the specimen increases its endurance. Apart from that, the smaller the diameter of the specimen, the more pronounced is the effect of stress concentration. ... The higher sensitivity to stress concentration of small specimens compared with the large specimens is due to the increased sensitivity of the former to the effect of the corrosive medium. Conversely, the larger the specimen, the smaller is its sensitivity to stress concentration, and in certain conditions its endurance in a corrosive medium may be even greater than in air. The favorable effect of the corrosive medium on the decrease in stress concentration is due to the corrosive erosion of the notch bottom (blunting) and to the appearance of a number of cracks which propagate simultaneously and lead to the unloading of the main notch. In small specimens these factors may not have a favorable effect on the fatigue process, since various corrosive and mechanical breakdowns lead to a large relative decrease in the working

cross section, and consequently also to a sharp increase in actual stresses leading to an intensification in the corrosion fatigue breakdown. The results described show that the supporting power of large notched specimens (shafts) operated in a corrosive medium should not be assessed from results obtained using small specimens, since this may lead to an excessive overestimation of the safety factor, and consequently to excessive weight of the structure."

Kowal and Lemiesz (1974) write (Summary, p. 129): "In the paper are defined the conditions which should be fulfilled in order that there was a probabilistic model similitude between the tie in the object and that one in the model. The authors prove also the theorem, due to which the safety of non-uniformly tensioned tie in the object may be determined in terms of safety of the tie in the model. These considerations are exemplified and some directions are given how to apply in practice the results obtained for determining safety of tie constructions, especially the suspension bridges used in industry communication and transport." If the conditions of similitude are satisfied and if the safety of the model (the probability that it will not plasticize or rupture under a non-uniform load) is P, then the safety of the tie in the object is P^n, where n is the ratio of the lengths of the ties.

Kreider (1974) writes (pp. 17-23): "A simple expression has been used...to relate composite strength to constituent properties: $\sigma_C^* = \sigma_F V_F + \sigma_M V_M$ (12) where σ_C^* is the ultimate strength of the composite expressed as a stress based on the original area, σ_F is the average stress on all the fibers, and σ_M is the average stress on the matrix at failure. V_F and V_M are the volume fractions of fibers and matrix. If there is no porosity or third phase, $V_F + V_M = 1$. If all the fibers have approximately the same strength and the remaining matrix cannot sustain the load at fiber failure, σ_F can be equated to the average strength of the fibers and σ_M can be interpreted as the stress on the matrix at a strain equal to the fiber strain at failure. ... The effective strength of the fibers that should be used in Eq. (12) is not as simple to determine in the case of brittle fibers having a significant range of tensile strengths. Although the average fiber strength can be used when the fibers have similar strengths, this average strength does not predict composite ultimate tensile strength in the case of brittle fibers such as boron. For example, Rosen (1964) has determined the composite strength as a function of fiber-strength coefficients of variation for composites with test gauge lengths equal to the critical stress-transfer length. A plot of this type is given in Fig. 11 [not reproduced here]. Note that a coefficient of variation of 20% would lower the composite strength about 35% according to this theory. Three significant modifications are generated when weak fibers fracture. First, the cross-sectional strength at the location of the broken

fiber is lowered by the loss in strength of that fiber. Second, a static stress concentration around the crack generated by the broken fiber can lower the effective strength. Third, the dynamic stress wave generated as the broken fiber unloads can shock the composite, thereby lowering the instantaneous load-carrying ability of that cross-section. ... Analytical treatments of the concept of the bundle strength of fibers and how it relates to the statistical probability of failure in brittle fibers have been given by Corten (1967) and others. This analysis has been based primarily on the Weibull function, which relates the probability distribution function of flaws in a brittle elastic solid to the size of the specimen. ... In summary, the relationship between constituent properties and composite strength is much more subtle than the one that relates the elastic moduli, because the strength is a point function rather than an average material constant. Although the rule of mixtures can be applied to composites in which the effective fiber strength can be well predicted, it is not nearly so accurate when brittle reinforcing fibers are used."

Lee and Harris (1974) write (Abstract, p. 359): "Unidirectional and cyclic tensile stress-strain testing has been carried out on continuous tungsten fibre reinforced copper composites, with fibre diameter from 11 to 48μm at a volume fraction of 0.37. In tensile tests the composites showed positive deviations from the rule of mixtures, the amount increasing with a decrease in fibre diameter and, therefore, interfibre spacing. This matrix strengthening continued to failure and was shown to be related in part to the matrix grain size. In the cyclic stress-strain tests the matrix strengthening was approximately the same for all the composites and was greater than for the tensile tests. The strengthening could be accounted for by the formation of a substructure during cycling of approximately 0.5μm cell size."

Madison and Irwin (1974) write (Summary and Conclusions, p. 1347): "Using 3-in. (76-mm) deep, 12 in. (300-mm) long single-edge-notched specimens of A441 steel in bending, K_c [plane-stress] values can be determined from the maximum load measurement without serious inaccuracy until the value of $(K_c/\sigma_{YS})^2$ [σ_{YS} = uniaxial tensile yield strength] becomes larger than 1.5 in. (3.8mm). Although larger specimens would be desirable for some purposes, the size of the starting crack of the specimen, 1 in. (25mm), is clearly large enough to represent a crack of significant size in a service component. Thus the specimen size employed in this project is large enough for a wide range of practical applications. The method permits evaluations of K_c based upon initial crack size and the observed maximum load of the fracture test. However, the method deviates significantly from the ASTM K_{Ic} [plane-strain] method with regard to certain testing practices which, if used, might tend to give lower fracture toughness values. From present evidence the writers believe that the testing method con-

sidered herein provides toughness estimates that are conservative relative to most active service cracks. Furthermore, the method is applicable to full thickness tests of plate materials, up to 2 in. (51mm). ..."

McClintock (1974) writes (Abstract, p. 93): "From a simple statistical model of occasional cracked grain boundaries, a statistical distribution of strength is derived which does not fall into any of the three asymptotic forms of extreme-value distributions. The size effects for this new extreme-value distribution are similar to those of the third asymptote [Weibull distribution] with an exponent [shape parameter] of about m=4, but it is necessary to drop 5 or 6 standard deviations below the median in order to reduce the failure probability to 1 in 10^6, which corresponds more to the first asymptote, m=∞. Stress gradient effects, leading to notch insensitivity, are reviewed for the third asymptotic distribution, and a method for correlating scatter in strength with position of failure in three-point bend specimens is derived and illustrated."

Metcalfe and Klein (1974) write (pp. 129-131): "Cooper and Kelly (1968) [pub. 1969] have divided the mechanical properties of composites into those affected by the tensile strength of the interface σ_i, and those dependent on the shear strength τ_i. For longitudinal tensile loading, they conclude that the tensile strength of the interface is not critical, but the interface shear strength controls the following properties: (1) Critical or load transfer length (the filament length required for the longitudinal stress in the filament to reach its fracture stress). (2) Composite fracture under conditions of fiber pullout. (3) Deformation of the matrix in fracture. The critical or load transfer length ℓ_c is given by $\ell_c = \sigma_f\, d/(2\tau_i)$ (1) where d is the diameter of the filament, σ_f is the strength of the filament, and τ_i is the shear strength of the interface. This length decreases as the interface strength increases and reaches a minimum value when the shear strength of the interface equals the shear strength of the matrix (this neglects any difference in shear area for interface failure and for failure in the matrix a small distance away from the interface). The load transfer length influences the fracture of composites with noncontinuous filaments in regard to filament pullout, fracture energy, and fracture path. The noncontinuous filaments may arise from filament fractures or be present in the original composite. A composite with strong interfaces and uniform properties throughout fibers and matrix will fracture on a plane normal to the direction of the applied stress and the fracture will be smooth. If the filaments are nonuniform due to weak points or discontinuities, then the crack will be deflected so as to link the weak points. ... The distribution of these weak points or defects along the filament (length-strength effect) and their relative intensities control the failure

of filaments and have an important influence on fracture and fracture energy. In the extreme case the length-strength effect must include filament breaks leading to discontinuous filament composites. Although Rosen (1964) [pub. 1965] and others have treated the case of fiber strength characterized by Weibull distribution, the assumption of a strong interface was made. Cooper and Kelly (1968) [pub. 1969] have treated the simple case of filaments with uniform properties but containing weak points. If these weakened points have strength σ^* in filaments of uniform strength σ, then the filament will break at a distance y from the fracture plane if $y<[(\sigma-\sigma^*)/\sigma](\ell_c/2)$ (2) where ℓ_c is the critical length and is a function of the interface strength (Eq. 1). When y is greater than the distance given by Eq. (2), the interface will be strong enough to reduce the filament strength below the reduced strength σ^* at this distance from the fracture plane. The critical spacing of weak points along the filaments is half this value, or $d_{crit} = [(\sigma-\sigma^*)/\sigma]\ell_c$(3). If the actual spacing along the filament $d<d_{crit}$, all filaments break at these weak points. If the spacing $d>d_{crit}$, then that fraction of the filaments with $d\leq d_{crit}$ fails out of the plane of the crack. Cooper and Kelly (1968) [pub. 1969] developed simple equations for the effective reinforcing strength of the fibers, the average pullout length and the mean work of fracture per fiber."

Moses (1974) writes (Introduction, pp. 1813-1814): "Reliability-based specifications and design procedures are usually aimed at design of individual elements such as beams and columns. ... The individual elements, however, are almost always part of a structural assemblage in which member and system interact so that additional reliability-based analysis is required to further rationalize the design. For example, it is useful to determine whether the overall structural system reliability will be safer or less safe than the reliability level assigned to the member. This factor depends on indeterminacy, ductility, failure mode characteristics, and the general 'fail-safe' nature of the design. Fig. 1 [not reproduced here] shows two structural examples and reliability models that characterize their respective safety. The first model...is a 'weakest-link' chain type model in which system failure occurs if any element fails, as in a statistically determinate structure. The second model...is a parallel system ('fail-safe'), as occurs in collapse analysis of a statically indeterminate structure in which failure results only after several elements reach their strength capacity. For practical applications the reliability analysis is further complicated by the fact that most structures exhibit characteristics that are a combination of the weakest-link and fail-safe models. ... Although a design goal may be to optimally proportion structures for a specified overall structural reliability, in the foreseeable future design will still need to be on a member of [sic; or (?)]

element basis. System reliability analysis will be needed to develop 'partial' safety factors to relate member to system reliability. This partial safety factor is defined as the quantity by which the element safety factor should be changed to account for its presence in a structural assemblage so that the overall failure probability remains roughly equal to what is desired of the element. The partial safety factor could also be adjusted to account for consequences or costs of assemblage failure as opposed to typical element failures. ..."

Niyogi (1974) writes (p. 1700): "It is known that for a given material, differences in size and shape of specimens have a marked influence on the observed strength of the material -- the strength falls as the size of specimen increases. The influence of size variation on the local bearing strength of concrete as a result of change of dimension of the specimen in one dimension, either lateral or vertical, has already been reported earlier [Niyogi (1973)]. The effect of size variation of the specimen as a whole or scale effect has been considered herein by using geometrically similar test specimens, e.g. cubes... . Table 13 [not reproduced here] presents the relative values of n [bearing strength ratio] for different sizes of cubes in terms of n obtained for 6-in. (150-mm) cubes under similar R [ratio of areas of loaded surface of specimen and bearing plate.] Within the limitations of the size and type of specimens tested herein it would appear that n for a constant R decreases by approx 25%, 15%, 12%, and 4% with the increase in the size of specimens by scale factors 3, 2, 1.5, and 1.33, respectively."

Ohlson (1974) writes (p. 460): "The purpose of this study is to determine the influence of layer thickness and joint strength on the fracture toughness of laminated steels. The apparent fracture toughness K_Q as a function of the specimen thickness is illustrated in Fig. 2 [not reproduced here]. The yield stress, σ_S, of the material was 800 MN/m^2. Under these conditions, a steel plate of 2mm thickness fractures under plane stress [with $K_Q \simeq$ 200 $MN/m^{3/2}$], whereas a plate which is more than 10mm thick (approximately) fractures in plane strain [with $K_Q < 100$ $MN/m^{3/2}$]. It is then reasonable to expect that a specimen built up from five plates of thickness 2mm should display the same toughness as a single thin plate." After describing the experimental procedure, he writes (p. 465): "The fracture toughness K_Q is a function of three independent variables, namely, $K_Q = K_Q(\sigma_S, d, x)$ where d is the relative layer thickness when compared with the minimum thickness required for obtaining plane strain, $\delta_{PD} = 2.5(K_{Ic}/\sigma_S)^2$ and $d = \delta/\delta_{PD}$, δ being the thickness of one plate. x denotes the relative joint strength, $x = \sigma_{UTS\ joint}/\sigma_{UTS\ steelplate}$. The value of $\sigma_{UTS\ joint}$ was determined in a tensile test for each type of joint, whereas σ_{UTS} for the steel plate is difficult to determine and has been replaced by the yield stress

σ_s." He reports that, as expected, high fracture toughness was obtained for low values of x and d.

Padawer (1974) writes (Abstract, p. 333): "The notch sensitivity of boron film (B/PI) reinforced graphite fiber composites was compared to the behavior of fiber-only material by means of uniaxial tests on tensile coupons with two sizes of holes (.052 and .104 in.) drilled at midgage length, and on undrilled control coupons. The results indicated that B/PI additions to graphite fiber laminates reduced the notch sensitivity, while at the same time substantially raising both the tensile strength and the work-to-fracture of the material."

Pomey (1974) reports the results of experiments, on butt-welded mild steel specimens, of which one of the purposes was to determine the effects of the size and the shape of the pieces on the fatigue limit. He notes that the specimens tested are usually small, while the actual pieces in use may be very large. One expects a size effect not only because of the greater chance of a flaw in a larger specimen, but also because of residual stress in large pieces. The size effect is rarely known with precision but, in general, the endurance decreases as the dimensions increase. The possibilities of varying the dimensions of specimens in fatigue tests are very limited, so that extrapolation does not allow calculation of the size effect in large pieces. However, in the case of butt-welded joints in mild steel, the size effect can be stated precisely. Some experimental results which the author has tabulated indicate that the fatigue limit decreases as the width of the specimen increases up to about 200 mm. and as its thickness increases up to about 26 mm., but is unaffected by further increases in either width or thickness.

Radchenko, Shapovalova, Gorskii and Kovalenko (1974) report similar strength values for microscopic and 2 mm. diameter specimens of high-strength thin-walled tubes. They fit a parabolic (almost linear) equation relating strength to hardness.

Sedov, Cherepanov and Parton (1974) write (pp. 217-218): "Let us distinguish between two conceptions: the metallurgical strength and the structural strength. The former one is understood as a tensile strength value obtained while experimenting in a laboratory with smooth samples of a standard size cut off from a supplied material. The real strength of a structure made of this material (i.e. the structural strength) is far less sometimes. The influence of the structure configuration upon the structural strength is defined here as the 'scale effect'. The scale effect is absent if the tough fracture occurs. In this case the strength dependence upon the body configuration is to be determined by a calculation, based on a chosen model of the body and the local condition of fracture, in accordance with some theory of strength. For ideal elastic-plastic bodies, the necessity to make use of a special

strength theory does not arise so that the strength is calculated within the scope of the very model. If the brittle fracture is realized, the scale effect is always present. In this case the strength dependence on the configuration and size of a body (form and size of cracks and other crack-like defects being included) is to be evaluated according to the theory of Griffith and Irwin on the basis of the model of an elastic body." Concerning the physical nature of the scale effect, the authors write (p. 218): "The brittle fracture is a rather rapid process characterized by absence of necking and by the orientation of cleavage surfaces along planes with maximal tensile stress. Under the condition of tough fracture, when considerable plastic deformations occur, a neck is formed on a sample and the cleavage surface is oriented along a plane with maximal tangential stress. However, for real materials some combination of the brittle and tough types of fracture is always realized. When the scale of a sample increases and there are slits and concentrations a tendency toward rise of the probability of brittle fracture is observed." Concerning the statistical nature of the scale effect, they write (p. 220): "The strength of a material represents always a certain random quantity because, first, the exact location of all defects is unknown a priori and, second, if even this location were known, solving of the corresponding mathematical problem would be impossible due to its complexity. A simple consideration that the probability to come across the most large-scale and dangerous defect is larger for a large volume of a sample forms a background for explanation of the scale effect in terms of the statistical theory."

Signorelli (1974) writes (pp. 246-250): "Fiber size is another variable [besides wire strength] with which to increase composite strength. Since matrix-fiber reaction is the primary cause of reduction of properties and since the degree of property loss for refractory-wire composites has been related to the depth of penetration of reaction into the fiber, composite strength can be increased by increasing the unreacted fiber core area. As shown in Fig. 11 [not reproduced here], the depth of penetration of reaction was essentially the same for a smaller-diameter fiber as for a larger-diameter fiber. However, the area percent of unreacted core is considerably larger for the larger fiber. The smaller-diameter fiber has a higher rupture stress than the larger, so the two effects must be balanced. For very short-time service where the depth of penetration of reaction is very slight, the smaller fiber results in higher strength composites; for longer times, the larger is superior. ... It should be noted that the tradeoff between fiber size, reaction area, and strength is a function of the fiber properties."

Tsai and Hahn (1974) write (Abstract, p. 493): "Survey of available experimental results indicates that the applicability of LEFM approach is by no means universal and

depends on the type of composites and the loading and specimen configuration. The limiting factors associated with LEFM should be addressed and broadened to accommodate the inherent difference between homogeneous materials and composites such as anisotropy, heterogeneity and the lack of actual through cracks in composites. This paper discusses several variations of LEFM in conjunction with the unique fracture behavior of noncoplanar crack extension and K_I - K_{II} coupling in unidirectional composites. Most laminated composites show notch sensitivity, the degree of which reduces with the extent of subcritical damage before final failure. It seems necessary to understand the growth of this subcritical damage in order to assure satisfactory structural performance of composites." On p. 500, they write: "Fracture of laminated composites poses much more difficulty because the problem frequently becomes no more two-dimensional but three-dimensional, let alone heterogeneous. Depending on the type of material system and lay-up, there is an extensive variety of failure modes which seems to defy any simple characterization that the two-dimensional LEFM is capable of. As such, the current approach to fracture of laminates should be regarded as a methodology rather than as an attempt for a detailed explanation of fracture behavior. Irwin's idea of incorporating the effect of plastic zone into fracture toughness of quasi-brittle material was taken up in [a paper by Waddoups, Eisenmann and Kaminski (1971)] and extended to explain the hole size effect."

Whitney and Nuismer (1974) write (Introduction, pp. 253-254): "Experimental data...has shown that the tensile strength of laminated composites containing a circular hole depends on the hole size for diameters less than 1.0 inch. It is well recognized that such a phenomenon cannot be predicted by a classical stress concentration factor (SCF). Waddoups, Eisenmann and Kaminski [(1971)] used concepts of linear elastic fracture mechanics (LEFM) to explain the hole size effect. ... For purposes of tensile strength determination, Cruse [(1973)] modeled a circular hole of radius R as a crack of half length R*. The dimension R* was determined by comparing the stress distribution for a circular hole in an orthotropic material of finite width with that for a crack in the same material. ... Having determined the effective crack half length R*, the strength of the laminate was determined by using LEFM in conjunction with the critical stress intensity factor K_Q for the material, which was determined from an independent test. In the present paper two stress criteria are discussed for predicting the strength of laminated composites containing through the thickness discontinuities. The criteria result in the prediction of discontinuity size effects without applying principles of LEFM. Analytical results do, however, yield a direct relationship between Mode I fracture toughness and unnotched laminate tensile strength. Although the stress criteria could be applied to any geometric

discontinuity, the present work is limited to circular holes and straight cracks, by virtue of the availability of experimental data."

Zaitsev and Wittmann (1974) write (Conclusions and discussion, p. 109): "It can be concluded that strength of hardened cement paste and concrete is strongly dependent on the pore size distribution and on the distribution of pore shapes. The overall pore volume has only an indirect effect on strength under uniaxial and biaxial compression. With this statistical approach the experimentally determined scatter of strength values can be directly linked with differences in the distribution of pore size and pore shape. This result can serve as a basis for a better understanding of reliability of porous building materials."

Zweben (1974) writes (Conclusions, p. 9): "A method of analysis for notched unidirectional composites has been proposed based on an approximate model which includes effects of nonelastic matrix and interfacial behavior. The results of the present analysis agree reasonably well with those obtained from analyses based on models which consider an infinite array of fibers. The predictions of the present analysis provide fairly good correlation with experimental data for graphite/epoxy. In light of the limited amount of data, it cannot be concluded that the present analysis provides a general solution to the problem of analyzing notched unidirectional composites. However, the model does provide insight into some mechanisms of failure and effects of heterogeneity. It is evident that there is a need for additional carefully controlled experimental and analytical studies in order to establish methods for the characterization of the fracture behavior of composite materials."

Bansal, Duckworth and Niesz (1975) write (Abstract, DD Form 1473): "(1) Bend strengths of a commercial sintered alumina, 3M Company's Alsimag 614, are reported. Fracture stresses in specimens differing in each linear dimension by a factor of five were measured at room temperature under environmental conditions which either minimized or enhanced subcritical crack growth prior to catastrophic fracture. Strength was found to be dependent on specimen size under both test conditions. Fractographic identification of strength-controlling flaws, coupled with the analysis of strength-size data, indicated a limited applicability of Weibull's statistical approach.
(2) In interpreting the strength of ceramic materials, the size of the fracture-initiating flaw is invariably described by a single dimension, e.g., by the depth of a surface flaw. A variable factor, Φ, is then used to account for the flaw shape, and determination of Φ requires knowledge of the ratio a/c, where a and c are the axes of an ellipse which describes the flaw shape. The following presentation shows that this complexity is not necessary; a, Φ, and c can be eliminated, and the flaw area, A, used to describe the flaw size."

Batdorf (1975) writes (Abstract, DD Form 1473): "An approximate technique sometimes used for estimating the probability of failure of brittle materials under polyaxial loading conditions is to assume that the effect of each principal stress is independent of the presence of the others. It is shown that this assumption is generally unconservative, and that the errors can be large. An exact treatment is given in terms of Weibull theory for materials that obey Weibull's two-parameter formulation, and a simple conservative approximation is suggested for such materials."

Batdorf, Huddlestone and Pridmore-Brown (1975) write (Abstract, DD Form 1473): "The reasons why an improved understanding of the statistics of failure of brittle materials is important to the armed services are briefly reviewed. ... The tasks accomplished in FY75 are discussed in some detail... . The major items of interest are: (1) a literature search for fracture data useful for validating or correcting theories, (2) recent discoveries that relate the theory for surface-distributed cracks to that for volume-distributed cracks, (3) work on the graphite model of Buch, Zimmer, and Meyer, (4) effects of porosity and crack interaction, and (5) the significance of asymptotic forms (such as the Weibull theory) in extreme value statistics."

Bowen and Ayres (1975) report the results of an investigation of buckling and failure in compression of panels made from carbon fiber-reinforced plastics faced with aluminum alloy. They write (pp. 72-74): "A series of panels with differing widths, lengths and thicknesses were produced Table 2 [not reproduced here]. From these data, theoretical predictions of the buckling stress and load at failure were calculated. ...There were two panel lengths (228.6 and 330.2mm), five panel widths (101.6, 127.0, 152.4, 177.8 and 203.2mm), with three composite thicknesses. Differing gauges of aluminium alloy sheeting (0.4826, 0.5588 and 0.7112mm) determined the composite thickness. The core consisted of a balanced, eight-layered, cross ply laminate of 1.016mm thickness, with the outer fibers parallel to the direction of loading. The comparison of experimental and theoretical results may be found in Table 2... . ...A comparison of panels 1, 2, 3 and 4 with 8, 9 and 10 (similarly 5, 6, 7 and 14, 15, 16) shows that failure is primarily dependent on panel thickness, with panel width having a secondary effect." The dimensions were not varied over a sufficiently wide range to allow comprehensive assessment of the size effect.

Brassell, Horak and Butler (1975) write (Conclusions, p. 295): "The transverse tensile strength of filament wound/CVD [chemical vapor deposition] carbon-carbon composites is greatly reduced by increasing volume fraction of porosity. ...Parameters influencing the effective cross-sectional area and thus affecting transverse tensile strength besides volume fraction porosity include: pore size, geometry, and orientation. ..."

Broutman and Gaggar (1975) write (Introduction, pp. 1-3): "The fracture behavior of composite materials has been studied by a number of investigators in recent years. ...Recently, Owen and Bishop [(1973)] have carried out fracture toughness tests on a polyester resin containing various forms of glass reinforcement. Using the procedure outlined in ASTM E399-70T, they investigated the effect of crack length by testing geometrically similar specimens of increasing size. It was shown that the critical stress intensity factor K_c values were not independent of crack size but a method similar to the plastic zone correction in metals can be used to obtain K_c values independent of the crack length. They used the K_c values obtained from fracture tests in Bowie's solution to predict the failure strength of specimens containing a hole at the center. They finally concluded that applicability of LEFM [linear elastic fracture mechanics] to glass reinforced plastics depended on the type of reinforcement and the orientation of the crack in the specimen. Ellis and Harris [(1973)] studied the effect of specimen size and test variables on fracture properties of some fiber reinforced epoxy resins. They used carbon fibers and silica fibers to produce unidirectional composites and the starter crack was positioned in a direction perpendicular to the fiber mats. A variety of test methods and specimen designs were employed to measure the work of fracture values. It was concluded that the work of fracture values depended on the dimensions of the test specimen, crack length and the type of test employed. They reported good agreement between the fracture energy obtained from the compliance tests and the work of fracture values obtained from the slow notch bend tests. ... More recently, Mandell et al. [(1974)] have reported some results dealing with the effect of notch root radius, specimen thickness, notch configuration...on the candidate stress intensity factor for a number of composite materials. It was reported that the candidate stress intensity factor K_Q for a roving/mat and 181 style fabric reinforced composites were almost insensitive to the thickness of test specimens over a broad range. ... The K_Q values were also observed to be insensitive to notch configuration and notch root radius up to a notch root radius value of 0.1 inch. ... The objective of this investigation was to study the fracture behavior of random glass fiber composites. ..." The authors state (Conclusions, p. 29): "Notch bend tests and double edge tension tests provide K_Q values independent of the a/w ratio [a = initial crack length, w = specimen width] in the range considered in this study. ... It was also shown that the K_Q was insensitive to the thickness of the test specimen in the broad range of thicknesses considered in this study and thus the complications due to thickness variations which arise in most metals are not present in random fiber composites."

Campo (1975) writes (Abstract, DD Form 1473): "A limited number of bend as well as tension tests were performed at room temperature on specimens made of Norton HS-130 grade silicon nitride. By application of the two-parameter Weibull analysis for a material governed by volumetric flaw distribution, tensile properties of the specimen, based upon the data of both types of testing, were determined and compared. The results show, at least for the specimens tested, that the bend test tends to predict fracture stresses of the tension specimens approximately 8% higher than those obtained in the actual tension tests."

DeMorton and Ritter (1975) write (Summary, pp. 335-336): "The present state of plane strain (K_{Ic}) fracture toughness testing of metals has been reviewed with the aim of providing a comprehensive guide to testing procedure. Where possible, the review has centered on the efforts of the ASTM in developing standard methods of test such as E399 which have been adapted by BSI. ... It is shown how the choice of test specimen involves both shape and size which together must meet certain dimensional requirements on thickness and crack length to give a valid K_{Ic} test result, and must maintain the correct orientation relationship between microstructure, crack and applied stress in order to represent the practical situations. ..."

Dodd and Stone (1975) write (Abstract, p. 1): "The results of uniaxial tensile tests on thin-walled tubular steel specimens of varying gauge lengths are described. From these results it has been found that the ultimate tensile strength and the strain to fracture increase with decreasing gauge length but the lower yield stress plateau remains at a constant value of stress. It is suggested that the increase in the U.T.S. is caused by the constraints of the ends of the short gauge length specimens which produce approximately plane strain conditions in the wall of the tube. Optimum dimensions for thin-walled tubular specimens to be tested in uni-axial tension are suggested."

Green and Knott (1975) write (Summary, p. 167): "Critical crack opening displacement (COD) values have been examined for a range of specimen thicknesses. The COD at the initiation of fracture δ_I is found to be constant, given a plane-strain crack-tip stress-state, whereas the COD at maximum load δ_{max} decreases with increasing thickness. The loads required to produce instability are found to vary with thickness, in a way analogous to behaviour observed under linear elastic conditions. Crack growth under constant load for a range of specimen thicknesses has been examined, and failure has been found to occur at loads below that associated with δ_{max}; the minimum load per unit thickness required to cause failure decreasing with specimen thickness."

Hancox (1975) writes (Abstract, p. 234): "A simple compression test, suitable for quality control measurements on unidirectional carbon fibre composite, is described. The specimen, a plane bar, with aluminium end tabs attached, is compressed by applying shear forces over the ends. With either type 1 or type 2 treated fibre the failure mode is one of shear over a plane at approximately 45° to the fibre axis. With untreated type 1 material failure is due to delamination. The variation of the compression strength of treated material with fibre volume loading is linear, the values being considerably below those predicted by buckling theory. Increasing void content causes a steady decrease in compression strength, and off-axis strength values are above those given by the maximum work criterion. The present work supports the recently proposed view that the compression strength of unidirectional carbon fibre composites at room temperature is not governed by fibre buckling but is related to the ultimate strength of the fibre."

Heckel and Köhler (1975) report the results of an experimental investigation of the statistical size effect in fatigue tests, with unnotched specimens, in which the relation of specimen length to the cummulative distribution of fatigue life was determined. The specimens were 190 mm. length of austenitic chrome-nickel steel wire 5 mm. in diameter. Approximately 50 specimens each with proof-lengths 5 mm., 20 mm. and 70 mm. were subjected to vibration at a constant load in a high frequency pulsator. The numbers of cycles to failure were arranged in order from smallest to largest for the specimens of each proof-length, and the cumulative probability of failure of the mth smallest life (cycles) of n specimens of a given proof-length was taken as $m/(n+1)$. These probabilities were plotted against the life in cycles. The resulting graph shows a strong size effect on fatigue life, the median life (in thousands of cycles) for proof-lengths 70, 20 and 5 mm. being approximately 180, 210 and 270, respectively.

Jones (1975) discusses various statistical aspects of strength and load (including strength models and probability distributions, strength distribution functions for composites, and load-time histories for structures). He writes (pp. 38-42): "The statistical models used in the study of failure composite materials almost always take as a starting point Griffith's theory, which states that strength-reducing flaws reside within the material, weakening it. Accepting this concept, then the strength of a given specimen is determined by the smallest value to be found in a sample of size n, where n is the number of flaws. The problem of finding how the strength depends on the volume is equivalent to studying the distribution of the smallest value as a function of n, the sample size. During the last fifty years, considerable effort has been devoted to studying the distribution of extreme values (smallest or largest) in samples of size n drawn from a population possessing a prob-

ability density function $f(x)$. [See the Bibliography on Extreme-Value Theory at the end of this report.]... Insofar as application of the statistical theory to fracture problems is concerned, the primary interest is the distribution of smallest value in samples of size n (for large n). ... Epstein (1948[a]) concluded that the distribution of strengths due to flaws could be written in the general form $f(x) = A \exp(-B|x-\mu|^p)$ for large values of $|x-\mu|$, where A, B, μ and p are positive constants. On the basis of this, the most probable value of the smallest value in samples of size m must decrease as $(\log m)^{1/p}$ and the distribution of smallest values must be negatively skewed. This implies that given a physical phenomenon in which the weakest link hypothesis represents the true state, there is a statistical compulsion for skewed distribution of strengths of samples of the same size and an approximately semilogarithmic dependence of the most probable value of the strength (or whatever is equivalent to strength) on the size of the specimen or number of places at which a break may occur. This appears to account for the skewed distribution and semi-logarithmic relationship in studies of strength of materials. Two basic distribution forms are commonly used to reduce strength data for composite materials. They are the normal and Weibull distributions; the log-normal is frequently used to represent fatigue life scatter. ... The first person attributed [see, however, Chaplin (1880,1882) and Lieblein (1954)] with the realization that the strength of a specimen was mathematically related to the distribution of smallest values was Pierce [sic; Peirce] (1926) and he assumed a Gaussian distribution of strengths. This distribution was also assumed by Kontorova (1940) and Frenkel and Kontorova (1943) [translation of Russian paper by Kontorova and Frenkel (1941)]. ... Weibull (1939[a]) suggested that the probability of failure of a unit volume as a function of the stress σ is given by $F_0(\sigma) = 1-\exp[-(\sigma/\sigma_0)^m]$ where σ_0 and m are unknown parameters characteristic of the material under test. This distribution has found considerable application in describing the strength behavior of composites... . A frequent criticism of the Weibull distribution is that it is devoid of physical meaning, the normal distribution being viewed as Nature's own. At the moment, there is insufficient knowledge to say whether the Weibull distribution is less suitable as an _a priori_ distribution than one that is purely normal. Both involve two free undetermined parameters. The Weibull does implicitly involve the principle that it is the weakest link that determines fracture strength. It is easy to calculate the maximum (mode or most probable value) of the distribution of strengths...in each distribution." Jones also includes a section on design applications of the Weibull distribution (one being its use in study of composites) and an appendix on estimation of Weibull parameters.

Larder and Beadle (1975) write (p. 241): "Brittle fibers such as glass monofilaments exhibit decreasing strength with increased length. In the case of glass filaments, it has been shown that the strength is dependent upon submicroscopic flaws within the glass itself or on its surface as a result of handling damage. Experimental data show considerable scatter, and Herring [(1966)] describes a method using the Weibull distribution for characterizing this effect. Metcalfe and Schmitz [(1964)] present data for glass fibers showing that logarithmic plots of mean strength versus length are not linear, but exhibit a change in slope at a critical fiber length... . They suggest that the strength of glass fiber elements depends upon flaws falling into two or more distributions, where the average spacing between flaws varies with their severity. This note illustrates a method for determining a statistical distribution of strength exhibiting the change in slope as shown above. The work was prompted by the need to describe such a single glass fiber material behavior in a finite element analysis simulating the failure of parallel composites."

Liu (1975) writes (p. 110): "It should be pointed out that when using existing smooth specimens fatigue data to predict the fatigue life of a fatigue element the size effect is neglected. The size effect is also neglected when comparing the estimated fatigue life of the limiting case of a composite laminate with existing large specimen fatigue data, because the effect of specimen size on the fatigue life of a specimen with a stress raiser is relatively unimportant as compared to that for a smooth specimen. An explanation of the size effect on fatigue life may depend on the difference in the type of stress-raiser causing fatigue failure. For smooth specimens, fatigue damage starts at defects or inclusions of some kind which are presumably randomly distributed in the specimen. Therefore, the larger the specimen the greater is the probability of the presence of fatigue damage nuclei. This is also true to some extent in a notched specimen. But the volume of the material which is subjected to the peak stress at the notch tip is quite small even in a wider specimen. The probability of the presence of a randomly occurring defect within this volume with a stress concentration greater than that of the notch is slight. Therefore, it is expected that the notch rather than a pre-existing flaw is the controlling stress raiser and the fatigue crack starts at the notch tip."

Phoenix (1975) writes (Abstract, p. 287): "This paper discusses the asymptotic tensile strength distribution of a special class of classic fiber bundles. The bundles are assumed to be composed of several sub-bundles wherein the random variables associated with the loading and failure of any two fibers in the same sub-bundle are allowed to be probabilistically dependent. Derivation of the basic asymptotic results involves an adaptation of earlier analysis by Phoenix and

Taylor [(1973)]. Examples involve a modification of Daniels' [(1945)] earlier bundle model to include both random fiber and random sub-bundle slack. Second order approximate results are developed which indicate that random slack results in a loss in asymptotic bundle strength mean and a change in asymptotic variance both of which are approximately proportional to the random fiber slack variance. Coleman's [(1958)] earlier model is modified to demonstrate the effects of such slack on the strength distribution of bundles of brittle linearly elastic fibers. The results have implications in forecasting the tensile strength behavior of fibrous cables and unidirectional composite materials."

Rosen, Kulkarni and McLaughlin (1975) write (p. 20): "The high strength of contemporary uniaxial fibrous composites is well recognized and widely utilized. In contrast, the present understanding of tensile failure mechanisms and the influence thereon of constituent properties is limited. There are, however, several existing analyses which provide critical elements of a rational theory for tensile failure of fibrous composites. It has been shown that, for ductile phases, composite strength can be related to fiber strength by the 'rule of mixtures'. For brittle fibers, computing composite strength by the rule of mixtures can be misleading because of the statistical variability of fiber strength as well as effects resulting from order of magnitude differences between fiber and matrix moduli. There are two important consequences of the statistical distribution of the strength of contemporary fibers. First, the strength of a group of fibers will neither equal the sum of the strengths of the individual fibers nor even their mean strength value. Second, distributed fiber failures may occur at relatively low stress levels. This will result in a localized perturbation in the stress field and unstable crack growth may result at stress levels lower than expected. In addition, there arises the question of the specimen size effect on the strength because of the probability of encountering a larger number of weaker points in a larger specimen. Statistical models for tensile failure of unidirectional fiber composites have been developed in References [1, 2 and 3] [Rosen and Dow (1972), Rosen and Zweben (1972) and Argon (1972), respectively]. ... In Reference [2] [Rosen and Zweben (1972)], three possible modes -- the fiber break propagation mode, the cumulative group failure mode, and the weakest link mode -- are studied. The influence of the size effect and the fiber strength variability is considered and a critical discussion of these failure mechanisms is also presented." The authors also confirm analytically the anomalous hole size effect in composite laminates by employing a shear-lag analysis which considers the fiber and matrix phases separately, and extend the shear-lag analysis to model a composite laminate with through-the-thickness notches.

Shih and Wei (1975) write (Abstract, p. 46): "The effect of specimen thickness on delay in fatigue crack growth, produced by load interactions in variable-amplitude loading, was examined. Experiments were carried out on 7075-T6 aluminum alloy sheets. The results indicate that delay is a function of specimen thickness. This influence of thickness, along with variations in behavior among different materials, suggest that extensive extrapolation of existing data should not be made."

Tsai and Hahn (1975) discuss two problems concerning the strength reduction of composite materials due to notches, one being the size effect that is not explained by the linear elastic stress concentration and fracture mechanics approaches. They write (Conclusions, pp. 93-94): "Most work in the area of strength prediction of notched composites has been done for the cases where the load is applied along one of the material symmetry axes. When the notch is a crack, the loading is further restricted to the direction perpendicular to the crack plane. Study is then focused on the variation of strength with the notch size. Each method proposed so far in this area [inherent flaw model, point and average stress criteria, and Barenblatt-Dugdale model] is invariably based on the concept of a critical region of one kind or another at the crack tip and relies on the linear elastic stress analysis solution. Noting that the stress intensity factor at ultimate failure asymptotically approaches a constant -- the fracture toughness K_c -- as the crack size increases, we can relate the dimensions of these critical regions to this K_c. These methods, however, show numerically insignificant difference from one another within the range of notch size for which they are intended."

Vardar and Finnie (1975) write (Abstract, p. 495): "It is pointed out that the Weibull multiaxial treatment of brittle strength contains limitations which are not present in the more familiar uniaxial formulation. Provided these limitations are satisfied, it is possible to use tension or bending data to predict multiaxial behavior when at least one principal stress is tensile. This is illustrated for the Brazilian disk test (diametral compression of a disk). Predictions based on bending tests agree well with observed strength values in disk tests on two types of rocks."

Vinson and Chou (1975) discuss, in sections 2.4.2 (pp. 66-74) and 8.2.2 (pp. 356-362), the effect of the length of discontinuous fibers on the strength of fiber-reinforced composite materials. In Section 8.4.4 (pp. 382-388), they discuss the statistical nature of brittle fiber-reinforced composites. They write (pp. 382-384): "High strength brittle fibres are non-uniform in their strengths. Glass and silica fibres as well as whiskers generally show considerable variation in strength. This is due to the high sensitivity of fibres to surface imperfections. ... The sensitivity of brittle fibres to surface imperfections is...reflected by their strength dependency

on fibre length. This is merely because a longer fibre has a greater chance of being weakened by surface imperfections. The understanding of the mechanics of the very brittle fibres in composite materials is best approached by examining the statistical nature of their failures. One of the earliest problems investigated was concerned with the strength distribution of unbonded fibre bundles. It was first pointed out by Peirce [(1926)] that the strength of a simple parallel array of filaments is not equal to the average strength exhibited by the component filaments when tested separately. A rigorous statistical theory of strength distributions for fibre bundles was first developed by Daniels [(1945)]. Coleman [(1958)] further investigated the implication of the work of Peirce and Daniels regarding the relative magnitudes of the strength of a large bundle and the mean strength of the component filaments. The following assumptions were adopted: (a) the long, continuous fibres in a bundle are uniform in cross-sectional areas; (b) the fibre tensile strength is independent of the rate of loading; and (c) the fibres come from a common source and the statistical properties of the strength distribution are independent of the distance along the yarn and thus free from long-range trends. Based upon these assumptions and, further, noticing that fibres break at their weakest cross-section, Coleman was able to show that the tensile strength distribution of separate long fibres obeys the Weibull distribution function [Weibull (1939a,b)]. The Weibull statistical strength distribution function is given by: $G(\sigma) = 1-\{1-[(\sigma-\sigma_\ell)/\sigma_u]m\}^{\omega}$. $G(\sigma)$ represents the probability of fracture of a fibre at a stress level equal to or less than σ. σ_u and σ_ℓ denote, respectively, upper and lower strength limits of the fibres. σ_u is believed to be affected by fibre composition and thermal history. The effects of surface damage may be measured by the parameter m. The parameter ω reflects the size effect [Corten (1964)]. Coleman then considered a bundle composed of a very large number of filaments of equal length. The bundle strength was defined as the force at break per unit initial area. ... Unless there is no dispersion in fibre strength, the tensile strength of a large bundle is always less than the mean fibre strength. The decrease in bundle strength relative to the average strength of a group of filaments is attributed to the progressive failure of filaments in the bundle at surface flaws. It is then obvious that the experimental results of single filaments tend to overestimate the strength of a bundle made of the same filaments. Statistical analyses are then essential to correlate filament strength with strand strength [Tsai and Schulman (1968)]. Furthermore, the interaction of fibre and matrix needs to be taken into consideration when the strength of a composite is investigated." The authors proceed to summarize the theoretical analysis by Rosen (1964,1965) of the "cumulative weakening" failure mode and Rosen's associated experimental results.

SECTION IV

ADDITIONAL LITERATURE

In this section, summaries are given of publications about which the compiler learned, or of which he was able to obtain copies, only after the portion of the manuscript in which they should have appeared, in accordance with the chronological arrangement, had been sent to the typist. The references are given under ADDITIONAL REFERENCES, along with a few references (marked with asterisks) which were inadvertently omitted from the main list, even though summaries were given in Section II or III.

We have seen [Timoshenko (1953)] that awareness of the size effect goes back much farther than we have indicated in Section I. In particular, Timoshenko discusses the contributions of Leonardo da Vinci (ca. 1500) and Galileo Galilei (1638). The former made experimental studies, described in some detail in his unpublished notebooks, of the effects of length on the strength of iron wires, of length and width on the strength of beams, and of length and cross section on the strength of columns. The latter studied the effect of the dimensions of geometrically similar structures on their strength, concluding that they become weaker and weaker as the dimensions increase. The reviewer (W.B. Stiles) of a paper-bound reprint of the English translation of Galileo's work states that conclusions concerning the relative effects of varying the length, width, and thickness of beams are correct except for hollow beams.

The work of Karmarsch on the relation between the diameter of metal wires and their strength, which culminated in the paper [Karmarsch (1859)] giving the empirical relation $F = A + B/d$ reported in Section I, actually began more than a quarter century earlier. Three earlier papers [Karmarsch (1833, 1834, 1858)] report qualitative and tabular results, but the empirical equation is found only in the 1859 paper.

Hodgkinson (1838) reports the results of experiments on crushing and on trans-

verse loading of cast iron. For the latter he found (p. 364), according to Todhunter & Pearson [(1886), p. 518], "that the assertion of practical men, that if the hard skin at the outside of a cast-iron bar be removed, its strength, comparatively with its dimensions, will be much reduced, is not true." Pearson, the author of this section of his joint work with Todhunter, goes on to remark: "Hodgkinson's experiments seem hardly conclusive; even if the strength be not reduced (which is in itself questionable) there can be little doubt that the change in the elasticity towards the surface would effect [sic] the strain (e.g. deflections) within the limit of elasticity."

Hodgkinson (1840) writes (p. 386): "In order to ascertain the laws connecting the strength of cast-iron pillars with their dimensions, they were broken of various lengths, from five feet to one inch; and the diameters varied from half an inch to two inches, in solid pillars ... ." On p. 395, he turns to the strength of long pillars as dependent on their dimensions (diameter d and length ℓ). Finally (p. 400 et seq.), he takes $d^{3.76}/\ell^{1.7}$ as a comparative measure of the breaking weight for pivoted struts with $d/\ell < 1/15$ and $d^{3.55}/\ell^{1.7}$ for flat-ended struts with $d/\ell < 1/30$. [See Todhunter & Pearson (1886), pp. 519-525].

Hodgkinson (1845) writes (p. 26): "From the experiments on the two series of pillars [stone prisms of square cross-section, with the sides of the bases being 1 inch and 1-3/4 inches and the heights varying from 1 inch to 40 inches] it appears that there is a falling off of strength in all columns from the shortest to the longest; but that the diminution is so small, when the height of the column is not greater than about 12 times the side of its square, that the strength may be considered as uniform; the mean being 10,000 lbs. per square inch or upwards. From the experiments on the columns 1 inch square, it appears that when the height is 15 times the side of the square, the strength is slightly reduced; when the height is 24 times the base, the falling off is from 138 to 96 nearly; when it is 30 times the base, the strength is reduced from 138 to 75; and when it is 40 times the base,

the strength is reduced to 52, or to little more than a third." [See Todhunter & Pearson (1886), pp. 776-777].

Wade (1856) gives (pp. 281-304), according to Todhunter & Pearson (1893, p. 696 of Part I), "a great deal of information...as to the effect of...the size of the casting on the tenacity [tensile strength]...; thus gun-head samples have hardly half the strength of small bars cast with the guns, and as a rule less strength than small bars cast in quite different moulds."

Barlow (1857) gives results [reproduced in tabular form by Todhunter & Pearson (1893), p. 639 of Part I] on the tensile strength T_1 and the resistance to flexure T_2 of cast iron beams of lengths 48 and 60 inches and various cross sections. The results show, as pointed out by Todhunter & Pearson, that the large bars are relatively weaker than the smaller.

Bell (1857) draws, from extensive experimental data on the strength of cast- and wrought-iron beams under flexure, the following conclusions [as summarized by Todhunter & Pearson (1893), pp. 732-733 of Part I]: "(i) For slight strain theory and experiment coincide. (ii) The ordinary theory of rupture practically coincides with experiment for wrought-iron beams, especially those of large size. [It should only do this {writes Pearson} if Hooke's law practically holds for wrought-iron up to rupture.]...(iv) There is a divergence between theory and experiment in the case of small cast-iron bars whose transverse strength is compared with their direct tensile strength, but the coincidence between these strengths for large girders is nearly exact. I [Pearson] think the latter statement requires further demonstration. We should not expect such equality because Hooke's Law does not hold for cast-iron, even in the case of small strains, and certainly not up to rupture."

Fairbairn & Tate (1859) give experimental results [cited by Todhunter & Pearson (1893), p. 595 of Part I] for the tensile strength of flint glass, as follows: 2286 lbs/in^2 for cross-sectional area 0.255 in^2 and 2540 lbs/in^2 for cross-sectional area 0.196 in^2.

Mallet (1859) discusses problems concerning the strength of wrought-iron. Todhunter & Pearson (1893) write (p. 735 of part I): "This is an interesting paper dealing principally with the influence of the bulk of a forging on its elastic and cohesive properties. The author on p. 298 states the three principal points of his inquiry as follows: (i) What difference does the same wrought-iron afford to forces of tension and of compression, when prepared by rolling, or by hammering under the steamhammer, the bars being in both cases large? (ii) How much weaker, per unit of section, is the iron of very massive hammer forgings that the original, or integrant iron, of which the mass was made up? (iii) What is the average, or safe measure of strength, per unit of section of the iron composing such very massive forgings, as compared with the acknowledged mean strength of good British bar-iron in moderate market sizes?"

Kirkaldy (1862) notes [see Todhunter & Pearson (1893), p. 744 of Part I] that the size of the bar in rolled iron has far more influence on the absolute strength of "inferior" iron than of iron of "superior" quality.

Barba (1880) reports the results of an experimental study of the effect of changes in the dimensions of metal specimens on their tensile strength. When either the length or the diameter of the specimen is decreased and the other is held constant, or when both are decreased proportionately, the strength tends to increase.

Todhunter & Pearson (1886) review the early work of Karmarsch (1833,1834) and other pre-1850 work on the strength of materials, including that of Galilei (1638) on the law of similarity and that of Hodgkinson (1838,1840,1845) on the size effect.

Todhunter & Pearson (1893) review the literature on the theory of elasticity and of the strength of materials which was published after 1850, with emphasis on the period 1850-1860. Several of the publications reviewed [Wade (1856), Barlow (1857), Bell (1857), Fairbairn & Tate (1859), Karmarsch (1859), Mallet (1859), and Kirkaldy (1862)] are relevant to the size effect. Of these, the author of the present report has seen only the paper by Karmarsch; in summarizing the others, he

has relied entirely upon the comments by Pearson. (The numbers of all the sections in which these publications are discussed are enclosed in brackets, which indicates that they are the work of Pearson, who completed and edited the manuscript after the death of Todhunter.) Concerning the paper by Karmarsch (1859), Pearson writes (pp. 737-738 of Part I): "Karmarsch...determines the constants a and b [in the equation $F = aD^2 + bD$] for a great variety of metal wires, but his method of selecting the results from which a and b are to be determined seems to me very unsatisfactory. He ought to have proceeded by the method of least squares, but he calculates a and b from a number of selected experiments by taking the arithmetical means. The process of annealing reduces the values of both a and b. The coefficient b can amount in the case of ordinary iron or platinum wire to as much as one half of a and sinks in the case of lead to zero, or to an insensible quantity. The relation of the absolute strengths of annealed and unannealed wires is not the same for the same metal, but varies with the diameter of the wire. Further for wires of the same diameter but of different metals it varies from a maximum with platinum to a minimum with iron."

Rudeloff (1894) reports the results of an experimental study of the effect of length of test specimen on the tensile strength of hemp rope. Data given in both tabular and graphic form shows that the strength tends to increase as the length decreases from 3.0 m to 0.25 m.

Stanton & Batson (1920) report a scale effect observed in impact tests. The present writer has not seen their paper, but their results have been summarized by Cook (1932) and Docherty (1932,1935) [see summaries on the following pages].

Taylor (1924) describes a method of drawing fine metallic filaments (down to a diameter of 10^{-5} cm or less), and discusses their properties and uses. He points out that such filaments have greater tensile strength than wires of ordinary size, but offers no theoretical explanation.

Pearce (1928) writes (Summary, p. 89): "The number of variables encountered in

foundry practice in the raw materials, in melting methods, and in moulding practice, combined with the range of tests by which strength is measured and the number of standard bars of any one test, have hitherto prevented the correlation of strength with size and of strength with composition, because the connection has been obscured. It is suggested that by concentration on the new standard cylindrical transverse bar, and by casting bars of varied sizes and compositions it becomes possible to chart relations between size and strength, and between composition and strength. It is shown that the transverse rupture modulus increases continuously as the test-bar diameter diminishes until the point is reached at which the metal ceases to be grey. The size-strength curve is a useful index to the behaviour of cast iron in thick sections... . The transverse strength is closely related to tensile and compressive strengths, but this relationship is only shown if the transverse strength is expressed as rupture modulus, based on area at fracture. It is hoped that the expression of transverse strengths as rupture moduli will become general. The change of the rupture modulus with the diameter is so rapid that the use of conversion formulas to bring a breaking load on a cast bar to a breaking load on a specified diameter may introduce serious errors. If the size-strength curve for a particular mixture and melting régime is known, deviations from the specified size can be corrected with much greater accuracy."

Davidenkov (1929) points out (p. 180 of the first edition) that it is possible in some cases to obtain all gradations from plastic to brittle fracture in impact testing of the same metal, by varying continuously one of the factors (form of notch, width of specimen, impact velocity, and temperature) which affect the yield point. He discusses the effects of the form of notch (pp. 180-183) and the width of the specimen (pp. 183-187), noting that the theoretical role of the width of the specimen has been explained earlier (p. 163). He cites the work of several authors who observed a decrease in strength with an increase in the width of the specimen. He also discusses the effects of length and thickness of the specimen (pp. 227-229) and

the depth of the notch (pp. 229-232), as well as the limited applicability of the law of similarity in impact testing of notched specimens (pp. 233-238) and the inconsistency of results obtained during impact testing (pp. 238-248).

Cook (1932) writes (Summary, p. 107): "The paper describes an investigation carried out to determine for three samples of mild steel, (1) the relation between the stress at the yield point in simple torsion and in the non-uniform distributions produced by torsion, flexure and internal pressure in a hollow cylinder, and (2) the stress distribution in each of the latter cases in the early stages of overstrain. ... It is shown that the maximum shear stress at the initial yield point is consistently higher in the non-uniform distributions than in uniform tension. In the cylinder a pronounced scale effect was observed. ..." On pp. 142-143, he writes: "The results of the yield point determinations are...consistent with a supposition that a surface layer exists, having an elastic limit greater than in the body of the metal. It will account fully for the apparently increased yield points in all the cases of non-uniform stress examined, and for the scale effect observed in the cylinder tests. The hypothesis is, however, by no means free from difficulty. A consequence, for example, should be a scale effect also in torsion and flexure tests. This is a matter for further investigation, all the specimens in the present series being of the same size. The author is not aware of any test results which show such an effect, except in the fracture of cast iron by flexure [Pearce (1928)] and this is probably due to a different cause. It is noteworthy, however, that a scale effect has been observed by Stanton and Batson [(1920)] in impact tests. Although similarity was maintained both in the specimen and in the testing appliances, the energy per unit volume (which has the dimensions of stress) was found to increase as the size was reduced. It appeared, moreover, that the effect was greater in steel of lower carbon content, a tendency also shown by the present tests. No explanation of the effect was given, but it was suggested that it might possibly lie in the fact that similarity had not been extended to the microstructure."

Docherty (1932) writes (p. 645): "When geometrically similar test pieces of the same material are tested under 'similar' conditions, the usually accepted principles of similarity indicate that, for similar strains (i.e., in this case at the same angle of bend), the loads applied should be proportional to the square of the linear dimensions of the specimen. Diagrams of load on a base representing strain should, therefore, have heights proportional to the square of the linear dimensions and lengths proportional to the linear dimensions of the specimens. The areas of the diagrams, and, therefore, the energy absorbed, should vary as the cube of the linear dimensions. The diagrams should be of similar form, and would be identical if load : (size of specimen)^2 were plotted on a base representing strain : size of specimen. The results of notched bar bending tests do not appear to follow this simple law, even when the tests are carried out on carefully prepared specimens of uniform material. Usually, the energy absorbed is found to vary with a power of the size which is less than 3, and which differs in different materials. The problem has been investigated by several experimenters [including Stanton & Batson (1920)]." The author describes experiments whose results led him to the following conclusions (p. 647): "If the energy per unit volume (or energy : ℓ^3), be plotted on a base representing the linear dimensions, the graph obtained is a horizontal straight line for very ductile metals such as M [Table II (not reproduced here)]. For the less ductile metals the energy per unit volume decreases with size in a curve which, over small ranges of size, can be satisfied very approximately, by either of the empirical relations $E = C \cdot \ell^n$, n generally lying between 2 and 3, or $E = A \cdot \ell^2 + B \cdot \ell^3$. It is suggested that this curve is merely a transition curve connecting two horizontal straight lines, the upper one representing the purely ductile action, $E = \alpha V_1$ ..., which would be found in all metals if the size of specimen were sufficiently reduced; and the other representing the much smaller value, $E = \beta V_2$, which would be found only in specimens much larger than any of those used in this experiment. (Figs. 5 and 6, in Stanton and Batson's paper already referred

to, seem to show that their larger specimens had nearly reached this condition). In metal M, the horizontal straight line, $E = \alpha V_1$, seems to extend up to the 12-mm. [by 12 mm.] size, and in metal L to 4 mm. [by 4 mm.]. In other metals it is less than 4 mm. No indication can be given as to the size at which the energy would once again become proportional to the volume, i.e., at which the line $E = \beta V_2$ would be reached, but a few isolated experiments recently carried out by the author seem to show that in some metals it may be possible to reach it within the limits of ordinary testing machines."

Reinkober (1932) reports that the elastic and torsional moduli of quartz glass were determined for a large number of thin threads of diameters 1 to 100μ, and found to increase strongly with decreasing thread diameter, as is known to be true for the tensile strength of thin threads [see Reinkober (1931)]. Unlike the tensile strength, the elastic constants do not change with the age of the materials. Several measures of the breaking strength were obtained from bending tests; the values of the tensile strength derived from them show the same dependence upon the thread diameter as those ascertained directly.

Schurkow (1932) reviews the results of Griffith (1920) and other authors on the strength of fine glass threads. He reports results of tests of the strength of quartz threads, 1 to 10μ in diameter, in vacuum, oil, alcohol, and water. The strength in these media differs somewhat from that in air, but the size effect is still present, and the relative decrease in strength as the diameter increases is about the same as in air.

Bussmann (1934) reviews the results of Faulhaber (1933) and other authors on the effect of test specimen diameter and notch depth on the fatigue strength of steels. He states that an attempt to show graphically the influence of the ratio of notch depth to specimen diameter on fatigue strength led to the conclusion that a simple relation between the two does not exist; for each material and for each diameter one finds a different curve with varying notch depth.

Docherty (1935) writes (p. 211): "The relation between the behaviour of structural materials in laboratory tests and the behaviour of the material when it forms part of a structure is a matter of fundamental importance. While discrepancies may be covered by the factor of safety employed in the design, every opportunity should be taken to collect data regarding the behaviour of materials in actual service. This is specially important in members of large size when discontinuities are present. For obvious reasons (e.g., capacity of testing equipment, cost, &c.), tests are usually carried out on specimens of comparatively small dimensions, and it has long been known that such tests often fail to give a true indication of the quality of the material when it is used in different, larger, sizes. This point was clearly brought out in some tests by...[Stanton & Batson (1920)] on geometrically similar notched-beam test pieces varying from 63-mm. by 63-mm. section to 5-mm. by 5-mm. section. These tests showed that, by whatever standard 'brittleness' might be measured, the larger test-pieces were definitely more 'brittle' than the smaller. It should be noted that these were impact tests, but the author's work already published [Docherty (1932)], as well as the results about to be described, show that the same tendency is found when the bending is carried out slowly, in a test occupying, perhaps, several minutes. ...If the principle of similarity...could be applied to these tests the comparison of results would be simple, because, since the loads carried at every stage of the test would be proportional to the squares of the linear dimensions, and the deflections proportional to the linear dimensions, the work done during the tests, which is equal to the energy absorbed, would be proportional to the cube of the linear dimensions, and the same degree of toughness or brittleness would be found for each size. As already mentioned, this is not found in actual experiment, as the energy absorbed is proportional to a power of the linear dimensions which varies with the material and which lies between the square and the cube in most cases." The author goes on to discuss existing test data and data from new slow bending tests on large notched steel bars (up to 100 mm. by 100 mm. in cross

section). The new tests confirm that the larger the specimen, the greater the tendency to early failure by brittle fracture.

Jurkov (1935) summarizes the results of a theoretical and experimental study of which the purpose was to find the reason for the increased mechanical strength in thin filaments and plates of brittle materials. After reviewing the work of earlier authors, including Karmarsch, Kvinke, Griffith (1920), Boyce, Reinkober (1932), and himself [Schurkow (1932)], he describes his own work. Contrary to the results of Reinkober (1932), he shows that the modulus of elasticity of quartz and glass filaments is independent of their thickness when the effect of deformation due to the weight is eliminated. He closes with the following conclusions (pp. 398-399): "This investigation has shown, that the rupture of brittle materials (glass, quartz) takes place according to Griffith's theory, i.e., the observed technical strength is determined by the superstresses in the discontinuities. The discontinuities, which play the part of Giffith's cracks, are distributed throughout the volume of the material, but the most dangerous amongst them are the discontinuities on the surface, at which the breakdown usually begins. By treating the surface of the sample (tests with fluoric acid) the surface discontinuities can be considerably changed, and consequently the strength increased, which may be of practical interest. The effect of the medium, in which the test takes place, upon the strength is in complete agreement with the general theory. It is pointed out, that the discrepancy between the individual values of strength for samples of one and the same thickness is of statistical nature with a sharply defined maximum, corresponding to the most probable strength. The difference in the probability of the individual values of strength corresponds to the difference in the probability of discontinuities of various degrees of dangerousness. To the most probable strength corresponds the most probable discontinuities. For a given length of sample the probability of meeting a surface discontinuity of a given degree of dangerousness decreases in proportion to the area of the surface. It follows that the most probable strength for cylin-

drical samples will be proportional to their diameter, i.e., it will be given by the relation $P = a + b/r$."

Roark & Hartenberg (1935) report the results of tests on plaster models performed to determine how the effects (on strength) of proportions, form of cross-section, stress concentration and scale compare with those for concrete and cast iron. Their results show that there is no gain in strength with decreasing slenderness comparable to that for concrete; that form factors for green plaster and cast iron are in reasonably close agreement; that the static tenderness of green plaster is almost the same as that of cast iron, while cured plaster shows the tenderness usually associated with a brittle material; and that there is little difference in the effect on the strength of either plaster or gray iron due to cast and machined notches. On the scale effect, they write (p. 25): "There is some reason to believe that the effect on static strength of stress concentration depends not only on the form of a notch or other irregularity, but upon scale, and that geometrically similar specimens, of the same material but of different size, will yield different stress concentration coefficients. A limited number of tests made by the writer indicate this to be true for the metals investigated and also for the green plaster. Other investigators question this scale effect, however, and pending substantiating tests, presentation of the results obtained in this investigation is deferred [see Nadai & MacGregor (1934)]."

Horger & Maulbetsch (1936) present data to show that plain cylindrical specimens of steel of the usual 0.3-in. diameter give 10 to 15 per cent increased fatigue strength over that given by specimens about 1 in. in diameter.

Buckwalter & Horger (1937) write (pp. 239-240): "The tests reported here are on 2-inch diameter axles, and the question is now: (1) will the favorable effect of surface rolling be as great for larger size axles and (2) how does the fatigue strength of specimens vary with size of specimen having different forms of stress concentrations. In order to answer these questions and others relating to forging

and heat treating practice and the general problems associated with large sections the Timken Roller Bearing Company is building a fatigue testing machine with a capacity sufficient to test axles of $13\frac{1}{2}$ -inch diameter. The question of size effect is of importance but it is a phenomenon that has been little studied and on which few test data exist. Here an interpretation of the present knowledge on this subject is presented for discussion. The test results may be generalized by stating that the fatigue strength of the material depends upon the stress distribution or stress gradient in the region of maximum stress. If the stress decreases rapidly from its maximum value the material does not fail as readily as when the same maximum stress values vary little over the cross section of the machine part."

Kuntze & Lubimoff (1937) review the German literature on the subject of the dependence of the bending fatigue strength of steel on the size of the specimen and the depth and sharpness of the notches. They plot a number of graphs showing this dependence, using experimental data given by Faulhaber (1933) and other authors.

Reinkober (1937) first reviews the results given in his two earlier papers [Reinkober (1931,1932)] on the strength and elasticity of thin quartz threads in relation to their diameter. Then he reports the results of a study of the effects of etching on these properties. These results have been summarized as follows (English abstract by F.P.P.): "Etching the surface increases the tensile strength and, to a lesser degree, the elastic limit of quartz fibers. There is a minimum thickness of surface that must be removed before maximum strength is reached; excessively protracted etching, however, will cause a reversion to the original lower tensile strength. If the tensile strength is measured with the etchant (HF) still on the surface, only a slight increase is observed. Absorbed surface films lower the tensile strength. The latter is increased, however, by removing the film with vacuum heating; no significant effect of such treatment on the elastic modulus was observed. The adsorbed layers alone are not capable of reducing the tensile strength, but do so through joint action with a surface film of the quartz itself.

Measurements of bending strength showed that surface irregularities tend to increase bending resistance with decreasing fiber thickness."

Davidenkov (1938) discusses, in Section 5 (pp. 12-17), three basic hypotheses concerning the physical nature of brittleness, two of which (Joffe's and Stepanov's) make use of Griffith's theory about the effect of submicroscopic surface cracks which cause stress concentration and reduce the average tensile strength. In Section 10 (pp. 26-29) he mentions the effect of specimen size on transition temperature. Section 20 (pp. 50-55) deals at some length with the scale factor (size effect) on material strength, because of which special care is required in dealing with articles whose size is much greater than that of the test specimens used to evaluate the quality of the material, which almost always occurs in practice. The author regards the existence of the size effect, which represents a departure from the law of similarity, as conclusively demonstrated, but its theoretical explanation as inadequate. On page 55, he suggests seven sets of experiments which should be performed in order to explain the nature of the scale factor. In Section 22 (pp. 60-63), he points out that, because of the relation of stress concentration to the scale factor, all theories concerning the former must be regarded as tentative until an adequate theory for the latter has been developed. In Section 27 (pp. 74-75), which deals with the layout of further experiments, he emphasizes once again the necessity of studying the influence of specimen size.

Roark, Hartenberg & Williams (1938) write (p. 24): "Although k [the factor of stress concentration in fatigue] increases with scale, that is, of two geometrically similar specimens of the same material, the larger will show greater notch sensitivity under repeated stressing... . To ascertain whether k_r [the factor of stress concentration at rupture, also called the factor of strength reduction] indicated a similar scale effect under static loading, tests were made on several series of specimens geometrically similar and of identical material, but varying in size through a wide range." They state the following conclusion (p. 51): "For

geometrically similar specimens of metal k_r increases with scale. The results of tests on Bakelite indicate that in the case of an amorphous material this scale effect is absent." They also discuss the effect of span/depth ratio on the modulus of rupture and the effects of notch radius and notch depth on k_r.

Anderegg (1939) writes (Summary, p. 291): "The strength of glass is reduced by discontinuities but, by thorough melting and proper methods of attenuation, the effect is greatly reduced so that strengths of the order of 400,000 pounds per square inch are being produced commercially with fine fibers for textiles. Generally, the finer the fiber, the greater the strength, the discontinuities apparently being pulled out lengthwise. The strength may be summarized from the 'bulk' strength plus corrections for the decreased cross section. [The author's analysis leads to an equation for the tensile strength T of the form $T = A + B/(d + e) + C/(d + e)^2$, where A is the bulk strength, B and C are positive constants, d is the diameter, and e is a positive constant included in the denominators of both corrections to prevent the corrections from becoming infinitely great as the fiber becomes very fine. If we set $C = e = 0$, the equation reduces to that of Karmarsch (1859); however, the author finds that better results are obtained by taking e to be about half a micron and C to be a positive value depending on the type of glass.] The scatter in strength results of such a material as glass is considerable, resulting from the variations in the number and in severity of discontinuities. The shorter the fiber tested, the higher the average results. ..."

Greenwald, Howarth & Hartmann (1939) write (Summary, pp. 21-22): "Methods of testing seven small pillars in place in the experimental coal mine have been described. All pillars were square in horizontal cross section. The data obtained may be summarized as follows: ...Three pillars including the entire height of the bed were tested with flow of the clay [floor] restrained. Ratios of lateral dimension to height of these pillars were, respectively, 0.50, 0.75, and 1.00. They failed, respectively, at pressures of 500, 600, and 695 pounds per square inch.

These pressures are proportional to the square roots of the ratios. ...Two pillars comprising, respectively, the upper and lower halves of the coal bed failed at pressures of 885 and 920 pounds per square inch. ..." The results show that a decrease in either the lateral dimension or the height tends to increase the strength.

Horger & Neifert (1939) write (Summary and Conclusions, pp. 736-737): "Tests given here represent a progress report on the study of the fatigue strength of the body section of railroad car axles. While exceptionally few fatigue fractures occur in actual railroad service in the plain body sections of car axles, when pressed or clamped-on members are not applied, it is important to note from these tests that only 17,500 psi is the endurance limit of the full-size axle [6 to 7in. in diameter] as compared to 33,000 psi. for that of the usual 0.3-in. diamter specimens machined from the surface of the full-size axle. Various explanations are discussed as to the probable reasons for this large decrease in strength of the full-size axle but it is important that the designer take cognizance of this reduction when using an untreated and forged material in the machined condition discussed here."

Igarasi & Fukai (1939) write [English translation of abstract, p. 98(30)]: "By using Ono's repeated bending test machine, effects of size and surface finish on light alloy test specimens in fatigue test results were investigated and the standard size and the standard surface finish for test specimens were determined from the results of the investigation. Influences of sand-blasting on the fatigue limit were also studied."

Afanas´ev (1940) reviews previous results on the statistical theory of fatigue strength, including those of Weibull (1939a), none of which he finds satisfactory. He proposes a new theory, according to which he finds that, with an increase in the radius r of the specimen the proportional decrease in the fatigue limit lies in the range from $\sqrt[k]{r}$ (for a strongly defective surface) to $\sqrt[k]{r^2}$ (for an ideally smooth surface), while an increase in the length ℓ must provide a drop in the fatigue limit (with uniform stress along the length) proportional to $\sqrt[k]{\ell}$. He shows how

to determine the value of k, which typically lies between 10 and 20.

Morrison (1940) writes (Abstract, p. 193): "The paper is a description of an investigation undertaken to determine, by tests under various conditions of uniform and non-uniform stress distribution, the criterion of yield in specimens of mild steel. Apparatus is described for heat-treating the material after machining in such a manner as to avoid surface decarburization, and for determining the stresses at yield of specimens treated in tension, compression, flexure, torsion, and combined tension and torsion. The results show that the material used is uniform and isotropic. No differences are found between the yield in tension and compression. Tension tests on thin tubes and on solid specimens of varying size indicate a progressive change from simple-crystal stress-strain characteristics to the normal characteristics of polycrystalline material. Tests in combined tension and torsion give results which accord with the theory of yield at a critical value of the shear strain energy stored per unit of volume of the material, but this is shown to be entirely due to the size of the specimens tested. In all cases of non-uniform stress distribution, the yield is shown to depend on the specimen size, being delayed until a shear stress not less than the shear stress at yield under uniform stress is applied to a thickness of material of the order of a few crystal diameters."

Afanas'ev (1941) writes (translation of conclusions, p. 358): "1. The stress distribution [cumulative] probability function [$F(z) = z^k/(A^k + z^k)$] is proposed for metal grains, and the stress-strain equation of the metal based on it is derived. 2. The derived stress-strain equation is checked for a number of different metals where an entirely satisfactory coincidence of the calculated and experimental data is obtained. 3. A method is presented for calculating the characteristics of the metal by its stress-strain diagram, including the parameter of the stress distribution probability function. 4. Theoretically the possibility of estimating the sensitivity of the metal to the stress concentration and the variation in dimensions under variable load by its stress-strain diagram is demonstrated. 5. The conclusion

is theoretically drawn that the metals sensitive to the stress concentration under variable load have low sensitivity to the variation in size and vice versa. 6. The relation is theoretically obtained between the nature of the transition curve of the metal from brittle to plastic (for variation of temperature) during impact testing and sensitivity of it to size under impact loading. 7. The theoretically obtained conclusions are in need of experimental checking which must at the same time establish the correctness of the constructed theory."

Buchmann (1941) reviews the work of several previous authors, including Peterson (1930), Faulhaber (1933), Faulhaber et al. (1933), Mailänder & Bauersfeld (1934) and Kuntze & Lubimoff (1937), on the size effect on the fatigue strength of steel, as well as some previous work on light metals. He reports the results of tests on the fatigue strength (especially in bending) of magnesium-aluminum alloy rods of various diameters. The results show a size effect similar to that for steel reports by earlier authors.

Föppl (1943) and Hempel (1943) discuss the effect of cross-sectional area and form on the fatigue strength, with the former considering the case of unequally distributed tension.

Kontorova (1943) adapts the statistical theory which she previously developed, partially in collaboration with Frenkel, which was directed to the solution of problems involving the rupture of specimens in which the elements were effectively in series (as the links of a chain) to specimens in which the elements are effectively in parallel (as the wires in a cable). The problems studied include (1) the rupture of the first wire; (2) the rupture of all the wires; and (3) the degree of rupture for a given stress. It is assumed that the strength of each wire is uniquely determined by the most dangerous defect in it.

Moore (1944) writes (pp. 170-171): "It has been suggested that residual stresses in specimens and structural parts may have some connection with the 'size effect' observed when specimens of different sizes but of the same general shape are tested

under repeated stress. A well-defined size effect has been shown in rotating-cantilever specimens of steel varying in critical diameter from 1/8 in. up to 1 in. Beyond that up to 2-in. diameter the fatigue strength of the metals tested seemed to be independent of size, but data are lacking to show whether for still larger specimens the value of fatigue strength does not drop off again. An investigation was planned to determine the residual stresses in different-sized specimens of a metal and to see whether the magnitude and location of such residual stresses could be correlated with the size effect, but the loss of laboratory assistants [due to war conditions] made this plan impossible and it was possible only to make fatigue tests of different-sized specimens of one type of steel on specimens which had been very thoroughly annealed (and all residual stress presumably removed) to see whether these specimens showed size effect as did unannealed specimens of the same material. The metal available for this study was SAE 1035 steel, obtained in round bars of $3\frac{1}{2}$-in. diameter. ...Specimens were machined with the following sizes of critical diameter: 1/8, 1/4, 1/2, and 2 in. ...The specimens were tested in rotating-cantilever fatigue machines... . Both notched and unnotched specimens were tested. ...The S-N diagrams show distinctly a 'size-effect,' the diagrams for the smaller specimens being distinctly above those for the larger specimens, both for notched and unnotched specimens, except that the S-N diagrams for the 1/2-in. notched specimens and the 2-in. notched specimens are practically coincident. This series of tests seems to indicate that there may be 'size effect' present in metal parts in which the residual stresses are negligible in amount or, in other words, residual stresses are not a necessary accompaniment to 'size effect'."

Murgatroyd (1944) writes [Summary of and Conclusions from Both Parts I and II (pp. 396T-397T)]: "It has been demonstrated that the values of the elastic constants and the viscosity are greatly reduced when the [soda-lime-silica] glass is drawn into fine fibres, whilst at the same time the strength increases. ...The striking fact that the modulus of rigidity is affected at low temperatures [below

400^{o}] has been interpreted to be the consequence of a re-formation of lateral bonds between chains of molecules in the fibres. It is considered that the act of drawing a fibre from a piece of glass at high temperature produces a predominantly chain structure with relatively few lateral bonds; and even when the fibre is heated to 550^{o} the process of forming a structure of the same type as exists in massive glass cannot go forward to completion in the fine fibres due to the very considerable distortion produced by drawing and the dimensions of the resulting fibre. ...The distribution curves of the breaking-strain tests are in accordance with a random distribution of flaws in the fibres, and it is possible to indicate approximately the effect of increasing the area of the test-piece on the strength. Heat treatment of the fibres is shown to cause changes in the distribution curves which are compatible with the appearance of a greater number of flaws as the elastic constants increase and the viscosity is raised. The general conclusion reached is therefore that the great strength of fibres of glasses of the type described is due to a reduction in the severity of the flaws present in the glass; it is not possible to decide whether the most severe flaws are absent because they are not formed or because their frequency of occurrence is small. In either case the fundamental cause is a constitutional change in the glass itself, and the observations are in agreement with the supposition that this change is from a rigid three-dimensional structure to a chain-structure with weak lateral bonds, and that this is not the most stable molecular configuration for a soda-lime-silica glass. Heat treatment causes changes in the constitution toward the more rigid structure with properties similar to those of the glass in massive form."

Puchner (1944) summarizes the work of previous authors, including Peterson (1930), Faulhaber et al. (1933), Mailänder & Bauersfeld (1934), Kuntze & Lubimoff (1937), Buchmann (1941), von Philipp (1942) and Hempel (1943), on the effect of specimen size on the strength of steel, light metals, and cast iron. He reports the results of further tests on the fatigue strength of cast iron in bending and

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torsion, performed on both plain and notched specimens of various sizes. The results show that the fatigue strength decreases steadily as the diameter of the test specimen is increased from 10 to 50 mm., except in the case of the torsional fatigue strength of notched (transverse drilled) specimens, where there appears to be little or no size effect.

Daniels (1945) writes (Abstract, p. 405): "A group of parallel threads of equal length, clamped at each end so that all threads extend equally under tension, is called a bundle, and the maximum load which the bundle can support is called its strength. The object of the work is to study the probability distribution of the strength of bundles whose constituent threads are sampled randomly from an infinite population of threads in which the probability distribution of strength is known. The relation between the strength of a bundle and the strengths of its constituent threads is first discussed [see below], and results are stated for bundles so large that the proportions of threads of different strengths approach their expectations. The properties of the probability distribution of bundle strength are next developed in detail, attention being confined in the present paper to the case where all threads have the same load-extension curve up to breaking point. Finally, the asymptotic behavior of the distribution for large numbers of threads is studied, and it is shown that in the commonest cases the distribution tends to assume the normal form." The author reviews the work of Peirce (1926) on the strength of bundles, and notes that Weibull (1939b) considered that of materials composed of independent parallel elements. On the relation between a bundle and its constituent threads, he writes (p. 406): "It is assumed that for each thread there is a definite extension and load at which it breaks, so that threads which yield rather than snap are excluded from the discussion. Whatever assumptions are made about the elastic properties of the threads, the following sequence of events occurs when a given load is applied. Suppose the load is increased gradually from zero to its final value. At first it is distributed in some way between the n threads, and equilibrium may be reached

without any thread giving way. If the load is great enough, however, one of the threads breaks at some stage and the load is redistributed among the remaining n-1 threads, each bearing a somewhat larger share of the load than before and so being more likely to give way. Similarly, a number of threads may break successively until either a point is reached where the remaining threads have sufficient strength to maintain between them the final load, or no such point is reached and all the threads ultimately give way, in which case, of course, the bundle is broken. To formulate the problem more precisely, it is necessary to know how the load distributes itself between the individual threads of the bundle, and this depends on the elastic properties of the threads. [The simplest case is that in which the unbroken threads share the load equally.]... "

Gurney (1945) writes (p. 273): "It is of interest to derive the distribution of strength of rods the length of which differs from those tested. Consider first the strength of rods which are n times as long as those tested. ...If $F_1(x)$ and $F_n(x)$ are the probability integrals for the strength of test pieces of length 1 and n units, respectively, the distribution of the strength of the long test pieces is obtained by calculating the chance that n unit test pieces chosen at random shall all be stronger than a given strength; for this is the chance that the weakest of n test pieces shall exceed the given strength. Thus $1-F_n=(1-F_1)^n$. This result was given by Pierce [sic; Peirce (1926)]." The author shows that this result holds not only for n an integer, but also for n = 1/m, where m is an integer. Assuming that it holds for all values of n and that the strength is N(0,1) [normally distributed with zero mean and unit standard deviation] when n = 1, he plots the mean μ, the standard deviation σ, and the skewness $\sqrt{\beta_1}$ of the distribution of strength as functions of $\log_{10}n$. His graph indicates that μ, σ, and $\sqrt{\beta_1}$ all decrease as $\log_{10}n$ (and hence n) increases; however, $\sqrt{\beta_1}$ actually increases as n increases. The author has taken the wrong sign for $\sqrt{\beta_1}$ throughout, an error which probably would not have occurred if he had denoted the skewness by α_3 instead of $\sqrt{\beta_1}$. Gensamer, Saibel and Lowrie (1947) comment that Gurney has not produced sufficient data to verify his

theoretical solution.

Moore (1945) writes (Synopsis, p. 507): "In this paper the effect of size of specimen on the endurance limit of steel specimens (or parts) is studied from a slightly different viewpoint from that of the Third Progress Report on Size Effect [Moore & Morkovin (1942-44)]... . The size effect in plain (unnotched) specimens is computed on assumption that a fatigue specimen which fails under cycles of reversed flexure behaves as if a fatigue crack started slightly below the surface of the specimen, where the nominal stress is slightly lower than at the surface. The further assumption is made that at this point below the surface the nominal stress at failure is also independent of size of specimen, and that the depth of this assumed starting point below the surface is also independent of the size of specimen. The test results for six different steels gave endurance limits deviating from the results of computation based on the above assumptions ranging from +8.50 per cent to -7.05 per cent with a mean deviation of 4.19 per cent. The second part of the report takes up the problem of notch sensitivity of various steels tested... ." On the limitations of the test results obtained, he writes (pp. 519-520): "The tests so far studied are all tests of steel specimens. Whether these methods of estimating size effect and notch sensitivity can be applied to non-ferrous metals awaits actual tests of such metals, and tests of at least one non-ferrous metal are now in progress. The capacity of the largest rotating-cantilever machine available for these tests has limited the maximum critical diameter of specimen to 2 in., and extrapolation of these results to larger sizes is not to be recommended as safe practice, especially in view of reports of larger specimens failing at much lower stresses than those reported in these tests. Whether these results are to be ascribed to 'size effect' or to large variations in the strength of the steel throughout the cross-section of these large specimens, or to other causes, is not clear at the present writing. However, the presence of very definite size effect for diameters or thicknesses of steel up to 2 in. has been shown and approximate methods

suggested for predicting such size effect and also for predicting notch effect within the limits stated above." The discussion includes comments by R.E. Peterson and T. McLean Jasper, mostly on notch sensitivity, and a closure by the author, who gives a diagram for determining reduction of fatigue strength due to size effect.

Obert, Windes & Duvall (1946) write (Summary, pp. 49-50): "Fifteen physical tests have been described which were adapted to, or developed for, the determination of the following physical properties of mine rock: ...Compressive strength, Modulus of rupture, Tensile strength, Scleroscope hardness, Abrasive hardness, Impact toughness... . The tests were so designed that the test specimens could be obtained from diamond-drill core, supplied either from field drilling or from rocks cored in the laboratory. From [sic; For(?)] the majority of tests any standard size of core could be used, that is, NX core (2-1/8-inch diameter), BX core (1-5/8-inch diameter), AX core (1-1/8-inch diameter), and EX core (7/8-inch diameter). The measurement of the elastic properties required specimens approximately 8 inches long; the remainder of the tests required pieces 6 inches long or less. ...With the exception of the abrasive hardness test, all tests gave results that were independent of the diameter and length of the diamond core specimen, provided that the length:diameter ratio was unity for the specimens used in the compression and impact-toughness tests. [For these two tests, the authors give equations to convert values for other ratios to equivalent values for a 1:1 ratio]. For the abrasive hardness test, a correction factor was established that allows the data for various core sizes to be converted to an equivalent 1-1/8-inch (AX)-diameter [the most convenient diameter for testing]."

Shevandin & Manevich (1946) write (translation of Summary, Section 5): "Experiments with the static tension and bending of smooth specimens of phosphorous iron in liquid air indicate the statistical nature of the scale effect and give good confirmation of the theory of Weibull. This is namely: a) the index of uniformity according to Weibull obtains from experiments on tension and bending sufficiently close values: 23.5 and 25.4; b) the computed (with the help of the index) theoretical

values of the corresponding brittle strength for small specimens are distinguished from the experimental values by approximately 3%; c) the ratio of brittle strength with bending to brittle strength with tension is equal from the experiment to 1.40 as opposed to 1.41 according to calculation; d) the dispersion of the particular values for large specimens, as follows from the theory, is less in the experiment than for small specimens. ..."

Vitman (1946) describes experiments regarding the dependence of the ductile-to-brittle transition temperature of geometrically similar specimens of mild annealed steel, under impact elongation and bending, upon their size, and also experiments on geometrically similar notched specimens under conditions of static bending. An increased tendency toward early manifestation of brittleness is observed with an increase in specimen size. He examines the results obtained from the viewpoint of the hypothesis concerning the statistical nature of the scale factor, and derives the correctness of this hypothesis, which explains the scale (size) effect on the basis of the random distribution of inhomogeneities and material defects in the material, which implies that dangerously large inhomogeneities are more likely to occur in large specimens than in small ones.

Weibull (1946) writes (English summary, p. 64): "Reversed bending endurance tests with rotating test bars of two different effective lengths (50 and 25 mm) but in all other respects exactly similar have shown that the time of endurance of the short bar is considerably longer than that of the long bar. The results correspond with the author's statistical theory of strength."

Brown, Lubahn & Ebert (1947) write (Abstract, p. 554-s): "The influence of the tested section size on the properties of a fully Si killed 0.25 carbon steel plate was investigated for geometrically similar notched bars tested in static tension. Specimens had cylindrical diameters ranging from 1/4 to 4 in., and possessed 50%, 60°, sharp circumferential V-notches. All specimens were cut from 4-in.-thick plate and were located in respect to the thickness direction in such a manner that their

average ductilities were equal. The results revealed a considerable decrease in notch strength from 110,000 psi. for the smallest specimen to 88,000 psi. for the largest specimen. The corresponding decrease in notch ductility was much more pronounced, being from 20% to only 2%. Experimental data obtained by the authors are compared with those reported previously by other investigators for steel, and the possible explanations for the size effects are discussed." The authors also write (Conclusions, p. 559-s): "From a consideration of the authors' data and results reported previously by other investigators, the following conclusions may be drawn regarding the effects of section size on steel: 1. Section size, particularly when combined with the effects of other well-known embrittling factors, may be a severe embrittling factor itself. 2. An increase in the tested specimen size may result in a pronounced decrease in the unit properties of presumably geometrically similar specimens. 3. With an increase in section size, the metal properties decrease at a continuously decreasing rate. 4. The effects of section size as an embrittling factor can best be explained by a statistical theory that considers the metal volume to contain defects of various degrees of severity which determine the local properties."

Heywood (1947) writes (Introduction, p. 81): "In designing an aircraft engine, the strength of the individual components is of critical importance. ...The inherent dynamical nature of an engine produces vibrations and inertia forces so that components are subjected not to steady stresses, but to fluctuating stresses which lead to fatigue failures. Hence the fatigue strength becomes the critical factor in design rather than some other mechanical property, such as the ultimate strength or the proof strength. The use of the endurance limit as a criterion of strength brings many difficulties in its train. It is not always possible to predict the endurance limit of a part even though this may be known for the laboratory test specimen. However, some progress in linking the behaviour of such specimens with the engine component has been made. The laboratory tests have shown that it is nec-

essary to adjust the value of the endurance limit according to the shape and size of the part. Thus large specimens possess a lower endurance limit, so also a notch in the material is found to be particularly susceptible to failure. In this article, an attempt is made to throw some light on to these quantitative effects of fatigue. It is desired to coordinate the effect of size and type of notch with the fatigue strength, and it will ultimately be shown that these quantities may be linked by a simple formula." He states the following conclusions (p. 84): "The conventional use of the notch sensitivity index, q, appears to have only a limited application, since it cannot be generally related to notches and sizes of specimens. The new theory described assumes that the fatigue strength of the notched specimen depends on the elastic stress concentration factor [K] multiplied by a factor which depends on the absolute size of notch. The smaller the size of notch, the greater the fatigue strength. The theory leads to the formula $\sigma_{fn} = (\sigma_f/K)(1 + a/\sqrt{b \cdot r})$, where the augmenter factor 'a' is a constant for a given material and the factor 'b' depends only on the type of notch [σ_f = endurance limit of unnotched specimen, σ_{fn} = endurance limit of notched specimen, and r is a dimension which determines the absolute size of the notch]. Remarkably consistent results are obtained by the use of this formula with each group of materials. This formula will also account for the size effects observed in unnotched Wöhler specimens of different dimensions."

Laurent (1947) writes (pp. 719-720, translation): "...The fatigue limit depends upon the form and the dimensions of test specimens, as has been stated by numerous authors. If f is the fatigue limit calculated from the formulas of the strength of materials and f_t is the fatigue limit in tension-compression, experience shows that $(f-f_t)/f$ is a linear function of the inverse of the dimensions of the specimen... [as shown by the results of Buchmann for a magnesium alloy]; analogous conclusions are valid for steels... [Moore & Morkovin (1942-44)]. None of the theories accepted up to the present permits the explanation of these results; we propose, in this Note, to show that the theory of stress concentrations provides

their justification."

Oding (1947) discusses, in Chapter VII (pp. 122-157), the effect of the form of article on fatigue strength. He reviews theoretical and experimental work reported by various authors, including Peterson, Peterson & Wahl, Faulhaber, Afanas'ev, Buchmann, and Puchner, and presents new results of his own. On the statistical theory of strength, he writes (pp. 145-146, translation): "Recently a series of researchers solved the problem of the brittle and fatigue strength of metals, together with that of the scale factor, with the aid of statistical theory. The first attempt at the quantitative solution to this question by statistical methods belongs, apparently, to Weibull. Then this solution was considered by Kontorova, whose theory was modernized by Kontorova & Frenkel. If all these authors relate their conclusions to the brittle strength of real crystals without considering directly the fatigue strength of materials, then, in the work of Afanas'ev..., statistical theory is utilized for the direct solution of the problem of the fatigue strength of alloys. The authors of the statistical theory of strength construct their derivations exclusively on the inhomogeneity of the mechanical properties of micro-volumes of metal and its differing strength, depending upon the size of the inhomogeneities, utilizing the theory of probability for the quantitative solution of the problem. This new method for the solution of the problem of the strength of metals must undoubtedly play a very large role. It can become a very powerful implement in the hands of metallographic researchers and can explain many facts not otherwise explained. Even now qualitatively the results of the researchers coincide with a series of experimental data. Their derivations show that the fatigue strength must decrease with an increase in the dimensions of the specimen, not only with an increase in the cross-sectional dimensions, but also in the length. The effect of the latter on the endurance limit has up to now not been studied experimentally. However, it is necessary to warn that the attempt to explain the generation of the scale factor exclusively by the severity of the inhomogeneities and by differences

in the mechanical properties of microvolumes of metals can lead to false results. Therefore the comparison of the conclusions of the statistical theory of strength with the results of data from experiments (for which the procedure was completely free from the effect of other factors not taken into account by statistical theory) cannot be very persuasive even when the results of these experiments coincide very well with the conclusions of statistical theory. ...Now it is difficult even from the available data to establish the quantitative effect of the inhomogeneity of the mechanical properties of metals in the matter of the scale factor in examining the problem of the fatigue of metals. For this, new investigations, taking into account other possible explanations of the scale factor, are required. Only after this will it be possible to establish how well the statistical theory encompasses all the most important causes, not only qualitatively but quantitatively." The author explores the effects of technological factors (including damping and grain sizes), and reports experimental data which indicate that these effects can sometimes nullify or even reverse the direction of the size effect as predicted by the statistical theory.

Puchner (1947) writes (translation of summary, p. 253): "Several years ago, the author [Puchner (1944)] performed tests on three kinds of cast iron, using cylindrical specimens of 12 and 50 mm diameter in bending and bars of 12 and 45 mm in torsion. This work has been complemented by new measurements on specimens 24 and 22 mm in diameter. It follows from these tests that the existing hyperbolic relation, for light metals and steels, between the fatigue strength and the diameter of the specimen, can be considered as being equally valid for cast iron, in the case of bending as well as in that of torsion. However, weak variations can enter into account for small dimensions, probably as a result of eventual flaws in the material. With the increase in the diameter, if the specimens are drilled transversely or if they present abrupt changes in cross section, the fatigue strength again decreases very rapidly in comparison with that of normal bars. As a result,

the notch sensitivity increases rapidly in proportion as the dimensions become very large."

Dolan & Hanley (1948) write (Abstract, pp. viii-ix): "The purpose of this study was to investigate the effect of size of test specimen on the fatigue strength of unnotched and notched specimens of SAE 4340 steel heat-treated to an ultimate tensile strength exceeding 160,000 psi. Rotating cantilever beam fatigue tests were made of five sets of unnotched and of notched specimens whose diameters in the critical test section were 0.125", 0.25", 0.50", 1.00" and 1.75", respectively. The notched specimens were geometrically similar; each had a semi-circular notch whose radius was 0.08 times the diameter of the specimen at the root of the notch. The same value for the endurance limit was obtained from the test data for the two largest sizes of unnotched specimens, the value being about 74,000 psi. The endurance limits increased with decreasing diameter of specimen, to a value of 82,500 psi. for the 1/8" unnotched specimens. For the notched specimens, the endurance limits ranged from 42,000 psi. for the 1-3/4" specimens to 51,700 psi. for the 1/8" specimens. However, it is believed that the endurance limit for the notched 1-3/4" diameter specimens should have been about 45,000 psi., if the material in the critical test region had been metallurgically the same as that in the other specimens; further experiments are being made to check this tentative conclusion. The 1/4" and 1/2" notched specimens both had an endurance limit of 48,000 psi., and the 1" specimens an endurance limit of 46,000 psi. A brief discussion is included to outline some of the factors that may cause a decrease in fatigue strength of large members; empirical equations are also presented for calculation of the approximate fatigue strengths of large notched or unnotched members from test data on small samples."

Higuchi, Leeper & Davis (1948) write (Summary, p. 1030): "Peirce's equation, which relates observed tensile strength of textile fibers with their length, was found to be applicable to rubberlike material if specimen volume is used in place of specimen length. Experiments in which the tensile strengths of GR-S and natural

rubber compositions were determined for a range of specimen volumes yielded results in close accord with theory. A tenfold increase in the volume of the material results in a decrease of 308 and 339 and of 204 pounds per square inch in the observed tensile strength of GR-S and comparable natural rubber stocks, respectively. The *numerical magnitudes* of the slopes of the straight lines obtained when tensile strengths were plotted against the logarithms of the relative specimen volumes are shown to bear direct relationships to the homogeneity of the stocks under test. The use of a dumbbell sample with a constricted center should result in a relatively simple means of measuring quantitatively the degree of homogeneity of rubber compositions."

Lubahn (1948) treats primarily the notch effect, but writes as follows on the size effect (pp. 118-119): "Section size has been shown both theoretically [Fisher & Hollomon (1947), Weibull (1939a)] and experimentally [Brown et al (1947), Davidenkov et al (1947), Docherty (1935)] to have a considerable effect on the fracturing characteristics of metals, particularly in notch tensile tests. It was found that making all the linear dimensions of a notch tensile specimen 16 times as large caused a reduction in the average strain at fracture from 20% to 2% [Brown et al (1947)]. According to Fisher and Hollomon [(1947)], the fracture stress should be higher but should show more scattering when the volume of metal involved is small than when it is large. The 'volume of metal involved' (or what might be called the 'effective volume') is a rather intangible quantity when related to notch tests, but qualitatively may be said to include all elements of volume that contain conditions sufficiently near to those in the element of volume in which fracturing ultimately begins. When fracturing begins at the surface, the effective volume will be a more-or-less torus-shaped region adjacent to the notch bottom. When fracturing begins at the center, the effective volume will be a more-or-less disk-shaped region that has its plane of greatest extent in the plane of the notch section and that is somewhat smaller in diameter than the notch section. Sachs...has made a first attempt

at eliminating the size effect by keeping the notch diameter constant. However, the axial dimension of the disk-shaped or torus-shaped effective volume will depend on the notch radius and the notch angle (on the notch radius especially if fracturing begins at the notch bottom). Consequently, the only series of tests for which there is any possibility of eliminating the size effect is one in which the notch diameter, notch radius, and notch angle are maintained constant while the notch depth is varied by changing the diameter of the cylindrical bar into which the notch was machined. Even in this case, the size effect will not have been completely eliminated, since it is quite likely that the stress and strain distributions, and in particular the shape of the distribution curve of triaxiality across a diameter of the notch section, will vary with notch depth."

Moore, Dolan & Hanley (1948) write (Summary, p. i): "This document includes two final reports by the University of Illinois on the fatigue characteristics of two metallic materials. Part I deals with data in reversed bending (rotating cantilever beam) and in reversed torsion of a high strength aluminum alloy 75S-T6... . Part II deals with data in reversed bending (rotating cantilever beam) of S.A.E. 4340 alloy steel heat-treated to 160,000 psi. ... . In general the reversed bending data indicate a decrease of fatigue properties with an increase in size of test specimen within the range investigated, namely 1/4 to 1-7/8 inch diameter. Empirical equations are presented for calculation of the approximate fatigue strength of large notched or unnotched specimens from test data on small specimens of a similar material in the same metallurgical condition. The theoretical stress concentration factor K_t, for the notched specimens as determined from the Neuber diagram was 2.00 and 1.50 for bending and torsion respectively. The notch sensitivity in bending of the aluminum alloy and heat-treated 4130 steel shows a tendency to decrease with increase in size of specimen but that of the heat-treated 4340 tends to increase with increase in size. Comparison is also made with data from previous work on S.A.E. 4130 steel heat treated to 147,000 psi."

Parker (1948) writes (Introduction, p. 82): "The effects of section size on the fracture strength of metals has always been of great interest to metallurgists and engineers but the high cost of making large-scale tests on specimens and models has greatly restricted the amount of experimental work in this field. With the onset of the war, the problem of section size became acute and the situation was particularly aggravated by the serious fractures which occurred in welded steel merchant ships. The reasons for the ship fractures were not immediately apparent and consequently a number of research projects were initiated in an endeavor to learn more about the behavior of large structures under load. The results from these projects are now being correlated and published in various technical journals. It is the purpose of this paper to present some of the salient features related to the effect of section size on the fracture strength of mild steel, drawing particularly upon the information obtained at the University of California. Many of the investigations were sponsored by the Bureau of Ships, United States Navy, and additional details may be found in government reports...and technical papers...[including Boodberg et al. (1948) and Wilson et al. (1948)], which present excellent summaries of the results. The present paper presents a discussion of the effect of section size on the fracture strength of steel specimens of the following types: (a) cylindrical specimens; (b) unnotched flat plates; (c) notched flat plates; (d) geometrically similar specimens; and (e) welded assemblies. No attempt has been made to summarize all of the information available in the literature because the purpose of this presentation is to illustrate facts or trends with one or two specific examples." The author states the following conclusions (p. 89): "In conclusion, it appears that the size effect in fracture is dependent not upon size as such, but only upon the local condition of stress and strain at geometrical discontinuities, which may vary markedly in similar specimens of different sizes."

Shearin, Ruark & Trimble (1948b) write (Introduction, pp. 167-168): "It would result in tremendous economies if the mechanical behavior of large parts could

be predicted from the behavior of small specimens. For many years, however, it has been recognized that the absolute size of a test bar has an influence on its mechanical properties. The value of almost any mechanical property of a metal may depend on specimen size. This lack of similitude is referred to as a size effect. In general the energy required to cause failure in a large test specimen is proportionally less than in a small specimen, if the failure in each case is of a ductile type. Furthermore, a brittle failure may occur in the larger specimen, whereas under the same test conditions a ductile failure may be experienced by the small specimen. In spite of prominent size effects reported by some observers, other investigators have obtained conflicting results because they have failed to provide for identical material, and for geometric similarity in heat treatments of thick and thin plates. True size effects are those which are encountered when we test geometrically similar specimens, taken from very homogeneous material. True size effects in ordinary tensile tests of ductile engineering materials are so small that they are difficult to detect and still more difficult to measure. We shall dismiss them with brief discussion here, and then pass on to the main subject of the paper -- scale effects in slow bend tests of notched bars, which are better adapted to reveal brittle behavior and its dependence on size. ...In our laboratories tensile testing devices of high precision were developed and constructed. Results for a thick nickel-chromium steel gave some evidence of small size effects in maximum true stress (load/least cross section) and in breaking stress. However, unavoidable errors were large enough to mask the effects and make their reality uncertain. An unmistakable size effect was found in the reduction of area." In seeking an explanation of size effects, the authors consider three factors: the weakening effect of flaws, the nature of the crack, and the effects of the pattern of applied stress.

Isibasi (1948) writes (Introductory, pp. 1-2): "It is generally known that the notch-factor β of a ductile material i.e. the ratio of the fatigue limit of plain specimen to that of notched one of which the form-factor (the theoretical stress con-

centration factor) is α, is less than α itself and the difference of these two factors is small for specimens with small α, while β of specimen with large α is very small. From practical point of view, therefore, the knowledge of α will be sufficient for specimens with small α and in case of notched specimen with larger α the value of β itself is important. As a reason of these facts it is generally thought...that when a notched specimen with large α is loaded small local zone near the root of the notch will yield to the load and this yielded zone effects as if the geometrical radius of the root of the notch is increased and eventually the concentration of stress is released. Since these interesting relations between α and β seem to concern with the problem in fatigue of metals of the effect of size of specimens, that is with the so-called <u>scale effect</u> which is recently recognized to be very important in this field of investigation, the author made an attempt to explain these facts from a view point that the <u>occurrence of yielding or of fracture at the root of a notch is determined by the stress condition of a second point near by</u>. This assumption makes it possible to show that the relation between the curvatures of the root of a notch and the reciprocals of fatigue limits of geometrically similar specimens must be in parabolic relation; which coincides with the experimental results of American investigators [including Peterson & Wahl (1936)]. Above assumption gives also a satisfactory explanation of the fact that β must always be smaller than α. Thus to verify the appearance of such yielded zone become important. For this purpose an experiment was made with steel pipes each of which has a diametrical circular hold of various radius. From the results of this experiment we concluded that in some cases there appear small local yielded zones near the hole and in other cases there does not."

Peterson (1949b) presents a statistical approach to the size effect problem (pp. 68-73): "As pointed out in the main body of the paper, the test data available to the designer usually are obtained from standard test pieces of rather small diameter. We would like to have a method of estimating the properties for large di-

ameter pieces. While some tests are being made where size is the variable, it is desirable that parallel studies be made to seek a fundamental basis for interpretation of the results. Otherwise the amount of testing would be endless. Furthermore, it is clearly out of the question to run fatigue tests of sizes represented in Fig. 1 [ratio of diameters 100 to 1]; our only hope is to develop relations which will enable us to extrapolate with more confidence. In recent years, statistical analysis has been used to explain the lower strength obtained in specimens having a larger volume. In the engineering field, Weibull [(1939b)] seems to have been the first to have demonstrated this possibility, although evidently unknown to Weibull, a similar approach was used by Peirce [(1926); see also Chaplin (1880,1882)] in explaining the differences in strength obtained from textile thread of different lengths. Epstein [(1948a)] has shown that these and other papers are all based on the so-called 'weakest-link' theory which derives from a mathematical study of the distribution of the lowest values as a function of sample size. Hill and Schmidt [(1948)] have prepared a convenient chart...from which the breakdown voltage of large insulation areas can be estimated, starting with data from a small area. It is believed that this chart, which is based on a Gaussian 'normal' distribution, can also be used for certain fracture problems. ...Whether the above general method can be extended to fatigue problems is open to question [see Epstein (1948a)]. It is hoped that the present attempt will focus attention on a method of attack and will lead to other contributions in this field. In some respects a fatigue fracture resembles a static fracture of a brittle material -- yielding in the ordinary sense does not occur (broken pieces held together usually fit closely); cracking follows the normal stress field. Since fatigue starts locally and propagates by means of a progressively growing crack, it is not too unreasonable to expect that 'weakest link' theory might be applicable to the starting conditions of such a crack. Strictly speaking, comparisons should be made on the basis of a small detectable crack of given size, since the time of propagation through pieces of various size will vary."

Prot (1949c) writes (English summary, p. 1): "The mechanical properties of any material can be said to be well known after a certain number of tests have been carried out in order to obtain a fairly precise average value on the one hand and a dispersion index on the other. This fact is now accepted almost without dispute. The question now arises as to how many tests are to be carried out in practice. The tests of which the results are given in this report answer the question as fully as possible, i.e., they indicate what degree of precision can be reached in given test conditions when the number of tests is varied. These tests were carried out using specimens of different sizes, so that attention is drawn to the effect of the size of the test pieces on the results obtained -- a fact which is not well known; the dispersion index, in particular, appears to be a somewhat variable function of the similarity of test pieces. The practical conclusion of these experiments is that, if the traditional safety factors are raised to no purpose for large-sized pieces, they may on the other hand be inadequate for pieces of small thickness [since the dispersion as well as the average value of the strength increases as the size decreases]."

Helms (1950) writes (Abstract): "The performance of materials when subjected to axial, pulsating or repeated loadings is dependent upon many variables. The external physical dimensions of the material is one of these variables. By machining and testing a series of fatigue specimens, in which the physical dimensions are always proportional to some base dimensions, the effect of size can be evaluated for any given material. For the experimental data to be meaningful, however, all specimens must be made from the same heat of material, where internal structure, heat treatment and other previous work history are constant. Axial loadings using complete reversal of stress are one measure of size effect. The determination of endurance limit stress in the various sizes of specimens tested is the property used in determining size effect and size effect is stated as a percentage variation of the endurance limit stress of a given size of test specimen as

compared with the endurance limit stress of some base size test specimen. To assure true evaluation of size effect, a metallurgical investigation of internal structure should show constant structure composition of all sizes tested. Be this the case then, any examination of structure after testing which shows cracking, cold working or any other reorientation or disturbance from the untested structure will be an indication of the causes and factors which contribute to initiating and consummating fatigue failures in a material. This thesis determines the effect of size in an SAE 4140 steel subjected to axial, complete reversed loading. An analysis of microstructures is included to determine the method of propagation of fatigue failures and to point out disturbances in internal structure which might contribute to or be the result of consummation of the fatigue failure. It was desired also to show that microstructures of all sizes of test specimens were similar."

Various problems concerning the strength of concrete, including the effect of specimen size, are addressed by l'Hermite (1950). In order to determine the size effect, the strength of a number of specimens of various sizes (cubes with edges 10, 14, 20 and 30 cm) was measured, and the author reports the results of these tests along with previous test data for cubes whose edges range from 5 to 40 cm. The experimental results show that the strength increases as the edge of the cube increases from 5 to 14 cm., reaches a maximum for edge between 14 and 20 cm., and decreases as the edge increases above 20 cm. The author points out that theoretically the strength should decrease monotonically with an increase in the size of the specimen. He attributes its failure to do so to the coarseness of the aggregate, whose influence on the strength is proportional to $(1 - d/D)$, where d is the diameter of the coarsest aggregate and D is the edge of the specimen.

Yen (1950) writes (Abstract, p. iii): "In order to formulate an explanation of 'notch effect' and 'size effect' in fatigue tests of metals, it was assumed in the light of recent fatigue theories, that the criterion for fatigue strength for any kind of specimen was based upon two material properties: (1) an 'intrinsic endur-

ance limit'; and (2) a critical amount of fatigue damage (closely related to the work-hardening capacity). After several trials, an expression for the critical amount of fatigue damage was formulated and equations were derived to express the property of notch-sensitivity and the fatigue limit for any rotating beam specimens, large or small, notched or unnotched. The resulting equations generally agreed well with the general conclusions drawn from the observations of the phenomena of notch effect and size effect; other things being equal, the notch-sensitivity increased with increase of diameter or of material homogeneity of specimen, and decreased with increase of the relative stress gradient or the work-hardening capacity. Empirical methods were proposed to evaluate the material constants in the equations so that one may use the equations to predict the endurance limit for any rotating beam specimen. The available test results for several sizes of notched and unnotched specimens of six different steels gave endurance limits which deviated from those computed from the equations by amounts within $\pm$ 10.1 per cent."

Petersen (1951) writes (translation of summary, p. 170): "From the formula of R.B. Heywood [(1947)] a somewhat simpler formula for the influence of the stress gradient on the fatigue strength of smooth and notched specimens is derived, and it is shown that it fits the experimental data the best of all propositions to date. Further, the theory of artificial notches going back to D. Morkovin and H.F. Moore [Moore & Morkovin (1942-44), third progress report (1944)] is further developed and it is shown that this modified formula of R.B. Heywood can be deduced from it. Suggestions for practical evaluation and application are given and the domain of validity is bounded."

Craemer (1952) writes (English summary, p. 145): "The dependency of the strength...upon the test specimen size is explained by the nonuniformity of the building material. It is represented by a function of the standard deviation of strength values. The standard deviation is lower for large specimens. Longer pieces have a lower mean value of the strength. If the cross section is made larger,

the behaviour depends upon the stress-strain law; for a brittle material, a larger cross section has the same effect as a greater length, whilst for plastic materials the mean value of the strength remains unchanged. Practical minimum strength values depend little...upon the size of structural parts." The author's results are based on the "weakest-link" theory. In a postscript, he acknowledges the priority of Weibull (1939a), of whose work he became aware only after his own paper was written.

Herring & Galt (1952) report that tin crystals (whiskers), approximately 1.8 x 10^{-5} cm in diameter, are several orders of magnitude stronger than the ordinary bulk metal. They deduce from the product of strain and elastic constant that stresses of the order of tens of kg/mm^2 were present in their sample, as compared with a yield stress of approximately 0.15 kg/mm^2 for bulk tin.

Horger & Neifert (1952) write (Synopsis, p. 70): "This paper summarizes the results of rotating-bending fatigue tests on shafts 0.3- to 6-in. diameter from SAE plain carbon steels of 0.39 and 0.54 per cent carbon content in both the 'as-forged' and normalized-and-tempered conditions. Shafts were tested with stress concentration, as represented by both fillets and press-fitted members, as well as plain specimens. Shafts with stress concentration produced by fillets or press-fitted members from either untreated or normalized-and-tempered steels exhibited little or no size effect. An endurance limit ranging from 17,500 to 25,000 psi was found from tests on 101 plain specimens of 6-in. diameter untreated steel, while filleted shafts (r/d = 0.14 and 0.29) [r = fillet radius, d = shaft diameter] of the same material had endurance limit values within this range. A similar comparison exists on normalized-and-tempered steel, but here less extensive data are available. Some effect of size was indicated on plain specimens 6 in. in diameter, which had a minimum endurance limit 35 per cent lower than that found for the conventional 0.3-in. diameter plain specimens from untreated steel. Data are reported on various groups of small specimens machined from large forgings and $2\frac{1}{4}$-in. bar stock."

Kuhn & Hardrath (1952) write (Summary, p. 1): "Neuber's proposed method of cal-

culating a practical factor of stress concentration for parts containing notches of arbitrary size depends on the knowledge of a 'new material constant' which can be established only indirectly. In this paper, the new constant has been evaluated for a large variety of steels from fatigue tests reported in the literature, attention being confined to stresses near the endurance limit; reasonably satisfactory results were obtained with the assumption that the constant depends only on the tensile strength of the steel. Even in cases where the notches were cracks of which only the depth was known, reasonably satisfactory agreement was found between calculated and experimental factors. It is also shown that the material constant can be used in an empirical formula to estimate the size effect on unnotched specimens tested in bending fatigue."

Bateson (1953) writes (p. 302T): "In his classic work on glass fibres, A.A. Griffith [(1920)] attributed the discrepancy between the theoretical and practical mechanical strength to flaws or cracks of varying severity. Because of the flaws which occur at the surface of a fibre, atmospheric attack would be expected to wield a further influence, a phenomenon which has been amply demonstrated. One of Griffith's principal arguments in favour of the flaw theory was a dependence of mechanical strength upon sample size. Thus, he found that the strength of his glass fibres rose rapidly with decreasing diameter, and indeed this has been demonstrated by others since that time. Now the dependence of strength upon the stressed area is a most attractive corollary to a statistical theory of flaw structure. In spite of all this, evidence has accumulated which, while it does not invalidate Griffith's flaw theory, profoundly modifies its meaning; moreover, the diameter-strength relationship begins to lose its significance and becomes incidental rather than fundamental." He proceeds to cite evidence from the literature that the strength increase of alumino-boro-silicate glass fibres corresponding to a decrease in diameter from 11 to 5 μ is 100% for bushing-drawn glass, but less than 7% for rod-drawn glass. He attributes the difference to the fact that the structure (and hence the

strength) of a fiber is profoundly affected by its thermal history during attenuation. Rapid cooling tends to preserve the structure (and the high strength) of the high temperature glass, while slow cooling allows the structure to rearrange itself and a stress gradient to build up, which weakens the fiber.

Gillett (1953) writes on size effects in heat treatment as follows (p. 14): "Massive sections of steel and alloy castings cool more slowly than do the smaller test bars and as a result freeze more slowly. This permits the formation of relatively large crystals, causing a weaker aggregate in the piece than is shown by the test bar. Consequently, conventional test specimens of small size fail to represent the properties of the larger pieces and it is necessary to cut up the actual section in order to obtain true design values. The structure of wrought steels, as-rolled, as-forged, or normalized, depends on the temperature at which austenite, the high-temperature form of steel, transforms to ferrite and pearlite, the structural constituents of constructional steels at lower temperatures. The heavier the section, the slower the cooling, the higher the temperature of transformation, the coarser the metallographic structure, and the lower both the yield strength and the ductility. Steelmakers know, and take into account, the fact that a 2-in. steel plate will have lower mechanical properties than will the same heat of steel rolled to $\frac{1}{2}$-in. plate. In quenched and tempered steel, an even more marked size effect appears. It also stems from the transformation of austenite at varying temperatures. ..."

MacKenzie (1953) writes (Synopsis, p. 224): "The paper reports a statistical investigation on steel plates produced in two rolling mills. The first object was to determine the effect of plate thickness and production variables on the V-notch Charpy transition temperature, and the second to determine the general level of notch ductility of special qualities of mild steel. The results of the investigation are discussed and it is concluded that it is desirable that the notch ductility of thick mild steel plates should be improved."

Piper & Roth (1953) report remarkable strength properties, similar to those found by Herring & Galt (1952) for tin crystals, of crystals (needles up to about 5×10^{-3} cm in diameter and several millimeters long) of zinc sulfide, for which they estimate Young's modulus to be about 7×10^{11} dynes/cm^2.

Richards (1953) notes that Nakanishi reported the results of an investigation in which he found that rectangular mild steel beams yielded at bending loads as much as 50% above those corresponding to the tensile yield point, an effect which he attributed to a non-uniform stress distribution, while other authors have reported results ranging from no raise in the yield point to Nakanishi's high of 50%. In seeking an explanation of the discrepancy, the author noticed that the authors who reported the largest raises were those who used the smallest specimens, while those who reported little or no raise were those who used the largest specimens. In order to verify the hypothesis that the differences are due to a size effect, he develops a statistical ("weakest-link") theory of the size effect on strength for ductile materials, analogous to that of Weibull (1939a) for brittle materials, making use also of the results of Griffith (1920), Kontorova & Frenkel (1941) [Frenkel & Kontorova (1943)], Davidenkov et al. (1947), Epstein (1948a), and others. He reports that the average yield strength observed in both bending and tension experiments for specimens of various sizes is in close agreement with that predicted by the statistical theory.

Wells (1953) writes (Summary, p. 34r): "... A notch brittle material has been shown experimentally to possess, at a given temperature, a minimum surface energy of crack propagation, as first suggested by Irwin. Thus, if a fracture is initiated in a structure, but the rate of release of stored elastic energy with increase of length of the crack is insufficient to provide the minimum surface energy of the material, then the crack will not propagate. On the other hand, the surface energy of the material can have values above the minimum, so that the type of brittle breaking strength formula based on surface energy, and first proposed by Griffith,

cannot be used indiscriminately. In general, it appears that the brittle breaking loads of large specimens are determined by the initiation process, where the influence of stresses other than those due to external loads is excluded, while the breaking loads of small specimens are controlled by minimum surface energy. ..."
The above statement concerning brittle fracture of large specimens is confirmed by experimental data reported by Boodberg et al. (1948) and by Wilson et al.(1948) for notched 3/4 in. ship plates ranging in width from 12 in. to 72 in. For nine steels, the ratio of the net average breaking stress at 12 in. width, to that at 72 in. width, is 1.16, whereas if the breaking strength were controlled by surface energy, the ratio should be $\sqrt{72/12} = 2.45$.

Chechulin (1954b) writes (translation of Conclusion, p. 1109): "1. A critical analysis is presented for the present-day existing explanations and theories of the scaling effect observed during static testing of polycrystalline metals giving the plastic nature of rupture. 2. It is demonstrated that the geometric similarity of the dimensions and the test conditions of the samples for static deformation rates insures similar microscopic occurrence of the plastic flow. This confirms the known similarity law of plastic flow. ... 3. On the basis of a generalization of the data on the effect of plastic flow on the internal structure of polycrystalline metals and the micrononuniformity of the occurrence of plastic flow, the conclusion is drawn of the statistical nature of the scaling effect in the case of plastic rupture. 4. The means of applying to plastic rupture the basic conclusions of the statistical theory developed for application to brittle rupture are proposed; satisfactory coincidence of the proposed formulas with the experimental data is demonstrated."

Gyulai (1954) gives a simple method of growing NaCl needle crystals. He reports results of measurements of the tensile strength of NaCl needle crystals, which in several cases is found to be in close agreement with the theoretical strength. The thickness of the crystals studied was from 0.5 to 10 μ. The author plots a graph of the tensile strength as a function of the thickness. The tensile strength de-

creases very rapidly as the thickness increases for small values of the thickness, then more slowly for larger values.

Hikata (1954) writes (Summary, p. 8): "1. The fatigue strength under reversed direct stresses of geometrically similar specimens of from 6 to 34 millimeters in diameter, containing a hyperbolic grove, was greater for the smaller size of specimen. 2. It is confirmed that the size effect in fatigue is mainly due to the stress gradient in the specimens and that the critical factor to determine the fatigue strength of notched specimens is the stress slightly apart [at distance ε] from the tip of the notch in the direction along which the stress gradient is maximum. 3. The distance ε for this material [mild steel of 53 kg/mm^2 ultimate tensile strength] is equal to 0.1 millimeter approximately and independent of the sizes of specimens. 4. The relation between $1/\sigma_n$ and $1/\rho$ is parabolic [σ_n = fatigue limit of notched specimen, ρ = root radius of notch]. ..."

Horger (1954) writes (p. 77): "Very little is known regarding the fatigue characteristics of large sections. Strength assumptions are made by the engineer which in effect recognizes lower fatigue values and/or the application of higher factors of safety in the design of large components than for small members. There is no accepted definition of what constitutes a small or large member but here an arbitrary classification of any component having a sectional depth exceeding a few inches is considered a large section. There are several reasons for this selection. Most of the available fatigue data pertains to specimens of less than 0.5-inch in diameter with some systematic tests on parts up to 2 or 3 inches in diameter." After reviewing the work of American investigators of small sections [Peterson (1930), Horger & Maulbetsch (1936), Buckwalter & Horger (1937), Moore & Morkovin (1942-44), Moore (1945), Moore et al. (1948), and Yen (1950)] and work on statistical and size effects [Freudenthal & Gumbel (1953), Horger & Neifert (1953), and other authors], the authors present extensive fatigue data on large sections.

Sears, Gatti & Fullman (1954) write (pp. 727-728): "It has been reported that

whiskers of tin [Herring & Galt (1952)] and zinc sulfide [Piper & Roth (1953)] have elastic properties closely approaching those which have been predicted for perfect crystals... . In this note the unusual elastic properties of iron whiskers, which have been grown in this [General Electric Research] Laboratory... , are reported. These whiskers are believed to grow by the same mechanism which has been proposed for mercury whiskers... . Accordingly, they are presumed to be perfect crystals except for an axial screw dislocation. Their unusual strength, however, only demonstrates that they are perfect or near-perfect crystals. ...The largest observed elastic strain in a smoothly bent section [of a 15 μ iron whisker] is 1.4 $\pm$ 0.1 per cent. It was established by X-ray diffraction...that an iron whisker is α-iron bounded by four (100) planes parallel to the axis. Young's modulus for α-iron normal to the (100) plane is 1.9 x 10^7 p.s.i. ... . The maximum elastic stress is calculated to be 270,000 p.s.i. in a smoothly bent section. The stress is to be compared to an elastic limit of 4,000 p.s.i. for a pure iron single crystal. Although no chemical analysis is available to attest the purity of the iron whiskers, impurities would not raise the elastic limit beyond 20,000 p.s.i. ... It has been observed that iron whiskers lose their unusual strength after exposure to air for a few days. The loss of strength occurs before visible signs of oxidation appear. This fact indicates that edge dislocations can be introduced into α-iron by oxidation."

Beams, Breazeale & Bart (1955) write (Abstract, p. 1657): "The tensile strength of silver films are determined as a function of their thickness. The films are electrodeposited on the complete cylindrical surfaces of small rotors and the rotor speeds necessary to throw them off are determined. Both the tensile strength and the adhesion of the films are obtained by using rotors of different radii. ...The tensile strengths of the films thicker than about 6 x 10^{-5} cm are found to be independent of thickness and approximately equal to that of bulk silver at the corresponding temperature. For thickness from 6 x 10^{-5} cm to 2.5 x 10^{-5} cm the data

show some scatter, but below 2.5×10^{-5} cm the tensile strength increases many-fold. The region where the tensile strength increases very rapidly is slightly dependent upon the electrodeposition current but within experimental error is independent of film temperature."

Deeg & Dietzel (1955) review previously published data [Griffith (1920), Reinkober (1931), and Murgatroyd (1944)] on the results of experimental investigations of the dependence of the strength of glass fibers on their diameters and on the time that has elapsed since they were drawn. They relate these results to new results on the causes of anomalous mechanical properties of glass fibers.

Dorey & Smedley (1956) write (pp. 254-255): "The available experimental evidence has been employed to substantiate the belief that the criterion of true strength of a filleted shaft is represented by the equation $\sigma = A\text{-}BD + C\sqrt{r}$ ... where A,B, and C are constants for the material ... [D = shaft diameter, r = fillet radius]. ... The criterion accounts for a number of phenomena which have been observed in practice. These include: (1) the decrease of notched fatigue strength with increasing diameter of the shaft; (2) the apparently low fatigue strength of notched and sizeable components made of high tensile steels; (3) the apparently greater notch sensitivity of certain sizes of steel shaft under either reverse bending or push-pull loading than under reverse torsion. While the new theory has many desirable features, two immediate difficulties arise which require explanation. These both relate to size effect and can be expressed by two questions: Are the results of small-scale fatigue tests satisfied by the equation? For many large shafts with relatively small fillet radii, the size effect term could be greater than the other two terms. Can the theory be applied to shafts of very large diameter? ... The method proposed by Kuhn & Hardrath (1952) appears to be the most reliable for treating the small-scale results. No matter how small the fillet radius, a very large shaft must retain a real fatigue strength. In its present form the equation does not satisfy these conditions. One of the following theories may account for the

discrepancies: (1) There is a lower limit of fatigue strength comparable with the upper limit, i.e., the unnotched fatigue strength. No matter how sharp the fillet radius or large the shaft, a fatigue crack will not form or propagate below this stress level. (2) Size effect, BD, reaches a maximum value and then remains constant. Above the diameter corresponding to the peak value, either (a) the fatigue strength-fillet radius relationship remains unchanged or (b) C or r or both terms become functions of D. (3) The unnotched fatigue strength falls rapidly with increasing shaft diameter; the values of the constants change so that σ never approaches zero." The authors give reasons to believe that theories (1) and (2b) are more plausible than (2a) or (3), but state that there is no experimental evidence that either is correct, so the problems must remain unanswered until systematic tests are made.

Gaddy (1956) writes (Abstract, p. 5): "The study concerns the relationhsip between the absolute size of test samples and their ultimate strength. To investigate this effect, cubes of Pittsburgh coal ranging in size from 2-inches to 64-inches on an edge were tested in compression machines. The best correlation of the test results was obtained when the ultimate strength was compared to the absolute volume of the sample. Expressed mathematically: $Z = kV^{-1/2}$ where Z = ultimate strength in pounds per cubic inch, V = volume of cube in cubic inches, k = coefficient depending upon the chemical and physical properties of the coal. 'Z' may be expressed in pounds per square inch merely by multiplying both sides of the above equation by the edge dimension of the cube and simplifying thus: $S = kD^{-1/2}$ where S = ultimate unit strength in pounds per square inch, D = edge dimension of the cube in inches, k = coefficient depending upon the chemical and physical properties of the coal. Subsequently, four other coal beds were sampled and cubes ranging in size from 2-inches to 9-inches were cut from the samples and tested. Similar relationships were found to exist. Since the coal beds tested represent the range of physical and chemical characteristics encountered in bituminous coal beds, it is

concluded that the above equation, which states that the ultimate unit strength of cubical blocks of coal varies as the square root of their edge dimensions, can be used for bituminous coal beds other than those from which samples were obtained. However, the value of 'k' must be determined for each coal bed and locality in which the equation is to be applied."

Greene (1956) writes (Introduction, p. 67): "Careful observation of a considerable number of breaks in glass objects of various sizes and shapes, ranging from rods to pie plates, has failed to reveal a single failure starting from the interior of a homogeneous piece of glass. ... A survey of the literature on the strength of glass also amply supports the generalization that fracture always starts at a place on the surface that is in tension. Hence, it seems logical to treat strength as a surface property of glass. ...If one considers tensile strength, not as an intrinsic property of the glass but rather as a distribution function of the probable stresses required to cause a failure to start at a specified location on a glass surface, the variation in strength with size of the test piece, which is recognized by almost all authorities in the field, becomes a necessary consequence."

Uzhik (1956) presents results which have been summarized by D.G. Sopwith as follows (Reporter's Introduction, pp. 16-17 of same Proceedings): "Uzhik... , discussing the mechanical aspect of size effect, draws attention to two possible effects of size -- the increases in the number of internal or external flaws in a larger specimen, and the change in the stress conditions. The geometrically similar specimens usually used in size-effect investigations have similar maximum stresses under similar loading, but the stress-gradient is inversely proportional to the radius r of the specimen. For similar stress conditions in large and small specimens the relative stress-gradient (ratio of stress-gradient at notch to maximum stress) should be the same. For a deep notch of radius ρ in bending, the author derives from Neuber...an expression for the relative stress-gradient which can be approximately represented by $1/r + 2/\rho$. Using either this or the exact expression,

specimens of different sizes can be designed in which the stress-gradient as well as the maximum stress are similar. Preliminary fatigue tests on such specimens indicate little size effect; if this is confirmed it will be possible to determine the fatigue strength of large components with stress concentrations using suitably designed small specimens."

Arnold (1957) writes (Synopsis, p. 1273): "A specimen has been developed which permits measure of sheet metal toughness over a range of temperatures using conventional pendulum-type testing apparatus and test procedures. Blanks are cut from the sheet and riveted together to form a laminate from which the specimen is machined to dimensions following those of the standard Charpy V-notch design. Toughness of the sheet is calculated by dividing the impact strength of the laminated specimen by the number of laminae which it contains. Tests have shown that the specimen yields highly reproducible data and is sensitive to metallurgical variables affecting toughness. Test of specimens fashioned from laminae machined from steel plate disclosed linear relations for (a) the impact transiton temperature and (b) the minimum energy of an individual lamina in ductile fracture <u>versus</u> thickness of an individual lamina. This linearity maintained over a range of thickness from 0.050 in. to 0.200 in. Test of laminated specimens prepared from titanium alloy sheet of various compositions indicated that the impact strength for a given composition at a selected temperature is closely proportional to sheet thickness, provided that sheet tensile properties are approximately the same. Sheet ranging between 0.020 in. and 0.137 in. thickness were tested."

Hoffman (1957) writes (Summary, p. ii): "Fine crystalline filaments -- commonly called 'whiskers' -- possess the highest strength known in materials, in some instances approaching the theoretical limit of atomic cohesion. This study deals with the possible exploitation of the phenomenal whisker strength for structural purposes. First, the feasibility of mass producing whiskers is established by surveying various growing techniques, means for ordering and aligning, and methods

for collecting and binding into a continuum. The theories and experiments that explain the strength of whiskers are reviewed, particularly the strength-diameter relation, and the dependence of the ultimate tensile strength on the modulus of elasticity. Criteria for choosing the least weight whisker materials are derived and used to determine the optimum materials for use at various temperatures. The structural properties of these hypothetical materials are calculated and used to compare weights of equal-strength tensions structures using whisker and conventional materials. In general, weight reductions to one-fifth appear possible, and the consequences of such drastic weight savings are discussed. The study is concluded with a list of topics for research."

Shevandin, Razov, Reshetnikova & Serpeninov (1957) write (translation of p. 1057): "The effect of the scalar factor upon the deformation capability and durability is detected in brittle [Weibull (1939a,b), Shevandin & Manevich (1946)], semi-brittle [Vitman (1946), Chechulin (1954[a])], and completely tensile [Chechulin (1954b)] disintegration. In the overwhelming majority of investigations, the role of this factor is treated from the viewpoint of statistical theory of durability [Alexandrov & Zhurkov (1933), Kontorova & Frenkel (1941)]. ... Davidenkov... submitted a proposal concerning the possibility of connection of the scalar effect in the semi-brittle breaks of steel (and an earlier emergence of crystalline areas in larger notched samples) with the effect of elastic energy accumulated by sample in loading. Later on, this idea, as far as we know, was proven experimentally in only one opus [Wells (1955)]. Nevertheless, this problem has not as yet been solved. ...The object of the present study was to determine the task of scalar effect for all the above-mentioned instances of metal disintegration. ..." In conclusion, they write (translation of p. 1060): "...In all cases which we have studied, viz. in tensile, semi-brittle, and brittle disintegration, the scale effect determined by the influence of elastic energy accumulated in the sample-machine system, at least in that usually studied and having practical meaning for a range of dimensions,

is not connected with the statistical factor. The scale effect, that is, the effect of a large amount of stored elastic energy is made manifest in large samples; in tensile disintegration -- in a quicker rate of disintegration of flaws and, correspondingly, by a lesser amount of real resistance to a break; at semi-brittle disintegration -- in an earlier appearance of zones of crystalline structure and increase in inclination to brittleness; in brittle disintegration -- in a decrease of the value of elastic strength. In all cases, it can be connected with the fact that in large samples -- in contradistinction to small samples -- an unstable state characterized by an acting force increased resistance to deformation arises at the formation of a flaw. Nevertheless, considering the results obtained, it is probable that the cause of the scale effect for all types of disintegration is taken out of a large amount of elastic energy liberated at the formation of a flaw in larger samples. A number of experimental facts attest that the scale effect at fatigue-induced disintegrations is also connected with elastic energy accumulated in a sample at loading. Consequently, the recommended interpretation on the nature of the scale effect has universal significance and this, mainly, in its positive aspect. A new elucidation of the mechanics of metal disintegration is an essential result of the research done."

Björklund (1958) writes (Section 2 of English summary, pp. 75-76): "Among the properties relating to the strength of the material the tensile strength must be considered the most important. It has been maintained that every kind of excessive strain to which a ceramic material is subjected and which results in a fracture really means that the tensile strength of the material has been exceeded in some part of the body. It is consequently of primary importance to elucidate the conditions governing the value of the tensile strength in ceramic materials. In this connection particular attention should be focussed on the following problems: a) The values generally show a considerable dispersion at experimental determinations, even when the materials exhibit a high degree of isotropy and the experimental con-

ditions are exceptionally favourable. This is a characteristic peculiarity of ceramic materials. b) The tensile strength is markedly dependent on the volume of the object subjected to the breaking strain. Particularly the size and qualities of the surface layers appear to exert a decisive influence. When the volume of the object (limiting surface, area of fracture) increases, the tensile strength decreases. c) The dispersion of values also depends on the volume of the object. The larger the volume subjected to load, the greater the dispersion. ...The classical theory concerning the strength of materials gives insufficient explanations regarding the properties exhibited by ceramic bodies as to their tensile strength. WEIBULL has put forward a theory giving quantitative expressions of the influence exerted by the phenomena mentioned above under 2a, 2b and 2c in the case of isotropic materials. In view of the paucity of experimental data it is impossible to make any pronouncement on the validity of the theory; it appears, however, to be highly promising."

Evans & Pomeroy (1958) write (Abstract, p. 5): "Plane crushing tests on cubes of Deep Duffryn and Barnsley Hard coals in the size range 1/8 in. to 2 in. have established that there is a considerable variability in strength, even for cubes of the same size. If P_a is the probability of a cube of side a surviving a particular stress and P_b the probability of a cube of side b surviving the same stress, the relation between them is shown to be of the form $P_b = P_a^{b/a}$. This expression is similar to that derived in the 'weakest-link' theory for the strength of chains. The relation between the mean crushing load (Q) and side of cube (a) is of the form $Q \propto a^{\beta}$. It is shown that a law of this form is in agreement, as a reasonable approximation, with deductions made from the weakest link theory. The observed values of β are a little less than 2, and there are reasons for supposing that the extreme permissible values are 1.5 and 2. The implications of the weakest-link theory on the physical nature of the breakage of coal have been examined. It has been shown that the theory, as well as explaining phenomena on a macroscopic scale, is consonant with the breakage of the ultimate structural elements of the coal according

to the Griffith crack theory; the elements being the 'links' of the statistical theory. It is estimated that the sizes of these elements lie in the range 1-1,000 Å in linear dimension and that the ultimate crushing strength of coal is of the order of 100,000 lb./in.^{2}."

Jellinek (1958) writes (Abstract, p. 797): "Tensile strength measurements on ice cylinders adhering to stainless steel have been made as a function of the rate of loading, thickness and cross-sectional area of specimens, and temperature. A rapid increase of tensile strength occurs as the volume is decreased. The data for a temperature of -4.5°C can be represented over a thousandfold range of volume by an equation as follows: $S = 2.74AV^{-0.84} + 9.4$ kg cm^{-2}, where S is the tensile strength, A the cross-sectional area in cm^2 and V the volume in cm^3. The experimental results are interpreted by means of a statistical treatment involving imperfections in the specimens. The statistics for a model consisting of a large number of parallel elements is elaborated. The final equation derived on statistical grounds approximates the equation found empirically, and reads as follows: $\bar{S}_{r,n}=kA^{1/\beta}V^{-1/\beta} + C$, where $\bar{S}_{r,n}$ is the number average of the tensile strength, k, β and C are constants and r is the number of imperfections in each of n parallel elements. This result is quite general and not confined to ice. The conclusion is reached that due to imperfections the tensile strength is a statistical function of the volume and cross-sectional area of the specimens. Superimposed on the statistical effect is a stress distribution effect, which becomes predominant for large volumes."

Walton (1958) writes (p. 66): "Griffith [(1920)] measured the tensile strength of glass fibres in the diameter range 0.00013-0.040 in. and his results are shown in Figure 1 [not reproduced here]. The strength found for his finest fibre was 4.8×10^5 lb./in.^{2}. His results were very similar to those obtained by Karmarsch in 1858 for metal wires, and for the corresponding diameter range, they were fitted by Karmarsch's derived expression correlating breaking stress and diameter. From this Griffith inferred that the mechanism of rupture in metals was similar to that

in brittle amorphous solids. He also postulated that isotropic solids contain randomly distributed cracks or 'flaws' which act as stress concentrations, so the average stress at rupture is lower than the theoretical cohesion. Fine fibres would therefore give a higher measured strength than larger ones as they would probably contain fewer severe flaws. Although Griffith's experimental results have been discussed at great length and have served to stimulate various theories of rupture mechanism and 'structure,' it should be pointed out that his correlation was not really valid, as his test pieces were not produced and tested under sufficiently controlled conditions. An interesting general observation from his work was that the strength of glass fibres was very much higher when tested within a few seconds of preparation than when tested a few hours later. The fibres tested for the relationship of Figure 1 were aged for 40 hours after drawing, but for specimens tested within a few seconds of drawing, he found strengths up to 2×10^5 lb./in.^{2} for fibres 0.020 in. dia." The author reviews work on the size effect on the strength of glass by various other authors, including Anderegg (1939), Murgatroyd (1944), and Otto (1955). She states the following conclusions (p. 76): "It is apparent...that much of the early work on the effect of specimen size on the strength of glass is invalidated by the insufficient control which was exercised over the conditions of specimen production. Otto's attempt to produce test specimens under nearly identical conditions is outstanding. When he pulled fibres at constant speed from a constant temperature melt, the diameter being varied by varying the size of the orifice, the fibres gave the same measured strength. However, these drawing conditions would not necessarily ensure that all his specimens had the same fictive temperature. The effect of specimen size on strength can only be resolved by producing specimens, of one chemical composition, which can be shown to have the same uniform fictive temperature. Long annealing treatments may help to achieve such uniformity within specimens and measurements of density or electrical conductivity would allow a comparison between specimens to be made. It is possible that the de-

pendence of measured strength on the size of a specimen is only an apparent effect, the independent variables being the thermal history and the atmospheric conditions of storing and testing. Otto's work did, however, crystallize the important commercial fact that fibres of different diameters can be produced with the same strength. The concept of flaws in glass has provided a convenient explanation of the apparent size effect on strength ... . That micro-inhomogeneities do exist in glass seems to be beyond question, but much more work is necessary in order to elucidate their precise nature."

Kuczynski (1959) reviews the literature on the strength of concrete. He includes several papers on the size effect of concrete and other materials, among them those of Johnson (1943), Tucker (1945a-c), Prot (1949c), l'Hermite (1950), and Price (1951). He presents results of new tests made in Poland on cylindrical and cubical concrete specimens of various dimensions and h/d ratios. He writes (English summary, p. 168): "The following conclusions may be drawn from the tests in Poland and abroad: 1. The difference between the results obtained by various investigators are considerable. This shows a great number of parameters [including specimen size and shape] influencing the strength of concrete. These parameters are not taken into consideration in the same way by all the investigators. Hence the difference. 2. Every generalization of test results is of an approximate character, typically statistical. The dispersion of the strength measurements is usually of 17-18% which is the characteristic value of the dispersion for concretes produced on building sites. 3. In spite of certain differences, the results of the test described are similar to those obtained in other countries and the coefficient ϕ [ratio of concrete strength determined by means of various test-pieces] obtained should be taken into consideration during the elaboration of Polish standards and in the works of the I.S.O."

Fridman (1960) writes (pp. 1179-1180 of translation): "Considerable discrepancies between the strength of small laboratory samples or parts can only be properly

explained when utilizing a reliable theoretical basis. In our opinion, the theory of dimensions and model analysis...can be such a basis, since only scientific model analysis permits *generalization* of results of single (laboratory) experiments and their extension to a wide array of *phenomena and processes* of deformation and failure of materials while being worked on and in service. Scientific model analysis is such a utilization of a phenomenon (in our case of elastic, plastic deformation and failure), where the fields of all physical variables are similar to fields of corresponding variables in the process being modeled. Strictly speaking, such modeling (similarity) is never observed in mechanical tests. Therefore, there should be no surprise that a scale factor exists, but rather that it is in many cases small, despite the fact that there are significant deviations from conditions of similitude. ...In conclusion, we will note that very often the change of scale is accomplished by completely extraneous phenomena, leading to fundamental differences in structure and properties of small and large samples (for example, as a result of different annealing or of crystallization conditions, or of differences in conditions of pressure working, etc.). In such cases, deviations from similitude are not the only cause for the scale effect, since differences in material of the small and large samples compared are also (often even more) significant. Only determination and application of theory and criteria of similitude can help to separate the numerous reasons for 'scale effect' into two groups which are entirely different in nature: 1) failure to observe physical similitude, and 2) change of structure and of properties of the material in small and large samples, which essentially leads to comparison of different materials. The performance of such a separation will help in taking the scale effect into account, and will also teach to correct it in desirable directions."

Parratt (1960) reports the results of a study of the strength of plastics reinforced with glass and other fibers. He states the following conclusions (p. 266): "(i) The reinforcing strength obtained from a fibre of given diameter is the strength

corresponding to a certain critical length of fibre. At this 'critical length' the strength of the fibre exceeds the maximum stress it can transmit. In addition, the apparent reinforcing strength of a system of fibres is lower than that of an average single fibre. This is because in any cross-section of reinforced material just before failure, some fibres will already have broken, increasing the load on the remainder. (ii) The thicker the fibre, the greater is the 'critical length.' In the normal way this means that thicker fibres make a weaker reinforcement. However, equally strong mouldings can be made of fibres up to 0.001 inch thick at least, providing care is taken in drawing and handling. (iii) Crimping and twisting of fibre bundles in the textile processes have often been blamed for abnormally low strength in fiber glass plastics. As well as eliminating such faults as these, it is concluded that to make full use of high fibre strengths for reinforcing, mechanical defects on each single moulded fibre must be at least 10^4 diameters apart."

Bolotin (1961), in Chapter I (pp. 1-26), presents the elements of probability theory and mathematical statistics, including the theory of extreme values. In Chapter III (pp. 49-77), he deals with statistical theories of fracture. After some preliminary remarks (Section 27, pp. 49-50), he develops the statistical theory of brittle fracture (Section 28, pp. 51-53), concerning which he writes (p. 51): "...The statistical theory of brittle fracture was first proposed by W. Weibull [(1939a)], Ia. I. Frenkel' and T.A. Kontorova [(1941)] to explain the influence of the body volume on its brittle strength. According to this theory, brittle fracture depends on the local stress at a point where the most dangerous structural defect occurs. The body contains a very large quantity of defects of different degree of latent danger, and these follow some statistical distribution. The more brittle the solid, the greater the probability of detecting a primary element of low strength, and the lower the strength of the body as a whole. If the stresses are distributed nonuniformly over the volume, then the volume of that portion of the body where the stresses are relatively large acquires greater importance. The origin of the defects is not important to the development of the statistical theory of brittle fracture;

it is only essential *that the strength of the body as a whole depend on the strength* of the most defective element, and that the properties of the primary elements be subject to some probability distribution. There are various approaches to the statistical theory of brittle fracture, which differ in the method of reasoning and in the postulated form of the distribution function for the strength of the primary elements [Weibull (1939a), Kontorova (1940), Kontorova & Frenkel (1941), Kontorova (1943), Kontorova & Timoshenko (1949), Chechulin (1959[a]), and Sedrakyan (1958)]. A development using an asymptotic distribution for the minimum values of sufficiently large sets...will be given below. This approach is a generalization of the Weibull method, being at the same time more justifiable from the theoretical viewpoint." In Section 29 (pp. 53-54), the author deals with the scale factor in brittle fracture, establishing the connection between the mathematical expectation of the strengths of a body and its volume. In Section 30 (pp. 54-61), he considers the variability of strength in brittle fracture; he shows that for $s_0 \neq 0$ [s_0 = minimum strength of primary element], the coefficient of variation of the strength limit decreases as the volume increases. Sections 31 and 32 (pp. 61-69) deal with generalizations and applications of the theory of brittle fracture. Sections 33-35 (pp. 69-77) deal with the statistical theory of fatigue. The author derives equations which permit examination of the questions of distribution of fatigue limits, of the influence of absolute size and stress concentration, of the connection between the fatigue limits for different stress fields, and of the connection between scale effect and the variability of fatigue limits. Results given by Afanas'ev (1940,1953) and Weibull (1949) are special cases of his formulas.

Gregory & Spruill (1962) write (p. 34): "The application of statistical theory to fracture of structural elements has been developed slowly over the last forty years. The beginning may be considered to be A.A. Griffith's [(1920)] work although this work was based rather solidly upon previous research. Griffith postulated that there exist flaws of random intensity distributed randomly throughout a body. This

means that there will be a distribution of strengths in a given specimen in the sense that a different stress level is required to originate fracture at various points in the body. If this flaw concept is accepted, then the strength of a given specimen is determined by the smallest flaw fracture stress in a sample of size 'n' where 'n' is the number of flaws in the given volume under load. Since 'n' increases as the volume increases, the relationship of strength to volume is equivalent to the distribution of the smallest flaw fracture stress as a function of 'n,' the sample size. F.T. Peirce [1926)] based his work upon the fact that the strength of a chain is that of its weakest link. This allowed him to pose the problem mathematically, and use the research of Tippett [(1925)] on the distribution of extreme values to obtain quantitative results. Investigations in this area must be considered basic research until the publication by Weibull [(1939a,b)]... . Here a unified theory of failure is developed and an important contribution is made. This is the introduction of a simplified empirical cumulative distribution that allows closed form solutions to be made. This distribution function was originally in the form $P = 1-\exp[-f(x)]$; $f(x) = (x/x_0)^m$, where P is the probability of failure, x_0 is a function of the mode of the distribution and m is related to the amount of scatter present in the data... . After experience was accumulated with this distribution it was found that its fit to many data samples could be improved by adding a lower bound. This new distribution [Weibull (1951)] was shown to apply excellently to seven different statistical samples from a wide range of fields... . In its revised form, $P = 1-\exp\{-[(x-x_u)/x_0]^m\}$. ...Epstein [(1948a)] attempts to bring this work into context with the general statistical theory and show how it fits into an underlying foundation of theoretical statistics." The author also gives a technique for surface effects. He writes (p. 42): "For cases where failure originates on the surface of a material in a large percentage of cases, the procedures of this investigation may be applied by assuming the volume which precipitates failure extends only over a surface layer of small thickness h."

Gücer & Gurland (1962) write (Summary, p. 365): "The relative strengths of an aggregate material are calculated for the following two modes of fracture: (i) by independent fracture of individual volume elements separated by a crack propagation barrier [dispersed fracture], (ii) by propagation of a crack from the weakest volume element in the absence of a propagation barrier [weakest link]. The first case is analysed by means of a model which consists of an assembly of layers in series, each layer representing a bundle of volume elements uniformly loaded in parallel. The strength distribution of the stacked layers is obtained by an application of the statistical theory of bundles of filaments after Daniels and others, and 'weakest link' statistics are used to yield the strength distribution of the cylindrical bodies. For comparison, the 'weakest link' statistics of Epstein and others are given for the second case which corresponds to the usual cleavage fracture model of brittle materials. The expected volume and shape effects of the two modes are compared." The authors state the following conclusions (p. 373): "(1) The strength of a large body failing by 'dispersed fracture' of its volume elements may be expected to be greater than that of a similar body failing by 'weakest link' fracture. (2) The statistical theory based on the 'dispersed fracture' model predicts opposite effects of length and cross-sectional area on the strength of a specimen in simple tension. (3) The difference between 'weakest link' and 'dispersed fracture' strengths, also the shape and volume effects, are most pronounced at small values of the homogeneity [Weibull shape] parameter β."

Serensen, Giatsintov, Kogaev & Stepnov (1962) give the results of fatigue tests for three wrought aluminum alloys used for the manufacture of rotor blades for helicopters. For their study of the scale factor (size effect), they chose the alloy AVT. Specimens tested were of three types -- solid with diameter 8 mm., solid with diameter 40 mm., and hollow with outside diameter 40 mm. and inside diameter 30 mm. With an increase in the diameter of the solid specimens from 8 mm. to 40 mm., the standard deviation of the fatigue life decreased by a factor of from 1.5 to 2. Up

to about 10^6 cycles, the strength of the alloy AVT was found to be relatively insensitive to the diameter of the specimen, but the fatigue limit for solid specimens 40 mm. in diameter was found to be 10-12 kg./mm.^{2} as compared with 14-15 kg./mm.^{2} for specimens 8 mm. in diameter. There was only a small difference between the strength of the solid and hollow specimens each 40 mm. in outside diameter, probably because of the fact that fracture always started at the (outer) surface.

Sakui, Nakamura, Nunomura & Fujiwara (1963) write (Synopsis, p. 672): "The effect of the specimen width on the Charpy test was studied with specimens of hot-rolled mild steel and quenched-and-tempered medium carbon steel, by recording the load-time relations under impact bending. Results obtained were summarized as follows: (1) The energy absorption in the ductile range was influenced by the specimen-width smaller than 4 mm -- the smaller the width, the lower the energy absorption per unit sectional area and it was found that the law of similarity was not satisfied in this case. On the contrary the maximum fiber fracture strength was almost constant for all the specimens, showing the applicability of the law of similarity. (2) Maximum fiber bending stress in fracture was the largest in the temperature range where the absorption energy was decreased almost to minimum value and the load-time curve of type I designated by the authors in the previous papers [published in 1960 and 1961 in the same journal] was obtained."

Corten (1964) considers (pp. 914-935) the influence of fibrous reinforcement on composite strength. After a brief introduction starting with the theory of Griffith (1920), he gives a statistical analysis of the strength of single filaments. He writes (pp. 915-918): "A single fiber of volume, V, will be viewed as an assembly of small equal volumes, v. The strengths of the small volumes of material constitute a population of strengths in which the probability of finding the strength of a volume, v, in the range from σ to $\sigma + d\sigma$ is $f(\sigma)\,d\sigma$. Thus the probability of a fracture of a small volume, v, of material at a stress level less than or equal to σ is given by $F(\sigma) = \int_0^\sigma f(\sigma)d\sigma$. ...A glass fiber will be considered to consist of

$\omega = V/v$ small volumes of glass. ...The statistical strength distribution for a fiber, $G(\sigma)$ is given by... $G(\sigma) = 1 - [1 - F(\sigma)]^{\omega}$. Weibull [(1939a)] has suggested a simple, realistic form for $F(\sigma)$, namely, $F(\sigma) = [(\sigma - \sigma_u)/\sigma_0]^m$, where σ_0 is the upper strength limit and σ_u is the lower strength limit... . The Weibull statistical strength distribution is given by $G(\sigma) = 1 - \{1 - [(\sigma - \sigma_u)/\sigma_0]^m\}^{\omega}$. This function appears to be particularly useful because of the direct physical interpretation of the parameters, σ_0, σ_u and m. The effects of the composition and the thermal history may be directly measured by the parameter σ_0. Surface damage may be measured separately by the parameter m. Size effect is inherently part of the analysis through the number ω of small volumes. ...The mean strength, $\bar{\sigma}$, of a group of single filaments is given by $\bar{\sigma} = \sigma_0\, \omega^{-1/m}\Gamma[(m + 1)/m] + \sigma_u$ (53) and the variance of σ...is $\mathrm{Var}\, \sigma = \{\sigma_0\omega^{-1/m}$ $\Gamma[(m + 2)/m] - \Gamma[(m + 1)/m]\}$...*. ...It is clear that the mean strength and scatter are a function of the parameters σ_0, ω and m. Thus for fibers of a given composition and thermal history (constant σ_0) and specimen length and diameter (constant ω), the mean strength $\bar{\sigma}$, and scatter, Var σ, are completely characterized by the Weibull m parameter. This means that m constitutes a one-parameter measure of the degree of surface damage present in a single filament... ." Next he considers the strength of a bundle of filaments. He writes (pp. 919-922): "Based on the statistical description of the strength of single filaments, the strength of a bundle of parallel filaments may be estimated. It is assumed that each filament in the bundle is stressed to the same axial tensile stress...and that the filaments are parallel and do not touch one another. For convenience the cross-sectional area of each filament is assumed to be the same... . The bundle strength, σ, ...is computed for the original N filaments and is $\sigma = \sigma_0(1/m\omega)^{1/m}e^{-1/m}$(61). ...It is of interest to compare the strength of a bundle of filaments with the average strength of a group of single filaments. For this purpose it is convenient to form the ratio $\sigma/\bar{\sigma}$, using σ from Eq. (61) and $\bar{\sigma}$ from Eq. (53). Thus for $\sigma_u = 0$, $\sigma/\bar{\sigma} = (me)^{-1/m}/\Gamma[(m + 1)/m]$. It is immediately clear that this ratio is a function of m only and that the values of the ratio $\sigma/\bar{\sigma}$

* This expression for Var σ is incorrect; for correct expression, see Corten (1967).

increases as m increases, as shown in Figure 54 [not reproduced here]." Finally, the author considers the influence of fiber diameter and length. He writes (pp. 923-925): "The strength of single filaments as a function of fiber diameters is shown in Figure 55 [not reproduced here]. The decrease in strength with an increase in diameter is observed with all data except curve 7. ...In Figure 56 [not reproduced here] data for single filaments illustrate the reduction of mean strength with increasing filament length. These data can be interpreted in terms of Eq. (53) arranged to read $\ln(\bar{\sigma}/\sigma_0) = \ln \Gamma[(m+1)/m] - (1/m)\ln\omega$ (64) where $\bar{\sigma}$ is the mean strength from a group of single filament tests, and for convenience, v will be taken as a cylindrical portion of a fiber with a length equal to one fiber diameter. Thus for a fiber of length ℓ, $\omega = \ell/d$. The data from Figure 56 are replotted in Figure 57 [not reproduced here] according to Eq. 64... ."

Metcalfe & Schmitz (1964) write (Synopsis, p. 1075): "The purpose of this work was to investigate (1) the importance of gage length in tension of glass fibers, including verification of a proposed damage model, and (2) the applicability of Weibull-type analyses to study flaws governing failure. E- and S-glass fibers, both virgin and from strands, were examined. A marked effect of gage length was found on the strength of glass fibers. The logarithmic strength-length plots were not linear over the entire length range from 0.025 to 30 cm. For a given fiber, a slope change occurred at a critical gage length due to varying contributions of mixed flaw populations to failure strength. Different flaws could be identified through a specially developed analysis of failure distribution plots. Single exponent failure distribution functions, such as the Weibull function, are inadequate to describe failure in glass for the conditions investigated. At least two different exponents are needed to fully describe the observed strength behavior. As a result, it does not appear possible to substantiate a generalized model to describe damage of glass fibers."

Rosen (1964) writes (Abstract, p. 1985): "This paper presents a theoretical and experimental treatment of the failure of a composite, consisting of a matrix

stiffened by uniaxially oriented fibers, when subjected to a uniaxial tensile load parallel to the fiber direction. The fibers are treated as having a statistical distribution of flaws or imperfections that result in fiber failure under applied stress. The statistical accumulation of such flaws within a composite material is demonstrated to be the cause of composite failure. An experimental program utilizing a new technique to observe the failure process is also described. The test specimens contain a single layer of glass fibers which enables microscopic evaluation of the internal failure process. Random fiber fractures are observed at loads significantly below the ultimate composite strength level." He also writes (Discussion and Conclusions, pp. 1990-1991): "A statistical analysis of the tensile failure of fibrous composites has been performed. The analysis attempts to simulate a failure mode based on distributed internal fractures prior to composite failure. Although an effort was made to approximate the most important parameters influencing failure, the model is not likely to provide accurate quantitative answers without further refinements. It is expected, however, that the nature of desirable improvements in constituent characteristics can be ascertained from the present model. The shortcomings of this model include failure to consider fracture involving parts of more than one layer, variation of ineffective length with stress level (for the inelastic case), and stress concentrations in fibers adjacent to failure areas and the initial state of stress. Further, it will be necessary to obtain an accurate statistical characterization of any given fiber propulation so that the link strength characterization can be inferred properly. On the positive side, however, the model represents the constituents in the major functions of fibers carrying extensional stress and matrix carrying shear stress; it includes the effect of fiber imperfections on fiber failure, and it accounts for the accumulation of internal cracks that combine to produce component failures. The latter follows the concepts of Parratt [(1960)], who suggests the influence of flaws and ineffective lengths on failure. The failure mode in the present analysis, however, results from an accumulation of cracks rather

than from the existence of fully ineffective fibers. In fact, the present results, which indicate typical fiber lengths at failure and which are an order of magnitude larger than the ineffective lengths obtained by Parratt and those of...this paper. The present analytical and experimental studies indicate that discontinuous fibers may exist in a composite of originally continuous fibers at stress levels well below the maximum load. The mechanical properties of the matrix material will then have a significant effect on the composite strength. This influence is measured by the efficiency with which the matrix transmits load around a fiber break. The experimental techniques developed to observe the internal failure process appear to give consistent results. The expanded use of such techniques appears to offer promise of increased understanding of the failure process."

Brenner (1965) discusses the effect of size on whisker strength. He writes (pp. 15-18): "Only a very small fraction of the whiskers exhibit near-theoretical strength. Two common characteristics of most whiskers are the large fluctuation of their strengths and a strong dependence of strength on size. The strong size dependence, first reported by Gyulai [(1954)] for NaCl and by the author [Brenner (1956b)] for iron and copper, has now been found for many other materials such as Ni, Co, Al_2O_3, Si, etc. Figure 4 [not reproduced here] gives Regester's data for Al_2O_3 whiskers, from which it can be deduced that there is a nearly linear relationship between the strength and the reciprocal of the diameter. These data agree well with the author's earlier data on Al_2O_3 in which only a small section of the whisker was tested. ...The strength-size relationship obtained depends on several factors including the type of test performed. For example, it has been observed that whisker strengths measured in bending are often much larger than those measured in tension. ...The strength-size relationship is also strongly affected by growth variables. This is especially true of whiskers grown by the hydrogen reduction of halides. The only quantitative study of the effect of growth variables on the strength-size relationship was reported by Gyulai and co-workers, the results

of which are shown in Fig. 9 [not reproduced here]. Similar studies should be made with ceramic whiskers that are considered for composite materials." The author also discusses the strength of "non-whisker" fibers. He writes (pp. 30-31): "The effect of size on strength is not unique with whiskers. Experimentally it has been shown that many materials increase in strength with decreasing size. However, there may be several different reasons for this effect. In brittle solids it is well established that the strength is governed by the distribution and severity of surface defects. The size effect in glass and quartz fibers has been demonstrated many times and has been attributed to the decreasing number of surface defects as the surface area is decreased [Griffith (1920), Reinkober (1931,1932)]. It is now recognized that surface defects can also be removed by careful etching and polishing and that brittle materials in bulk form are able to sustain the same stresses as thin fibers, that is, the size effect disappears. ...*Ductile materials in bulk form* also show an increase in yield strength with decreasing size. For instance, drawn wires exhibit very high strength and size effect is associated with the degree of cold work that can be achieved and the shape of the precipitate particles produced by the wire-drawing process. Upon annealing the strength of the wires decreases very markedly. Size effects have also been reported for annealed single crystals that have been thinned by chemical etching. ..."

Rosen (1965) reviews the statistical model of failure of fibrous composites discussed in his previous paper [Rosen (1964)]. He writes (pp. 40-41): "The model used to evaluate the influence of constituent properties upon the tensile strength considers that in the vicinity of an individual break a portion of each fiber may be considered ineffective. The composite may then be considered to be composed of layers of dimension equal to the ineffective length. Any fiber that fractures within this layer will be unable to transmit a load across the layer. The applied load at that cross section would then be uniformly distributed among the unbroken fibers in each layer. The effect of stress concentrations, which would introduce a non-uniform re-

distribution of these loads, is not considered initially. A segment of a fiber within one of these layers may be considered as a link in the chain that constitutes an individual fiber. Each layer of the composite is then a bundle of such links and the composite itself a series of such bundles. Treatment of a fiber as a chain of links is appropriate to the hypothesis that fracture is due to local imperfections. The links may be considered to have a statistical strength distribution equivalent to the statistical flaw distribution along the fibers. The realism of such a model is demonstrated by the length dependence of fiber strength. That is, longer chains have a high probability of having a weaker link than shorter chains, and this is supported by experimental data for brittle fibers, which demonstrate that mean fiber strength is a monotonically decreasing function of fiber length. For this model it is first necessary to define a link dimension by consideration of the perturbed stress field in the vicinity of a broken fiber. It is then necessary to define the statistical strength distribution of the individual links, which can be obtained indirectly from the fiber strength-length relationship. These results can then be used in the statistical study of a series of bundles and utilized to define the distribution for the strength of the fibrous composite. ..." The author proceeds with a statistical analysis of the model, which consists of a chain of bundles of fiber links, using results of Daniels (1945) and Weibull (1951). In appendices, he gives detailed discussions of effective fiber length (that portion of the fiber in which the average axial stress is greater than 90% of the stress which would exist for infinite fibers) and of statistical models (Weibull type) for fiber strength.

Sanders & Siess (1965) write (Abstract, p. 654): "The current ASTM specifications for large-size deformed steel bars for concrete reinforcement (A408-62T, A431-62T, and A432-62T), permit use of any one of three types of tension test specimens. Tests performed on specimens taken from bars meeting ASTM Specification A431-62T showed (1) the 3/4-in. diameter and standard 0.505-in. diameter specimens had ultimate tensile strengths 5 to 8 per cent higher than the as-rolled bar [2-1/4 inches in diameter],

(2) specimens taken from the outer portions of the bars had tensile strengths 5 to 8 per cent higher than specimens taken from center sections, and (3) the size and location of specimens had little effect on the modulus of elasticity."

Stepanov & Mikhailova (1965) write (pp. 746-747 of translation): "The 10 x 10 x 55 mm standard specimen obviously cannot be used in assessing the cold brittleness of sheet metal less than 10 mm thick. Using specimens with several simultaneously changing dimensions for determining the dependence of impact strength on thickness failed to produce satisfactory results. Apparently, it is better to change only one dimension of the specimen (thickness). It should be noted that for determining the cold brittleness of a thin sheet metal less than 5 mm thick use is occasionally made of composite specimens. Plain and composite specimens made from 24 mm thick MSt 3kp-steel plate were tested in the 'as delivered' condition. ...A series of 2, 3.3, and 5, and 10 mm thick plain specimens...and 10 mm thick (2 x 5, 3.3 x 3, 5 x 2) composite specimens were used in the evaluation. ...Plotting the impact strength [a_n] data obtained by us as a function of log b (Fig. 3a [not reproduced here]) shows that in the brittle-ductile region this curve is very close to the straight line a_n = C-k log b, where b is the thickness of the specimen and C and k are constants. Figure 3a also shows that the inclination of the a_n = f(log b) curve increases with decreasing temperature, i.e., the rate of decrease of impact strength increases with increasing thickness. This is supported by the results obtained in testing a different melt of MSt 3-kp steel (Fig. 3b [not reproduced here])."

Herring (1966) reports the results of a study made to determine selected mechanical and physical properties of 12 types of boron filaments. Among the properties studied was the effect of gage length on filament strength. The author writes (pp. 7-8): "No evidence of plastic deformation was observed in boron filaments under tension. Therefore, any discontinuity or flaw which exists in a stressed filament is associated with a stress concentration which cannot be relieved by plastic deformation. A particular stress concentration can only result in atomic separation

which depends on the location, orientation, and severity of the flaw. One effect of this lack of ductility in boron filaments is that tensile strength decreases markedly with increasing gage length. This result is expected since the probability of occurrence of a severe flaw increases as the volume of material increases. Figure 6 [not reproduced here] shows the results of a series of tensile tests of boron filament A with specimen gage length varying from 1 to 240 inches (2.54 to 610 cm). A particular group of 120 tests was chosen for this presentation because (of all specimens tested) this was the largest number tested consecutively from a single reel of filament. In figure 6 filament A exhibits behavior which is typical of all halide-process filaments studied. The effect of gage length on the tensile strength of filament A is compared with similar data for E-glass filament [Rosen (1964)] which is brittle, of nearly equal density, and in common use for the fabrication of filament-wound composite structures. Within the comparable range of lengths, the slopes of the two curves are nearly similar and therefore suggest similar occurrence of flaws in both materials."

Riley (1966) proposes a simple graphical method of predicting effective reinforcing strength of fibers in composites. A discussion of his results given by Parratt (1972) is quoted in the summary of Parratt's work given in Section III of this report.

Corten (1967) develops, in Section 2.3.3 (pp. 49-54), a statistical theory of strength of individual fibers and bundles (as a function of volume), based on the Weibull distribution. He takes an elemental volume equal to the entire cross section of a cylindrical fiber with length equal to the diameter, so that a cylindrical fiber of length ℓ and diameter d_f contains $\omega = \ell/d_f$ elemental volumes. The cumulative distribution function of fiber strength, σ, is $G(\sigma) = 1 - \exp\{-\omega[(\sigma-\sigma_u)/\sigma_0]^m\}$, where σ_u is the lower limit of strength (location or threshold parameter), σ_0 is the scale parameter, and m is the shape parameter. The mean strength, $\bar{\sigma}$, of a group of fibers is $\bar{\sigma} = \sigma_0 \, \omega^{-1/m} \, \Gamma[(m+1)/m] + \sigma_u$ and the variance, var $\sigma = S^2$ (where S = the

standard deviation), is $S^2 = \text{var } \sigma = \sigma_0^2 \omega^{-2/m}\{\Gamma[(m+2)/m]-\Gamma^2[(m+1)/m]\}$, so that the coefficient of variation, CV, is $CV = S/\bar{\sigma} = \{\Gamma[(m+2)/m]-\Gamma^2[(m+1)/m]\}^{1/2}/\Gamma[(m+1)/m]$, if one assumes that $\sigma_u = 0$. The author plots CV against the shape parameter, m, and shows that is is closely approximated (to within 5% for m > 5) by 1.2/m. He finds that, if one assumes that $\sigma_u = 0$, the strength, σ_b, of a bundle of N continuous filaments, based on the area of the original N filaments, is $\sigma_b = \sigma_0 (m\omega)^{-1/m}e^{-1/m}$, so that the ratio of the bundle strength to the mean filament strength is $\sigma_b/\bar{\sigma} = (me)^{-1/m}/\Gamma[(m+1)/m]$. He plots $\sigma_b/\bar{\sigma}$ as a (monotonically increasing) function of m. In Section 2.4.3 (pp. 78-92), he develops a theory of fibrous composite tensile behavior and modes of fracture. His equations contain the composite volume, V, thus introducing a size effect into the strength of the composite, in accordance with experimental observations.

Friedman (1967) writes (Abstract, p. 1): "An analytical model of tensile failure of uniaxially oriented discontinuous fiber reinforced composites is developed for loading in the fiber direction. The model simulates the failure mechanism resulting from an accumulation of fiber fractures. Statistical distribution functions [Weibull] characterizing fiber strength and geometry are employed in order that the model may be utilized in the study of tensile failure of whisker reinforced composites. Specific forms of the distribution function are used to obtain quantitative results regarding the effects of the properties of the constituent materials on composite strength. Tensile tests performed on single-layer glass and boron composite specimens indicate that the strength of a discontinuous fiber composite can approach that of the corresponding continuous fiber composite, as predicted by the analytical model. An example is presented which applies the model to failure of alumina whisker-epoxy resin composites, making use of the available data on whisker properties."

Rosen (1967) discusses, in Section 3.3 (pp. 113-119), failure mechanics of fibrous composites in relation to their strength, assuming an underlying Weibull distribution. He plots the normalized composite strength, for $\ell/\ell_c = 1$ and $\ell/\ell_c =$

1,000 [ℓ = fiber length, ℓ_c = critical fiber length], as a function of the fiber coefficient of variation, which we have seen [Corten (1967)] is a function of the Weibull shape parameter, m. For $\ell/\ell_c = 1$, the normalized composite strength (ratio of strength of composite to strength of the constituent filaments) is less than 1, and decreases as the coefficient of variation increases; for $\ell/\ell_c = 1{,}000$, the normalized composite strength is greater than 1, and increases as the coefficient of variation increases. He also studies the effect of the compositing process by comparing composites with and without a matrix, that is, comparing the composite and the bundle. He plots the normalized bundle strength, σ_b^*/σ_c^* (σ_b^* = bundle strength, σ_c^* = composite strength) as a function of the normalized bundle length, ℓ/ℓ_c. His graph shows that σ_b^*/σ_c^* drops from 1.0 for $\ell/\ell_c = 1$ to less than 0.2 for $\ell/\ell_c = 1000$.

Tsai & Schulman (1968) write (Abstract, p. iii): "Experimental results of bundle tests of single-end glass, with and without epoxy resin, are compared with the predictions of statistical theories based on the normal and Weibull distributions. The effect of gage lengths varying from 1 to 20 inches is also investigated. Based on the results, bundle tests may be considered for the determination of the strength and the coefficient of variations of monofilaments, as well as the axial strength of unidirectional composites."

Cooper & Kelly (1969) write (Abstract, p. 90): "The mechanical properties of a fiber-reinforced material are governed in part by the transfer of stress between fiber and matrix. This transfer occurs across the interface between the components, and the properties of this interface, therefore, will affect the properties of the composite. In this paper we consider some of the composite properties which are affected by the mechanical strength of the interface, both in tension and in shear. The strength of the interface in tension governs the transverse strength of the composite. Properties dependent on this include the longitudinal compressive strength and the resistance of the material to the presence of notches. Theories of the transverse strength of a composite are examined and compared. The shear strength of

the interface affects primarily the load transfer length of the fiber-matrix system. It thus affects the strength of composites reinforced by discontinuous fibers, and the work of fracture under conditions of fiber pullout. It is also an important parameter in determining creep and fatigue properties and in determining notch resistance."

Sen, Bhattacharyya & Suh (1969) discuss the limiting behavior of certain sample functions applicable to the strength of bundles of filaments. This report is essentially the same as a journal article, published four years later [Sen et al. (1973)], which has been summarized in Section III.

Armenakas, Garg, Sciammerella & Svalbonas (1970) write (Abstract, p. iii): "This report contains a review of the statistical theories of composite strength for conditions of static (Section I-VI) and quasi-static loading (Sections VII-IX). Starting with the strength distribution of a fiber (Sect. II), we go through the Daniels bundle theory (Sect. III), the Gücer-Gurland-Rosen model of cumulative weakening of the composite (Sect. IV) and the Zweben model of crack propagation in the composite structure (Sect. V). In Section VI, the case when a few isolated fiber breaks are observed prior to failure is briefly considered. The material contained in Sections VII-IX is mostly due to Coleman. In Section VII, the stochastic process for the breakdown of a perfect fiber is presented. Its similarity to the Gücer-Gurland-Rosen model of composite strength is pointed out. The theory of breaking kinetics for a first order ensemble of fibers (no memory effects) is discussed in Section VIII for a fiber and for an ideal and a tight bundle. Finally, in Section IX, the generalized theory of breaking kinetics (including memory effects) is outlined. The relevant statistics and probability are reviewed in Appendices I and II. Throughout this review, special attention has been paid to applications to practical problems. The limitations and advantages of various theories are discussed in this light."

Lifshitz & Rotem (1970) write (Abstract, p. iii): "Theoretical and experi-

mental investigation of the longitudinal strength of unidirectional fiber reinforced composites has been performed. The fibers are treated as having variable strength which results in fiber fractures prior to composite failure. The matrix is treated as a linear viscoelastic material. A new model based on the cumulative weakening model [Rosen (1964)] has been developed and analyzed mathematically. The size effect is included in the analysis and the proposed failure mechanism agrees with observed failure geometry. The effect of length on strength is found to be very small. Unidirectional glass reinforced epoxy and reinforced polyester specimens with 60% volume fraction of fibers were tested in tension at three temperatures (25^{o} C, 78^{o}C and 130^{o}C) and a wide time range; from creep (more than a month) to impact tests. Delayed fracture was observed and interpreted according to the theory developed in [an earlier report by the authors], with emphasis on the difference between the time effects of the two matrix materials. The composite strength under impact conditions was found to be about three times higher than under static conditions. This increase in strength is due to the glass fibers, the strength of which is rate sensitive. Scattering of results is quite high ($\pm$ 10% in strength values, and $\pm$ 5% in moduli values) and this is attributed to the structures of the composite material. The fibers and matrix materials were also tested separately at similar conditions to the composite material and their mechanical properties are presented."

Waddoups, Eisenman & Kaminski (1971) present data from tensile tests on a graphite-epoxy laminate over a range of hole sizes. They tabulate the static strength (psi) for coupon specimens 1.0 inch wide and 9.0 inches long with no hole and with holes 0.015, 0.031 and 0.062 inch in diameter, also for coupon specimens 5.0 inches wide and 38.0 inches long with no hole and with holes 1.0, 2.5 and 3.0 inches in diameter. In the case of large holes, corrections for finite width are given. The authors plot the ratio of the strength σ_0 without a hole to the strength σ_c with a hole as a function of the hole radius r, the latter on a logarithmic scale. The

ratio σ_0/σ_c increases from near 1 for $r < 0.01$ inch to near 3 for $r > 1.0$ inch. The observed behavior is explained on the basis of the theory of fracture mechanics.

Argon (1972) presents some interesting recent developments in understanding of the strength of filamentary and laminar composites, especially the latter. He concentrates on the statistical aspects of the problem. On the size effect in tensile fracture of laminates, he writes (pp. 96-99): "The strength of the laminate of N parallel layers of reinforcement coupled by interfacial tractions characteristic of the behavior of the matrix...will, in general, depend on the number of parallel reinforcing elements in the laminate. ...The monotonic decrease in the laminate strength...is primarily a result of the increasingly many sites from which the fracture cascade can start as N increases. ...The decrease of composite strength with increasing number of reinforcing elements at nearly constant volume fraction has been established in a series of painstaking experiments on glass fiber wound pressure vessels of increasing size by Kies... . We...show that the strength of the laminate has a maximum at a certain N, and that therefore for small N it increases with N. ...We note further that as the flaw density exponent m [Weibull shape parameter] increases, the critical crack length...at the maximum laminate strength decreases monotonically." The author also considers the effect of element variability on composite strength. He writes (pp. 102-103): "It is interesting to note that the average strength of the individual elements increases steadily with increasing m or increasing perfection of fibers (decreasing variability). The strength of the composite, on the other hand, increases with increasing m or fiber perfection only for small m (fibers of large variability) and levels off sharply with increasing m when m is large (fibers of small variability). The effect shows up even more dramatically in the ratio of the composite strength to the individual element strength. For values of m less than about 10, the composite strength is greater than the element strength. Here the coupling of the matrix is beneficial, the breaks at the bad flaws at small stress are effectively bridged by strong regions of the adjacent ele-

ments. When m becomes significantly larger than 10, element variability is small and bad flaws are negligibly few. The stresses are much higher when isolated fractures occur in individual elements. Now, however, the stress concentration more than likely also overstresses the adjacent elements and fracture propagates. The composite now is weaker than the average of the individual elements since its strength is governed by the few elements with low strength. Stated in other words, composites with uniformly strong elements are more susceptible to stress concentrations than those with elements with more variable strength." The author also compares the strength of unbonded bundles with the strength of matrix-bonded laminates as a function of element variability. The bundle becomes more and more efficient with increasing m, but since the asymptotic strength of the composite is relatively insensitive to length while that of the bundle decreases with length, there is a certain length L_{min} (which depends on m) above which the composite is stronger than the corresponding bundle. The author plots L_{min}/δ (δ = ineffective length) as a function of m.

Kowal (1972) writes (English summary, p. 590): "The influence of a deterministic length of a tight bundle of cables [one which suffers failure (plastic deformation) when and only when all elements of the bundle undergo plastic deformation in the same cross-section of the bundle] is shown on its load-carrying capacity as well as safety. The problem is looked upon as a homogeneous stochastic process. The formulae are derived for the safety and the ultimate stretch load [tensile strength] as a function of the length. Some examples are given."

Rosen & Zweben (1972) write (Summary, p. 1): "An analytical model of the tensile strength of fiber composite materials has been developed. The analysis provides insight into the failure mechanics of these materials and defines criteria which serve as tools for preliminary design material selection and for material reliability assessment. The model incorporates both dispersed and propagation type failures and includes the influence of material heterogeneity. The important effects of localized

matrix damage and post-failure matrix shear stress transfer are included in the treatment. The model is used to evaluate the influence of key parameters on the failure of several commonly used fiber-matrix systems. Analyses of three possible failure modes have been developed. The modes are the fiber break propagation mode, the cumulative group fracture mode, and the weakest link mode. In the former, adjacent fibers fracture sequentially at positions which are within a short distance of a planar surface. Eventually the propagation becomes unstable and the plane becomes the fracture plane. In the cumulative group mode distributed fiber fractures increase in size and number until the damaged regions have weakened one cross-section so that it can no longer carry the applied load. In the weakest link mode, an initial fiber fracture causes an immediate propagation to failure. Application of the new model to composite material systems has indicated several results which require attention in the development of reliable structural composites. Prominent among these are the size effect and the influence of fiber strength variability."

Serensen & Strelyaev (1972) write (Abstract, p. 412 of translation): "The basis of the statistical estimation of the strength of structural elements made from fibrous composites is considered; the statistical characteristics of the strength of these materials are described and the limit states are formulated. Special attention is given to the resistance to debonding. The conditions of fracture in this class of materials in plane stress are subjected to a statistical analysis." They close with the following summary (p. 424 of translation): "Stochastic models of failure by breakage of the fibers, shearing of the polymer matrix and debonding make it possible to describe the observed data in the light of the weakest link hypothesis. These models can be extended to conditions of failure in plane stress. The statistical treatment of the mechanical properties of polymeric composites and the carrying capacity of elements made from such materials makes it possible to reflect, in quantitative terms, the dispersion of the principal types of resistance to failure in strength calculations with statistical allowance for the role of initial defects

in the products and the uniformity of their properties in relation to technological factors."

Niyogi (1973) writes (Conclusions, p. 1488): "Based on extensive tests, satisfactory expressions were derived and diagrams drawn for the ultimate bearing capacity of concrete with geometry as the principal variables [sic], i.e.: (1) Relative dimensions of loaded surface of specimen and bearing plate thereon; (2) relative height of specimen... . 1. A general expression for the bearing strength of concrete of different shapes of loaded areas from strip to square, representing the two and three-dimensional stress conditions is represented by Eq. 1 [$n = 0.42\ (a/a' + a/b' + 1) - 0.29\{(a/a'-a/b')^2 + 5.06\}^{1/2}$], embodying a/a', a/b' as the principal variables [$2a$ = width of loaded surface of specimen, $2a'$ = width of bearing plate, $2b'$ = breadth of bearing plate, n = nondimensional ratio of ultimate bearing stress to the concrete cube strength]. The cube-root formula considerably underestimates the ultimate bearing strength for square loading while over-estimating slightly under strip loading. 2. The relative height of specimen influences the bearing strength of concrete. In general, the strength decreases with increasing height of specimen, the effect being more for larger loaded areas. With simultaneous decrease in the height of block and loaded area, the rate of increase in the bearing strength is progressively diminished and a stage is reached when for $h/2a < 1$ and $R > 8$, the bearing strength decreases with decreasing specimen height [h = height of specimen, R = ratio of areas of loaded surface of specimen and bearing plate]. ..."

Argon (1974) states (Introduction, p. 154): "The goal of this chapter is to elucidate the fundamental basis of the variability and size effect of strength of brittle and ductile single-phase materials, and especially of laminates of stiff, brittle, reinforcing materials embedded in extensible matrices." In Section II (pp. 154-161), he deals with the extreme value statistics of brittle fracture. He writes (pp. 154-155): "In a brittle material with no capacity to level down high local stress concentrations by plastic flow, the worst flaw generating the highest

stress concentration governs the fracture stress of the part. The total behavior is set by a single worst flaw, hence the name extreme value statistics. The theory of extreme value statistics of brittle fracture has been developed first by Weibull (1939a,b) and has been applied widely to problems of variability and size effect in fracture. In what follows we will give a simple development of the theory to demonstrate that given a distribution of flaws of different severity in the volume or on the surface of a material, the larger the part the lower and less variable its strength and vice versa." In Section III (pp. 161-162), he discusses briefly the statistics of ductile fracture, but states that it is not possible to describe quantitatively the relatively minor size effect accompanying this type of fracture. In Section IV (pp. 162-164), he discusses the statistics of fatigue fracture. He writes (pp. 162-163): "The statistical nature of the fatigue process has been recognized very early, and many of the developments on extreme value statistics of brittle fracture had initially been intended for fatigue fracture (Weibull, 1939a,b). ...In the case of conventional fatigue, variability in the fatigue stress at constant life (or more properly variability of fatigue life under constant imposed stress amplitude) is most often the result of inhomogeneities and stress or strain concentrating discontinuities such as small scratches, machining grooves, etc. In a phenomenological sense it is often possible to describe the effects of these discontinuities on the fatigue strength by means of a mechanically equivalent population of surface flaw density in the same sense as for the brittle strength of plastically nondeformable materials. Wherever this representation is valid it is possible to develop similar relationships between the fatigue strength of a prototype structure and the strength of model laboratory specimens. An example of this has been given by McClintock (1955)... . We will adapt his results to the problem of scaling from model to prototype. ...A special application of the scaling of fatigue strength between parts of different size is in the explanation of the so-called fatigue strength reduction factor. It has been observed that notches of the same stress

concentration factor have a larger fatigue strength reducing effect in large parts in comparison to small parts. As discussed by McClintock (1955), this is a direct result of the size effect of extreme value statistics... ." In Section V (pp. 164-185), the author discusses the statistics of fracture in composites. His treatment of the size effect in tensile fracture of laminates and of the effect of element variability on laminate strength is almost identical with that in his earlier paper [Argon (1972)]. He also considers scaling laws of tensile strength of a laminate in relation to the size effect.

Chamis (1974) divides theoretical methods for predicting single-ply strengths in composites from constituent properties into strength-of-materials methods, statistical methods, and advanced methods. In discussing strength-of-materials methods for ply-longitudinal tensile strength, he suggests using filament-bundle breaking stress instead of the (higher) single-filament breaking stress in the rule-of-mixtures equations for composites with relatively high elongation-to-fracture fibers, poor fiber-matrix interfacial bond, and relatively high fiber-volume ratio. He plots filament-bundle and single-filament breaking stress versus length. In discussing statistical methods, he writes (p. 118): "The mathematical models devised for the statistical methods have their origin in the following two observations: (1) the fiber strength is length dependent..., and (2) fiber breaks occur continuously with increasing load until a critical number has been reached at some section along the ply length to cause fracture. The early work in the statistical theory development (Rosen, 1965) was patterned after the bundle theory proposed by Daniels (1945). See also the work of Tsai and Schulman (1968) and Armenakas et al. (1970). Application of the bundle theory to ply strength requires definition of the in-situ fiber ineffective length as the in-the-matrix embedded fiber length beyond which the fiber has reached its fully stressed condition. ...We follow the development presented by Lifshitz and Rotem (1970) in our subsequent discussion."

Gurland (1974), in his treatment of the strength of composite aggregates, first

discusses the statistical nature of brittle strength, and then elaborates the statistical treatment of several types of models. He writes (p. 83): "It is necessary first to postulate a distribution of strengths of the individual components of the aggregate, which may be grains, particles, or simply volume elements. Consider the brittle constituent subdivided into unit volume elements of uniform size and shape. The tensile strengths of the individual elements are assumed to vary according to a Weibull distribution function: $F(x) = 1 - \exp(-\alpha x^{\beta})$ where $F(x)$ is the fraction of volume elements breaking at or below the stress x, i.e., it is the cumulative distribution function of strength. The material constant α relates to a reference strength $\alpha^{-1/\beta}$, and the material constant β is a measure of the dispersion of strength. The coefficient of variation (ratio of standard deviation to average strength) is infinite if $\beta = 0$, and if $\beta = \infty$ the coefficient of variation is zero, i.e., there is no dispersion in the strength of the elements. The most probable strength (or statistical mode) of the volume elements is given by $x^* = \alpha^{-1/\beta}[1 - (1/\beta)]^{1/\beta}$. Once the strength distribution of the individual elements is defined, it becomes possible, in principle, to evaluate the strength distribution of aggregates of such elements." The author proceeds to do this for the following types of aggregates: (1) completely connected aggregates, using weakest-link statistics [Epstein (1948a)]; (2) fiber bundles, using bundle statistics [Coleman (1958)]; (3) brittle phase dispersed in a ductile matrix, using combined weakest-link and bundle statistics [Gücer & Gurland (1962)]; and (4) parallel continuous brittle fibers embedded in a ductile matrix, using combined weakest-link and bundle statistics [Rosen (1965)]. In discussing statistical models, he writes (p. 88): "The value of the materials constant β has considerable influence on the strength of brittle materials. In practice β ranges from 2 to 4 for brittle ceramics to about 20 to 30 for metals with slight ductility. ...Taking $\beta = 10$..., one finds...that the predicted strength of the ideally dispersed aggregate is about seven times greater than that of the fully connected aggregate, which compares with a factor of 7 to 10 based on the extrapolation of ex-

perimental trends. So far, the statistical considerations provide a more philosophical than practical approach to the design of composites. In theory, the two statistical models of 'weakest-link fracture' and 'combined weakest-link and bundle fracture' correspond to idealized cases of brittle and dispersed fracture of composites, whose strengths are controlled by the strength of the brittle constituent alone. Brittle fracture occurs by propagation of a crack from a single source. Dispersed fracture implies the progressive formation of non-propagating cracks, as occurs in the ductile fracture of composites, but without the direct contribution of the ductile matrix to the load-carrying capacity. One should note that the calculated strengths of all the statistical models would be the same if the strengths of all the volume elements were equal, i.e., if β tended to ∞. The models illustrate the crack-resisting function of the ductile matrix and the very great practical importance of the state of aggregation of the brittle phase. In practice, the fracture modes are not always so clearly defined. Composites may fail by a combination of mechanisms... ."

Mandell, McGarry, Kashihara & Bishop (1974) discuss fracture criteria and specimen size effects for fiber reinforced plastics. They write (p. 3): "Figures 9(a)-9(h) [not reproduced here] give the results of notched tension tests on laminates constructed of Style 799 woven roving/chopped mat, woven roving alone, and chopped mat alone, with various orientations of the woven roving. Specimens of 2, 4 and 6 inch width were tested with 2c/w ratios of 0.25 and 0.50 [c = notch length, w = specimen width]. ...The variation of K_Q [critical stress intensity factor] with specimen width is plotted for several materials in Figure 12 [not reproduced here] using the data given in Figure 9. As described by Owen and Bishop [(1973)] for similar laminates, the value of K_Q is found to increase with the width of the specimen." The authors also discuss laminate thickness effects. They write (pp. 3-4): "The fracture toughness of metals is known to be a strong function of thickness...; thin specimens under plane stress conditions yield more readily at the crack tip and dis-

play higher toughness, while for thick specimens yielding is constrained under plane strain conditions, resulting in lower toughness. ...Figures 14 and 15 [not reproduced here] indicate that the value of K_Q obtained for 181-style fabric and roving/mat reinforced composites is almost completely insensitive to thickness over a broad range. ...The results in Figures 14 and 15 suggest that the complications which thickness effects incur on fracture toughness testing of metals are not to be expected for this class of composites. Figure 16 [not reproduced here] indicates, however, that thickness complications can exist for some composites, particularly those with unidirectional plies. ..."

Ikeda & Fujisawa (1975) write (Abstract, p. 1): "Fatigue tests under fluctuating tension (R = 0.1) were carried out on AF-126-2 adhesive bonded single lap joints of 2024C-T3 Al-clad thin sheets to determine the effect of the sheet thickness, the lap length and the distance from the lap end to the grip end at room temperature, and a specimen group was also tested at low temperature (-55°C) to determine the effect of the environmental temperature. The experimental results are summarized as follows: 1) The effect of the sheet thickness and the lap length on the fatigue strength showed similar tendencies as the results already carried out by other researchers [Jacobs & Hartman (1954)]. 2) Increasing the distance from the lap end to the grip end resulted in a slight increase of the fatigue strength and the static strength, and a decrease of the scatter for endurance (N) of the specimens, and increases of the distance over a certain value had no effect on the 'S-N' curve and the static strength. 3) At low temperature (-55°C), the endurance (N) under higher fluctuating tension load was shorter than the 'N' at room temperature, however the reverse tendency was obtained under lower fluctuating tension load."

The compiler wishes to thank S. Leigh Phoenix for reading the manuscript and bringing to his attention seven papers published by Bernard D. Coleman between 1956 and 1958. These papers, which deal with the stochastic time to failure of fiber bundles, will not be summarized in detail here, but they are listed as References 694-700.

SECTION V

SUMMARY AND RECOMMENDATIONS

1. Introduction

The statistical theory of extreme values ("weakest-link" theory) plays an important role in studies of the size effect on material strength. Chaplin (1880, 1882) applied this theory to the effect of length on the tensile strength of metal bars of constant cross-sectional area. Weibull (1939 a,b) also used extreme-value theory to give the first reasonably satisfactory explanation of the volume effect on material strength. In so doing, he introduced the statistical distribution which now bears his name. This distribution is also known as the third asymptotic distribution of smallest values, and presumably would be known exclusively by that name except for the influence of Weibull. Nevertheless, engineering statisticians owe him a great debt of gratitude for pointing out its broad applicability to material strength and many other phenomena. Tucker (1941) questioned the application of the "weakest-link" theory to elements in parallel, and proposed instead the so-called "strength-summation" theory, which he found to work much better for beams differing in width (presumably the same is also true for those differing in depth). Daniels (1945) reached a similiar conclusion concerning the strength of bundles of threads or fibers, where failure of one thread does not necessarily result in failure of the bundle, but only in redistribution of the load to the remaining threads. Still further complications arise in the case of composites with fiber reinforcement, since a portion of the load is carried by the matrix in conjunction with the broken fibers. Though the "weakest-link" theory is directly applicable only on condition that the failure of the first element results in failure of the entire specimen or structure, statistical considerations enter into determination of the static strength, whether or not that condition is satisfied, as well as the fatigue life.

One should not make the mistake of assuming that statistical theory alone can

explain all the size-effect phenomena that have been observed. Davidenkov (1960) enumerated four competing theories: 1. The statistical theory, which explains the influence of dimensions by the presence of chance inhomogeneities in a solid, where the largest of these inhomogeneities determines the strength. 2. The energy theory, which states that the brittle failure depends on the elastic energy which is stored in the specimen-machine system and contributes to the specimen failure. 3. The technological theory, which ascribes strength differences to unequal treatment conditions, for instance, to the effect of treatment by cutting. 4. The theory proposed by Lavrov (1958), according to which -- at least in bending -- failure is caused by the initial microcrack on the stretched side of a beam, the dimensions and deformation of which do not depend on the beam dimensions. Lavrov's theory has never received wide acceptance, but the energy theory and the technological theory must be synthesized with the statistical theory in order to attain a comprehensive theory which will explain all the observed phenomena, as proposed, for example, by Volkov (1960).

A survey has been made of the literature on the size effect on material strength. The results, which were reported in detail in Sections I-IV, are summarized in this section, with subsections on the effects of length, cross-sectional area, and volume (or surface area), on the strength of fiber bundles and of composites, and on the size effect in fatigue. Some unsolved problems concerning the strength of composite materials and the reliability of composite structures are identified, and recommendations for their solutions are presented.

2. Effect of Length

Timoshenko (1953) has traced the history of the strength of materials from the times of the ancient Egyptians, Greeks and Romans. Knowledge of the size effect goes back at least as far as Leonardo da Vinci (ca. 1500), who made experimental studies of the dependence of the strength of iron wires on their length and observed that

long wires are weaker than short wires of the same diameter. Trautwine (1872) noted the effect of the length of a specimen of constant cross section on its strength, and Chaplin (1880, 1882) applied the "weakest-link" theory to the effect of length on the tensile strength of metal bars of constant cross-sectional area. He stated that, according to the laws of probability, if the probability is P that the tensile strength of a bar of given length and cross section exceeds a certain value, the probability that the strength of all of n bars of the same dimensions exceeds that value is P^n. Equivalently, the probability that the strength of the weakest of the bars (or the strength of a single bar of the same cross section but n times as long) exceeds the specified value is P^n. He worked out a quantitative relation between the average strength of a bar of given length and one n times as long, assuming normality. If the former is S, the latter is S-ap, where $p = .6745\sigma \doteq .6745s$ and $a = \Phi^{-1}[\sqrt[n]{.5}]/.6745$, p being the probable error, σ the population standard deviation, and s the usual estimate of σ from the sample, with $\Phi(x) = \int_{-\infty}^{x} \phi(x)\,dx$ and $\phi(x) = e^{-x^2/2}/\sqrt{2\pi}$. Chaplin (1882) tabulated values of a for n = 1(1)30(5) 60(10)100(50)200(100)1000. These results were summarized in a textbook by Slocum and Hancock (1906), but then in the course of time they were apparently forgotten, only to be rediscovered by Peirce (1926), who observed that if specimens of length ℓ break under normally distributed loads, then specimens of length $n\ell$, where n is an integer greater than one, will break under loads whose distribution is negatively skewed, the more so the larger the value of n. Peirce determined the decrease in the mode and the median of the tensile strength corresponding to an n-fold increase in length, and estimated the corresponding decrease in the mean by making use of the fact that the difference between the mean and the mode is approximately three times that between the mean and the median. He also pointed out the applicability to this problem of the theoretical results of Tippett (1925) on extreme values of samples from normal distributions. Epstein (1948 a) summarized the work of Peirce, and stated that, as far as he knew, Peirce was the first worker to realize the close

connection between the strength of a specimen and the distribution of smallest values; apparently neither Peirce nor Epstein was aware of the work of Chaplin. Lieblein (1954) called attention to the two papers by Chaplin, summarized the first of them, and interpreted it in modern symbols and terminology. In a footnote, he acknowledged indebtedness to Churchill Eisenhart for calling attention to an old engineering text [Slocum & Hancock (1906)] in which these articles were cited. Lévi (1949) concluded [see Peirce (1926)], since the distribution of the smallest of n values ($n \geq 2$) drawn at random from a normal distribution is not normally distributed, that if the strength of the links of a chain (or of short segments of a long bar) is normally distributed, that of the chain (or bar) is not, and proposed use of the lognormal distribution as an approximation to the distribution of the strength of bars or chains. Weibull (1949, 1951), in his treatment of the effect of length, assumed a Weibull distribution of strength, which is better than assuming a normal or a lognormal distribution. This is true because, if the strength of a specimen of length L_1 has a Weibull cdf (cumulative distribution function) $F_1 = 1-e^{-k(S-U)^m}$, where U is the load for which there is no probability of failure, the corresponding cdf for a specimen of length L_2 is given by $F_2 = 1-e^{-(L_2/L_1)\ k\ (S-U)^m}$, which is the cdf of a Weibull distribution with the same location and shape parameters, U and m, respectively; only the scale parameter has changed. During the past quarter century, many authors have given expository treatments of the effect of the length of a specimen on its strength. Among the few who have made significant new contributions to the theory are Bartenev & Sidorov (1966), who proposed a statistical theory for strength of glass fibers which takes account of different types of surface defects and their distribution along the length of the fiber. They assumed that the probability of not encountering a defect in the length ℓ is given by exp $[-(\lambda\ell)^n]$, where n = 1 for an exponential distribution of the defects along the length, $n > 1$ for any other distribution, and $n = \infty$ for defects occurring at a fixed interval $1/\lambda$ along the length. Defects of two kinds -- microcracks and micro-

breaks of the surface layer -- are encountered. A fiber has strength σ_A if neither kind of defect is present, σ_B if a defect of the first type is encountered in it but not one of the second type, and σ_C if a defect of the second type is encountered. Bartenev & Sidorov showed that the mean strength of a fiber of length ℓ is given by $\bar{\sigma}_\ell = (\sigma_A - \sigma_B) \exp [-(\lambda_1 \ell)^{n_1} - (\lambda_2 \ell)^{n_2}] + (\sigma_B - \sigma_C) \exp [-(\lambda_2 \ell)^{n_2}] + \sigma_C$, which can easily be generalized to the case of m different strengths (m - 1 kinds of defects).

3. Effect of Cross-Section

Timoshenko (1953) has pointed out that Leonardo da Vinci (ca. 1500) made experimental studies of the dependence of the strength of beams on their width and of columns on their cross section, also that Galileo Galilei (1638) enunciated and elaborated upon the similarity principle, which states that geometrically similar structures become weaker and weaker as their dimensions increase. Karmarsch (1859) represented the tensile strength (per unit cross-sectional area) of metal wires by an expression of the form $F = A + B/d$, where d is the diameter and A and B are positive constants. Griffith (1920, 1924) reported theoretical and experimental results on the strength of glass and other solids. His theory is based partly on the "weakest-link" or "largest-flaw" concept, but he found this concept inadequate to explain the extremely high strength of freshly-drawn fine glass threads which he observed. To explain this phenomenon, he postulated that fracture results from extension of pre-existing microscopic or submicroscopic cracks (which have come to be called "Griffith cracks"), with stability controlled by a balance between released elastic-strain energy and absorbed surface energy (solid-state surface tension). His experimental values of the strength of glass fibers are approximated by $F = 22{,}400\,[(4.4 + d)/(0.06 + d)]$, where F is in lbs/in^2 and d is in thousandths of an inch. He noted that, within the range of diameters available to Karmarsch (1859), this expression differs little from $F = 22{,}400 + 98{,}600/d$, which is in the form given by Karmarsch for metal wires. Similar results have been given by Reinkober

(1931) and Alexandrov and Žurkov (1933), who attributed the effect of cross section on strength to the fact that the greater the cross section the greater the probability of a dangerous surface crack. Slightly different results were obtained by Kontorova (1940) and by Kontorova & Frenkel (1941), who developed a statistical theory based on the length of the most dangerous crack -- the larger the specimen the greater the probability that it contains a dangerous inhomogeneity. Tucker (1941) questioned the application of the "weakest-link" theory to elements in parallel, and proposed instead the "strength-summation" theory, which he used in analyzing the strength of beams differing in width, with excellent results. Gerald Pickett, in his contribution to the discussion of Tucker's paper, stated that he saw no reason why the "strength-summation" theory cannot also be used for beams of varying depth. Tucker (1945) asserted that the "weakest-link" theory predicts the effects of an increase in the depth of specimens subjected to flexure, while the "strength-summation" theory appears to predict reasonably well the effects of an increase in the cross-sectional area of specimens subjected to tension or compression and of an increase in width of a flexural specimen. Weibull (1949) asserted that the "weakest-link" theory applies to two identical metal bars coupled side by side, thus leaving the length unchanged but doubling the cross section, because if one of the bars fails, the load on the other will be doubled and it will almost certainly fail also. However, if the two bars are allowed to melt together to form a single specimen with a doubled cross section, the validity of the "weakest-link" theory is not at all self-evident; it depends on the manner in which the fracture is spread over the area and also on the homogeneity of the material, so it is not a trivial problem to determine the effect of changes in the cross-sectional area. Meyersberg (1952) and Weibull (1961) have pointed out that the effect of the cross section of metal bars or wires on their strength is not entirely statistical in nature, but depends also on differences in the cooling rates and in the effects of heat treatments for

large and for small specimens, and have made attempts to separate these other effects from the statistical size effect. Gücer & Gurland (1962) have proposed a "dispersed fracture" model which is based on the following assumptions: (1) Cylindrical bodies, loaded in simple tension in a direction parallel to the axis of the cylinders, consist of identical volume elements subject to a homogeneous, uniaxial stress. (2) The fracture strengths (fracture loads per unit area) of the elements are independent of each other and all elements fracture independently, i.e. the fracture does not propagate from one element to the next. (3) A body has failed if all volume elements intersecting a cross-sectional plane are broken. Under these assumptions, their analysis shows that, for a Weibull distribution of the strengths of the individual volume elements, the strength of a specimen of constant length increases with cross-sectional area, the increase being most pronounced for small values of the Weibull shape parameter. Experimental results, however, regularly show that the fracture strength decreases as the cross-sectional area increases, as predicted by the "weakest-link" theory, which leads one to believe that very few, if any, real materials satisfy the assumptions underlying the "dispersed fracture" model, especially as regards independence (assumption 2).

4. Effect of Volume

Weibull (1939a) presented a statistical ("weakest-link") theory of the effect of volume on the strength of materials. According to his theory, the probability of rupture (S) at any given distribution of stress (σ) over a volume (V) is determined by the equation $\log (1-S) = -\int_V n(\sigma)\, dV$, where $n(\sigma)$ is a function characteristic of each particular material. For statistically homogeneous materials, the material function may be expressed by the formula $n(\sigma) = [(\sigma-\sigma_u)/\sigma_0]^m$, where σ_u, σ_0 and m are constants characteristic of the material -- location, scale and shape parameter, respectively, of the Weibull distribution, whose cdf is given by $F(\sigma) = 1-\exp\{-[(\sigma - \sigma_u)/\sigma_0]^m\}$. The scale (threshold) parameter, σ_u, below which there is

no probability of failure, is often set equal to 0. Weibull (1939b) reported the results of experimental investigations intended to verify his theory. Somewhat different results were obtained by the Russian writers Afanas'ev (1940, 1941), Kontorova (1940), and Kontorova & Frenkel (1941). Afanas'ev assumed a distribution of strength having cdf $F(z) = z^k/(A^k + z^k)$, where A is a positive constant and $z \geq 0$, and Kontorova & Frenkel assumed a normal distribution. The latter authors also took account of the number of inhomogeneities and the size of the largest one. Fowler (1945) gave a statistical explanation of the size (volume) effect based on the Weibull theory. Davidenkov, Shevandin & Wittmann (1947) reviewed the work of Weibull and that of Kontorova & Frenkel and pointed out that both are based on the assumption that brittle fracture is determined by the local stress at the most dangerous defect; the larger the piece, the lower the strength of its weakest element and therefore the lower the strength of the piece itself. They gave the theory of Kontorova & Frenkel the advantage over that of Weibull in the sense of stricter physical foundation, but found the latter more convenient for practical purposes, and gave qualitative experimental verification of it for static tension and bending of cylindrical specimens of brittle phosphorous steel in liquid air. Epstein (1948 b) summarized the work of the above authors and related work on the statistical theory of extreme values, which is the basis of the "weakest-link" theory. Chechulin (1954) criticized the theory of Kontorova & Frenkel (1941) for its use of a normal distribution of local strength, which admits negative values for sufficiently large volumes and infinitely large values for vanishingly small volumes, and that of Weibull (1939 a) for its lack of a rigorous physical basis, while admitting that it does not give bad agreement with experimental results. He proposed a new theory, based on the Pearson Type III distribution, for which he claimed greater rigor than the previous theories, and showed that Weibull's theory is a special case of his own when the number of defects in the body is large. Bartenev & Tsepkov (1958, 1960) pointed out that the bending strength of glass and other brittle materials depends primarily on the surface area rather

than the volume, since failure starts at the surface, and suggested that, when this is true, the working volume V should be replaced by the working surface S in Weibull's theory. Robinson (1965) summarized the Weibull theory, and pointed out that the strength in uniform tension of specimens differing in size from the test specimen may be predicted if m and σ_0 remain unchanged (normally a valid assumption, since they are constants of the material), in which case the survival probabilities at the mean values are the same, so that (if σ_u is taken to be zero) we get $\exp[-(\sigma_1/\sigma_0)^m V_1] = \exp[-(\sigma_2/\sigma_0)^m V_2]$, and therefore $\sigma_1/\sigma_2 = (V_2/V_1)^m$. Thus the larger specimen is weaker and this size effect increases as m increases. Batdorf & Crose (1974) [see also Batdorf (1974, 1975)] introduced a new physically based statistical theory of fracture for isotropic brittle materials, which (they claim) eliminates in large measure the long-standing rift between "weakest-link" theories and the main body of fracture mechanics. Their theory allows extension of the theory of strength for a stressed volume under uniaxial tension to the cases of biaxial and triaxial tension, and gives simple expressions for the probabilities of failure under equibiaxial and equitriaxial tension, but is not applicable when the stress is predominantly compressive. Jones (1975) summarized the volume effect on strength in relation to the distribution of the smallest value as a function of the sample size, whose general features he summarized for rectangular, Cauchy, Laplace, Gauss (normal) and Weibull distributions. Vardar & Finnie (1975) pointed out that the Weibull multiaxial treatment of brittle strength contains limitations which are not present in the more familiar uniaxial formulation, and asserted that if these limitations are satisfied, it is possible to use tension or bending data to predict multiaxial behavior when at least one principal stress is tensile.

5. Strength of Fiber Bundles

Peirce (1926) considered the strength of cotton yarns consisting of parallel threads and Daniels (1945) studied the statistical theory of the strength of bundles

of threads. If a load S is applied to a bundle of n threads or fibers, it is assumed that each fiber carries an equal part of the load, S/n for each fiber. If one fiber, having strength less than S/n, breaks, the load is redistributed to the remaining fibers, S/(n-1) for each. This may cause more fibers to break. After r fibers have broken, the load on each of the remaining fibers is S/(n-r). The process of breaking and load redistribution continues either until all n fibers have broken or until k fibers, each having strength greater than S/k, remain. Daniels developed the probability distribution of the strength of bundles of n threads, and found that asymptotically (as $n \to \infty$), this distribution tends to normality. Winkler (1954) approximated the strength of bundles of fibers by taking the total length as the product of the number of fibers in the bundles and the test length, and applying "weakest-link" theory. He reported close agreement of the resulting theoretical strength with experimental results. Coleman (1958) applied the theory of Daniels to infinite bundles composed of fibers which obey the Weibull distribution. He found that, in general, the tensile strength (per unit initial cross-sectional area) of a large bundle has the same order of magnitude as, but is less than, the mean strength of the component fibers, and decreases monotonically with increasing dispersion of the fiber strength. Sedrakyan (1958) used elementary combinational probability methods to determine the probability, if a cable is braided of n wires of equal length and cross-section whose strength distribution function is given, that $c \leq n$ wires will break under the influence of a given external load. Corten (1967) plotted normalized bundle strength as a function of the Weibull shape parameter, m (a material constant), and Rosen (1967) plotted normalized bundle strength as a function of normalized bundle length. Sen, Bhattacharyya & Suh (1969, 1973) pointed out that, if a bundle of fibers breaks under a load L, then the inequalities $nX_{n,1} \leq L$, $(n-1)X_{n,2} \leq L$, $\ldots$, $X_{n,n} \leq L$, where $X_{n,1} \leq X_{n,2} \leq \cdots \leq X_{n,n}$ are the ordered values of the strengths of the individual fibers, are simultaneously satisfied and that, consequently, the bundle strength can be represented as $B_n = \max\{nX_{n,1}, (n-1)X_{n,2}, \ldots, X_{n,n}\}$.

They observed that $n^{-1}B_n$ can be written as $\sup_{x \geq 0} x[1-S_n(x)]$ where $S_n(x)$ is the empirical cdf for the strength of individual fibers, and proceeded to generalize the results of Daniels on the limiting distribution of bundle strength. Suh, Bhattacharyya & Grandage (1970) developed, by probabilistic argument, small-sample and large-sample properties, as well as certain moment properties, of the bundle strength B_n. Hoshiya (1972) approximated, by Monte Carlo methods, the probability that exactly i of m parallel cables with a single load S will fail, where the component strengths are mutually independent and identically distributed with lognormal distribution and share equally the external load S, also lognormally distributed. He considered both the case of brittle fracture, where the failed components become inactive, and the case of ductile failure, where they still carry loads equal to their yield strength. He tabulated the results of 2,000 trials each for ductile and brittle systems with m = 2(1)14. Phoenix & Taylor (1973) determined the asymptotic distribution of tensile strength of a bundle of parallel fibers in terms of the statistical characteristics of the individual fibers, extending the results of Daniels (1945), Sen et al. (1969, 1973) and Suh et al. (1970) to cover the case of bundles composed of a mixture, in fixed proportions, of several fiber types. They showed that such bundles can be weaker than similar bundles of any single component. Sen (1973 a,b) developed the theory of sequential confidence procedures and sequential tests for the strength of bundles of fibers. Kreider (1974) converted the plot of normalized bundle strength as a function of the Weibull shape parameter, m, given by Corten (1967) to a plot of normalized bundle strength as a function of the coefficient of variation of the underlying Weibull distribution (itself a function of m). Phoenix (1975) gave asymptotic results for the distribution of tensile strength of a special class of classic fiber bundles composed of several sub-bundles wherein the random variables associated with the loading and failure of any two fibers in the same sub-bundle are allowed to be probabilistically dependent. He modified Daniels' model to include both random fiber and random sub-bundle slack, and found that random slack

results in a loss in asymptotic bundle mean strength and a change in asymptotic variance, both of which are approximately proportional to the random fiber slack variance.

6. Strength of Composites

Peirce (1926) mentioned composite materials in connection with his study of the strength of fiber bundles, but serious study of composites did not begin until about 20 years ago. Parratt (1960) noted that plastics reinforced with glass and other fibers had gained acceptance as structural materials. He attempted to connect the mechanical behavior of individual glass fibers and the strength of glass fiber moldings, hoping to gain a more detailed mechanical picture on which to base work on glass fibers in resin and other fiber reinforced materials. He concluded that the reinforcing strength obtained from a fiber of a given diameter is the strength corresponding to a certain "critical length" of fiber at which the strength of the fiber exceeds the maximum stress it can transmit. The thicker the fiber, the greater is the "critical length", which means that thicker fibers make a weaker reinforcement. Bell (1961) reported the results of a study to determine the effect of glass fiber geometry (defined to include the length and cross-sectional area of the fiber as well as its orientation in the matrix) on composite material strength. A preliminary analysis indicated that, for current fiber diameters (0.0004 in.) and matrix shear strengths (1000 psi), a minimum fiber length of approximately one-quarter inch is required to develop full fiber strength. Rosen (1964, 1965) presented a theoretical and experimental treatment of the failure of a composite, consisting of a matrix stiffened by uniaxially oriented fibers, when subjected to a uniaxial tensile load parallel to the fiber direction. He treated fibers as having a statistical distribution of flaws or imperfections, which result in fiber failure at various stress levels. Composite failure occurs when the remaining unbroken fibers, at the weakest cross section, are unable to resist the applied load. Thus, composite

failure results from tensile fracture of the fibers. This model contrasts with that of Parratt (1960), according to which failure occurs when the accumulation of fiber fractures, resulting from increasing load, shortens the fiber lengths to the point that further increases in load cannot be transmitted to the fibers because the maximum matrix shear stress is exceeded, so that composite failure results when a shear failure of the matrix or the interface occurs. Rosen evaluated the composite strength as a function of the statistical strength characteristics of the fiber population and of the significant parameters defining composite geometry. Among these is the "ineffective length", δ, which is the fiber length, measured from an internal fiber break, over which the stress σ is less than some given fraction ϕ (= 0.9, say) of the undisturbed fiber stress σ_{f0}. Rosen's model consists of a chain of bundles of fiber links, each of length δ. After the numerical value of δ has been calculated from an equation given by Rosen and the distribution of fiber stress has been determined experimentally, the distribution of bundle strength can be found from the theory of Daniels; then the composite can be treated as a chain of bundles, and "weakest-link" theory can be applied to obtain the composite strength. Schuerch (1964) studied the effect of diameter upon elastic properties in thin fibers, motivated by the remarkable mechanical properties of filament-wound structures made from endless, thin glass fibers bonded with various organic resins. He pointed out that these properties result, in part, from the size effect (increase in strength observed in thin glass fibers as compared with the bulk strength of glass), combined with an effective crack barrier function of the bonding material. Riley & Reddaway (1968) derived a theory for predicting the tensile strengths and failure mechanisms of fiber composites where the fibers are aligned in the direction of tensile loads and are flawed to some extent. They pointed out that several modes of failure can occur, depending on the length of the fibers and the degree to which they are flawed; the presence of defects leads to size effects, with the strength of the fibers (and hence of the composite) decreasing as either the length or the diameter of the fibers

increases. Wadsworth & Spilling (1968) presented the results of a theoretical and experimental study of the transfer of loads from broken fibers in composite materials (high-modulus carbon fibers placed in a resin matrix on an aluminum backing). Zweben (1968) investigated two modes of composite tensile failure, one involving composites containing a planar array of parallel fibers which exhibit a large number of isolated fiber breaks before failure, and the other involving monolayer and multilayer unidirectional composites that do not display many isolated breaks before failure. He reported that the failure loads predicted by these analyses were significantly closer to experimental data than the predictions of other theories. He established failure criteria for each mode, and reported that the analyses, based on the "weakest-link" theory and making use of the Weibull distribution, predict, for both cumulative and non-cumulative failure modes, that composite strength decreases with the size of the body, whereas the theory of Rosen (1964) predicts that composite strength is essentially independent of size. McKee & Sines (1969) presented a statistical model for the tensile fracture of parallel-fiber composites based on a stress criterion for crack propagation. The stresses in fibers surrounding a crack nucleus and the number and size of such nuclei are evaluated, and failure is predicted when, statistically, a new break is expected at any crack nucleus. The predictions of this model are in good agreement with measured tensile strength, the variation of strength with size, the variability of tensile fracture strength, and the observed mechanism of fracture of fiberglass composites. The model predicts that reinforced fiber structures will exhibit a regular decrease in fracture stress as volume is increased and that the variability of tensile strength of large specimens will be only slightly less than that of the individual fibers, in agreement with experimental results. Zweben & Rosen (1970) compared the "weakest-link" model of Weibull (1939 a), according to which failure occurs by the propagation of a crack through the material, and the "dispersed fracture" model of Gücer & Gurland (1962), according to which failure occurs by the inability of the overall cross-

section to resist the applied load. They pointed out that these two criteria provide lower and upper bounds on expected material strength, but that the bounds may be too far apart for any practical use, especially for large specimens, since the former predicts a strong size effect and the latter is relatively insensitive to size. Zweben & Rosen suggested a new model, based on the multiple break criterion, according to which fracture propagation results from the first multiple break for continuous fiber composites or the first triple break for whisker-reinforced composites, and applied the results to composite materials, obtaining good agreement with available experimental data. They plotted a graph of the size effect predicted by the multiple break criterion. Anthony (1972) pointed out that, even though statistical models that essentially assume series behavior have given good fits to experimental data for composite materials, one cannot help being somewhat apprehensive when small specimen data are used in designing large assemblies. Fortunately, he wrote, scaling based on a series statistical model tends to be conservative if the real situation involves parallel behavior; however, unduly large conservatisms may result in serious penalties. Cruse, Konish & Waszczak (1972) attempted to adapt metals technology (in particular, linear elastic fracture mechanics) for use with composite materials. They pointed out that recent work has shown that scaling of failure loads for geometrically similar coupons, which can be done for metals, is not always possible for composite materials. They stated that the development of sufficient experimental data to correct for size effects should improve prediction capabilities, though conservative strength estimates can be and have been made without such corrections. They identified three characteristic material dimensions that may be important in fracture testing of composites: Ply thickness, fiber diameter, and size of "initial damage" zone. Eisenman, Kaminski, Reed & Wilkins (1972,1973) advocated development of a new set of design rules for composite structures, taking account of scaling and complexity, which are not addressed by the conventional approach developed over the past forty years for metallic structures. Lifshitz & Rotem (1972) pointed out that,

while some carbon/epoxy composites fail by fracture at a single cross section as assumed by Rosen (1964), experimental results with glass reinforced plastics (e.g., epoxy or polyester) show very complicated fracture surfaces. They proposed a new failure model that agrees with the observed failure geometry. The results of their analysis, based on this model, show that a small decrease in strength is expected when the size is increased by a few orders of magnitude. Parratt (1972) summarized the accumulated knowledge concerning various aspects, including the size effect, of the strength of fibers, fiber bundles, and fiber-reinforced composites. Rosen & Dow (1972) presented a similar summation, and proposed a new model for the strength of composites reinforced with brittle fibers. This model differs from that of Rosen (1964) mainly because it uses a different definition of "ineffective length" proposed by Friedman (1967) and includes the defined ineffective region on both sides of the break to establish the link length. Waddoups (1972) discussed the implications of various models for the strength of composites and their correlation with experimental data. Armenàkas & Sciammerella (1973) discussed the two best-known statistical models employed in predicting the strength of fiber-reinforced composite materials, the "cumulative weakening model" [Rosen (1964)] and the "crack-propagation model" [Zweben (1968)]. They compared the results given by the two theories with available experimental results, and concluded that there is a great need for an extensive, systematic experimental program to evaluate those theories. Owen & Bishop (1973) called attention to the possibility that glass reinforced plastics structural components may fail in a manner similiar to brittle fracture in metals; if the structural element is large enough the loaded element may store sufficient elastic energy to propagate a fracture from a defect, accidental local damage, or a design detail. As evidence, they cited two failures of large structures where the failures took the form of narrow cracks, with the material almost entirely damage free on either side of the fracture surface, in contrast with the progressive damage (debonding and resin cracking), extending through the stressed region, observed in testing small laboratory

specimens. Wright & Iannuzzi (1973) measured the strength of individual carbon fibers and used the results to calculate the lower bound of strength expected from carbon-fiber reinforced epoxy matrix materials. The tensile strength of the composite was found to be greater than one would expect by assuming that the fibers functioned as a simple load carrying bundle; in effect, a synergistic strengthening effect was observed when the matrix surrounded the fibers. The following experimental results were also observed: (1) The shorter fibers exhibited the larger mean strengths, as predicted by theory; (2) The strengths of bundles of fibers also increased as the fiber length decreased; and (3) The bundle strengths were always less than the mean strength of the individual fibers. Argon & Bailey (1974) showed that if the surface flaw distribution in the brittle reinforcement layers of a laminate is a Weibull distribution with shape parameter, m, less than 10, the average strength of the laminate is greater than that of the isolated element (for a given laboratory test length), while for $m > 10$, the inequality is reversed; in nearly all cases of reinforcement using glass, boron, or carbon fibers, $m < 10$ and the laminate is stronger than the elements. Hahn & Tsai (1974) gave a set of generalized graphs, for all practical laminates of a given composite, for determining changes in stiffness and strength resulting from varying the ply orientation and total laminate thickness. This knowledge is required in designing composite laminates.

7. Size Effect in Fatigue

Among the earliest authors to discuss the size effect in fatigue were Peterson (1930), Peterson & Wahl (1936), Weibull (1939 a,b), and Afanas'ev (1940, 1941). Freudenthal (1946) reported on a study of the statistical aspect of the fatigue of materials. He pointed out that the size effect in the crack initiation stage (the probability of rupture under a definite load amplitude increases with increasing number of bonds, which is proportional to specimen size) is opposed by a size effect in the crack propagation stage (at equal rate of propagation the small cross section

will be destroyed more rapidly); the resulting size effect depends on the relative magnitudes of the two opposing effects. Aphanasiev [Afanas'ev] (1948) noted that an increase in the dimensions (diameter) of test specimens lowers their fatigue limit, and that this phenomenon becomes more apparent when stress concentrations are present, as in notched specimens. He developed a theory to explain the greater size effect on fatigue strength for notched specimens as compared with unnotched ones. Epstein (1948 a) pointed out a fallacy in applying "weakest-link" theory to the study of fatigue of materials [see Freudenthal (1946)] -- the specimen is changing in time and its distribution of strengths due to flaws is also changing, so that any essentially static approach which uses the "weakest-link" concept without modification leaves out certain basic features of the process. Stulen (1951) summarized the elementary theory of extreme values as related to the size effect on the endurance limit. He pointed out that, for specimens in which the stress is not uniform, allowance must be made for the fact that different volumes are stressed at different levels. He estimated the effective volume of a specimen as that portion of the volume where the oscillating stress is within 15 per cent of the maximum stress on the specimen. Afanas'ev (1953) explained the statistical theory of fatigue stability of metals previously published by him (not the "weakest-link" theory, but one based on stresses in the crystalline structure of the metal) and related it to the size effect. Freudenthal & Gumbel (1953) [see also Derman, Kwo & Gumbel (1954)] presented a statistical interpretation of fatigue tests, including the size effect, based on the distribution of largest cracks (extreme-value theory). Weibull (1961) discussed, in some detail, the effect of size on fatigue strength -- a complex problem which depends upon both structural (metallurgical) changes in the material and statistical considerations. The metallurgical size-effect can result from differences in cooling rate and/or differences in the effects of heat treatment and fabrication processes for large and for small specimens. The "weakest-link" concept, initially intended as an explanation of size effects in brittle materials [see Weibull (1939 a,b)],

is directly applicable to the fatigue strength at an arbitrarily preassigned life N (cycles), but not to the fatigue life. When crack initiation occurs mainly on the surface, the surface area, and not the volume, is the appropriate "size" to be taken into consideration. Weibull stated that it is safe to conclude that the size effect in fatigue exists, but is not easy to establish its magnitude, because many irrelevant factors mask the result and, even more significantly, the laws of size effect are quite different for the pre-crack and the post-crack stages of the fatigue process. Grover (1966) claimed that, while statistical considerations are important in fatigue studies, the statistical approach cannot provide a wholly satisfactory theory of fatigue. Probability considerations imply lower strengths for larger specimens, but they do not directly imply the magnitude of the size effect or the factors which govern the magnitude -- fatigue strength might be low at a thickness of a few hundred atomic distances, with no further decrease of engineering significance as thickness increases over practical ranges. This information is not likely to come, in any unique sense, from the statistical approach. Moreover, material factors, such as differences in surface finish and in residual stress, play an important part in the size effect. It was shown by de Kazinczy (1969) that the ratio between the endurance limits of two flawed specimens of cast steel with different stressed volumes (w) can be expressed by: $\sigma_1/\sigma_2 = (w_2/w_1)^{1/m + n/p}$ where 1/m is the exponent (reciprocal of the Weibull shape parameter for the material) in the absence of defects, n is obtained from the size distribution of defects, and p is related to their notch effect. Buch (1972) surveyed the literature (32 references) on test results and analyses of the effect of the size of specimen (unnotched) on the fatigue strength in the case of fatigue tests under reversed bending, reversed twisting, and reversed tension - compression stresses; the maximum value of the plain geometrical effect of size, effect of the surface hardening, the surface roughness and the statistical material defects; and estimation of the fatigue strength of structural components. Harris & Lee (1974) examined existing data and data from controlled experi-

ments to determine the effect of fiber diameter on the fatigue strength of metal matrix composites. Several Russian authors have studied the size effect on fatigue strength of steel in sea water and other corrosive media; corrosion affects small-diameter specimens more severely than large-diameter ones, so the size effect may be opposite in direction to that observed in air.

8. Unsolved Problems and Recommendations for Solution

The statistical theory of the size effect on both static and fatigue strength has been well developed, but other theories (especially the energy theory and the technological theory) must be synthesized with the statistical theory in order to attain a comprehensive theory which will explain more of the existing phenomena, even in the case of static strength. In the case of fatigue strength, account must also be taken of the theory of fracture mechanics. Experimental results on the size effect on both static and fatigue strength of the older structural materials (such as metals, wood, and concrete) abound in the literature, but such results for the newer fiber-reinforced composite materials are inadequate. In connection with the current trend toward greater use of composite materials in large aircraft structural members, the following efforts are required: (1) Synthesis of the statistical theory of extreme values with engineering theories of fracture and fatigue to predict the size effect on the static strength and fatigue life of composites and other structural materials, and the reliability of large composite structural members; (2) Experimental testing of small and large composite specimens to verify the results given by the unified theory.

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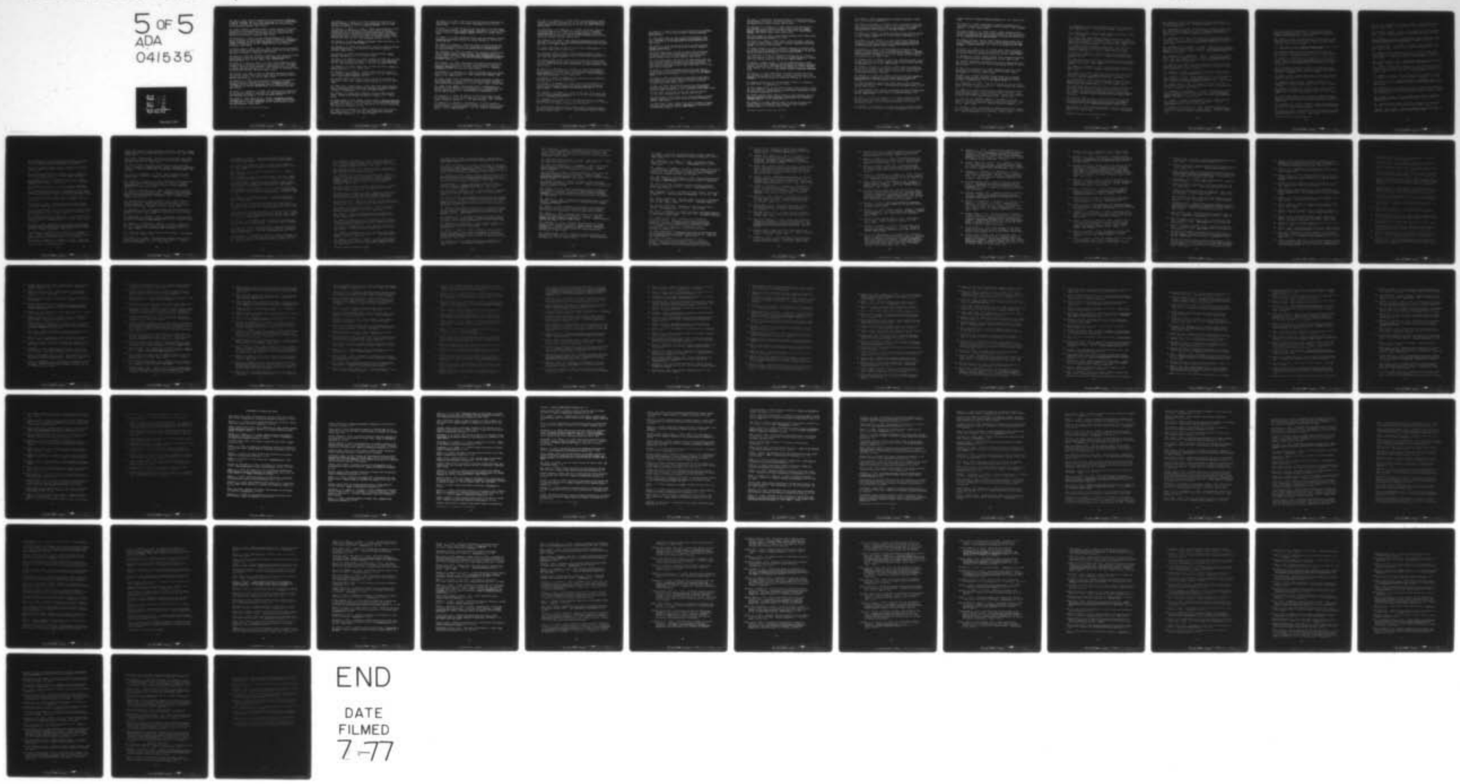

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